Soil mechanics1 : course

Abstract

This Pedagogical Book entitled “Soil Mechanics 1: Course” is proposed for first-year Master's degree students specializing in Engineering Geology and Geotechnics. its main objective is to provide the theoretical and practical tools needed to calculate stress distribution, estimate settlement, and analyze soil consolidation This book was divided into six chapters where each of them combines theory with practical examples to help students connect fundamental principles with their application in real-world engineering problems.

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Pedagogical Book

Soil mechanics1

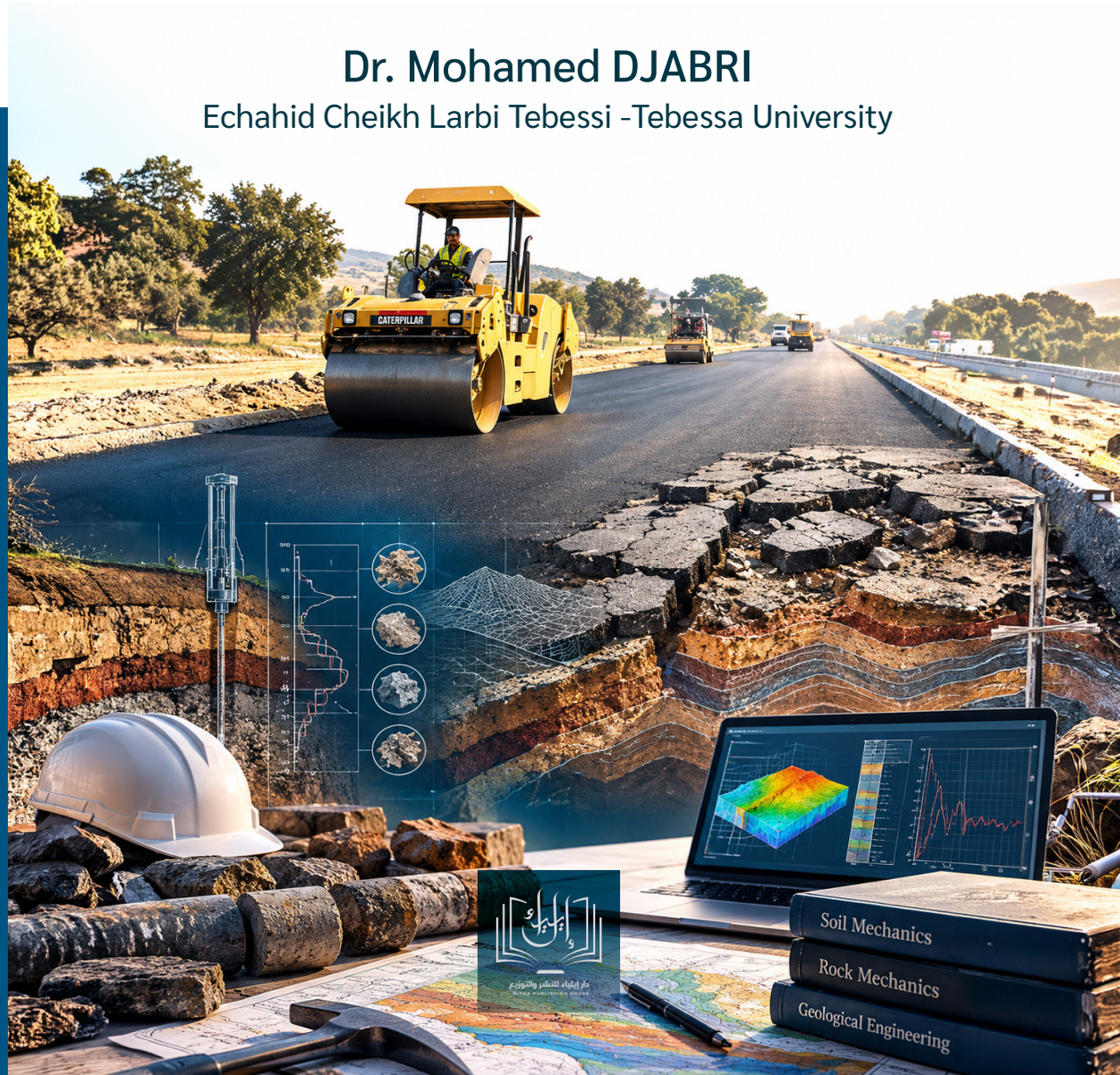
course

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Engineering Geology and Geotechnics

Dr. Mohamed DJABRI
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Soil mechanics1 : course



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Soil Mechanics 1: Course

Author(s):

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دار إيليا للنشر والتوزيع

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PREFACE

Soil mechanics, the mathematical discipline of geotechnics, focuses on the analysis, design, and construction of structures that interact with the soil. Some of the basic theories of soil mechanics include soil classification, effective stress, settlement, and soil permeability. Subsequently, soil mechanics has developed considerably thanks to research efforts resulting in the publication of numerous books. Successive international conferences and seminars have been organized to discuss many vital issues in this field. Today, geotechnical engineers aim to introduce new laws and concepts that describe soil behavior and address the stability of structures interacting with the soil.

This Pedagogical Book, entitled “Soil Mechanics 1: Course” is proposed for first-year Master's degree students specializing in Engineering Geology and Geotechnics. Its main objective is to provide the theoretical and practical tools needed to calculate stress distribution, estimate settlement, and analyze soil consolidation. The structure is clear, and the illustrations provided are helpful, serving both to capture the student's attention and to reinforce acquired knowledge.

This course material is prepared in accordance with the requirements of the Master's training program established by the National Pedagogical Committee of the Earth and Universe Sciences field "Ministerial Order of Compliance No. 318 of 23/03/2023".

The book is organized into six chapters. Each chapter combines theory with practical examples to help students connect fundamental principles with their application in real-world engineering problems.

The first chapter is devoted to the study of physical properties and the most commonly used methods for soil classification.

Earthworks are important operations in the construction of all types of structures; they are linked to compaction soil, which is covered in the second chapter. Although water is an important component of soil composition, the third chapter is devoted to the study of water flow through soils, such as soil permeability and seepage.

Chapter four explains the principle of effective and total stresses in soil caused by the weight of the ground and their calculations procedures. Chapter five presents the methods to calculate stresses induced by various applied loads with respect to their distribution in the soil. In the last chapter, the theory of one-dimensional consolidation is developed, and the methods used to calculate settlement due to consolidation are presented.

To achieving this educational book, it should be emphasized that I have drawn heavily on my experience teaching the course of soil mechanics, as well as on what I found in various bibliographic sources, to enrich its content. I hope it will be well received by the academic community, although excellent books already exist as references.

Faculty Scientific Council Visa: **May, 10, 2026**

Dr. Mohamed Djabri

Tebessa, May, 2026

CHAPTER 1:

PHYSICAL PROPERTIES AND CLASSIFICATION OF SOILS

1.1 INTRODUCTION

Soil is a three-state system of solids, water, and air (or gas). Most of the time, the water and air it contains are the determining factors in predicting how it will react. As a result, many soils do not have similar properties. Understanding and studying the physical properties of soil mainly play the dominant role in soil classification; soil identification involves determining the soil type through which crucial information about their properties can be obtained.

Different soils with similar properties can be classified into groups and subgroups based on their technical behavior. Soil classification systems primarily consider grain size distribution and Atterberg limits to express the general characteristics of soils.

1.2 GENERAL INFORMATION ON THE SOIL

1.2.1 Definition of rock and soil

Rocks are mineral aggregates that can be broken down into varying sizes elements without requiring considerable effort. In geotechnical language, a rock is a hard geological formation, i.e., the mineral particles that compose it are strongly welded and therefore have high mechanical characteristics (e.g.: Silica, Limestone, Feldspar, etc.).

- ***Noticed:*** *Rocky massifs are often, for builders, choice terrain for the foundations of structures.*

On the other hand, soils are loose formations that are formed from natural mineral particles that have low binding strengths and can be separated into small pieces without requiring considerable effort. Soils result from the chemical, physical or mechanical alteration of rocks (oxidation, temperature variation, erosion, etc.

- ***Noticed**: The mechanical characteristics of soils are weaker than rocks, they sometimes have foundation problems.*

1.2.2 Soil formation

In general, soils are formed by the weathering of rocks (Figure 1.1). According to the mode of deposition, soils can be classified into two categories.

1.2.2.A. Residual soil

It is a soil that was formed on site following the chemical or physical alteration of a rock (examples: Granite, Bazilt, etc.).

1.2.2.B. Transported soil

It is soil that was formed in one place and then transported to other following atmospheric phenomena (snow, rain, wind, floods, etc.).

1.2.3 Composition of soil

A soil is generally made up of three phases (Figure 1.1).

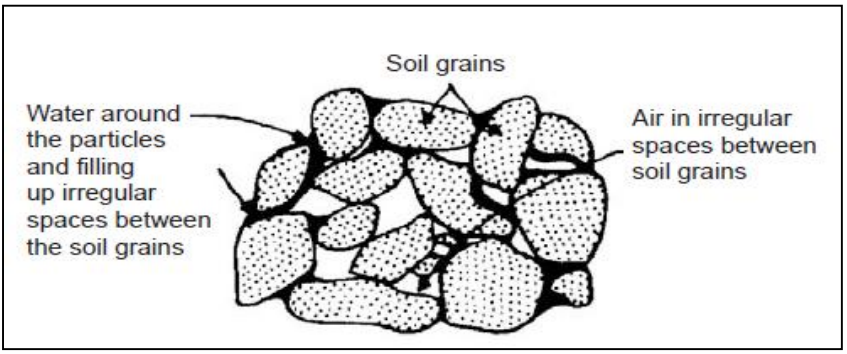


Figure 1.1. The different phases of a soil (soil mass)

1.2.3.A. Pores or voids (gas phase)

The gas phase generally consists of air it is the spaces between grains of different sizes.

1.2.3.B. Soil grains (solid phase)

The solid phase consists of essentially mineral grains and is the one that shows the greatest variability. These solid grains can be macroscopic or microscopic.

1.2.3.C. Water (liquid phase)

There are different forms of water in soils: interstitial water (free, capillary), adsorbed (bound) water and water of constitution in certain minerals.

- ***Noticed**: Adsorbed water has a negligible effect on sand, but it plays an essential role in the behavior of clays.*

- ***Noticed:*** *when the all pores are filled with air, then the soil is dry; where the pores are filled with water, the soil is saturated; and if some of the pores are filled with air and some are filled with water, the soil is partially saturated.*

1.3 STRUCTURE OF SOILS

Soil structure is defined as the way in which the various solid particles (grains) are organized. The mineralogical nature is formed by solid grains, such as Silica (SiO2) and Alumina (Al2O3), which represent 75% of the constituents of the soil solid. The dimensions of these particles can be uniform or varied, ranging from 10 cm pebbles to fine particles of less than a micrometer.

1.3.1 Main physical characteristics of soil particles

The main physical characteristics of soil particles are: grains (particles) size, shape and specific surface area.

1.3.1.A. Particle size

The particle size is measured by a equivalent diameter equal to the minimum square opening of the sieve through which the particle can pass. Square mesh sieves used in sieving granulometric analysis were used to establish the equivalent diameter.

1.3.1.B. The shape of particles

Although there are an infinite variety of shapes, two types are usually recognized: the voluminous shape and the leaf shape.

- *The voluminous shape:* The bulky shape generally characterizes particles of gravel, sand and silt. The equivalent diameter of bulky particles is generally greater than 0.001 mm (Figure 1.2).
- *The sheet form:* When the ratio of its length to its thickness is greater than 10, a particle is considered to have a sheet shape. This shape is especially the characteristic of clay particles.

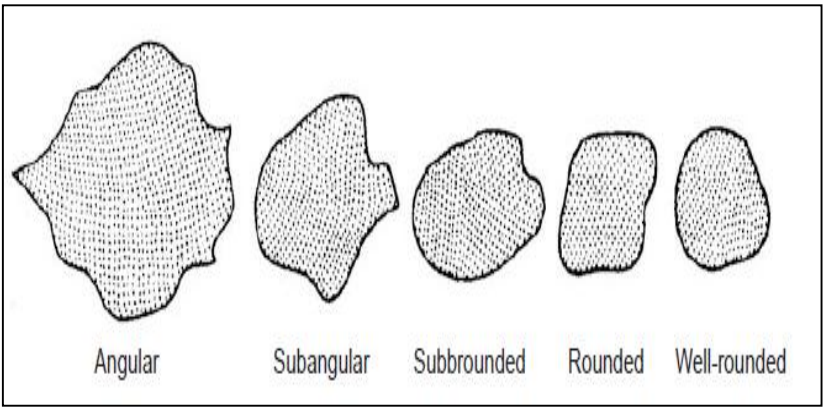


Figure 1.2. Some typical shapes of particles of soil

1.3.1.C. Particles specific surface area

The soil particle surface effect is represented by an index called the specific surface area of the grain. By definition, the specific surface area (Sp) also called "Mass area" represents the total surface area (As) per unit mass (M):

$$S_p = \frac{\text{Surface}}{\text{Mass}} \left(\frac{m^2}{kg}\right)\left(\frac{cm^2}{g}\right) \qquad (1.1)$$

It can be expressed as surface area per unit of volume:

$$S_p = \frac{\text{Surface}}{\text{Volume}} \left(\frac{m^2}{mm^3}\right) \left(\frac{1}{mm}\right) \qquad (1.2)$$

➢ ***Noticed:*** *The specific surface area of soil is inversely proportional to the grain size.*

Table 1.1 shows the method of calculating the specific surface area with some average values of particles from different types of soils.

Table 1.1. Specific surface area of particles in different soil types

Soil particle	Equivalent diameter (cm)	Surface area (cm^2)	Mass (g)	Average specific surface area (cm^2/g)
Gravel	2×10^{-1}	1.13×10^{-2}	1.3×10^{-1}	11.1
Sand	5×10^{-3}	1.77×10^{-7}	7.9×10^{-5}	444.4
Silt	2×10^{-4}	1.13×10^{-11}	1.3×10^{-7}	11.1×10^{4}
Clay	2×10^{-4}	8.48×10^{-15}	6.3×10^{-8}	7.4×10^{6}

1.3.2 *Arrangement* of particles

In soil mechanics, we distinguish between coarse-grained soils (granular soils) and fine-grained soils (fine soils).

1.3.2.A. Coarse-grained soils

The grains of these soils are formed as a result of the physical weathering of rocks which were initially angular and then during the weathering process, they changed shape (examples: pebbles and gravel). Pebbles and blocks have an equivalent diameter greater than 80 mm. They are characterized by very high permeability.

Gravel and sand are made up of rock particles with equivalent diameters ranging from 0.08 mm to 80 mm. Generally, they have good permeability. The arrangement of solid particles in this type of soil is formed under the effect of the earth's gravity, where we distinguish: loose soils, dense soils and cellular soils (Figure 1.3).

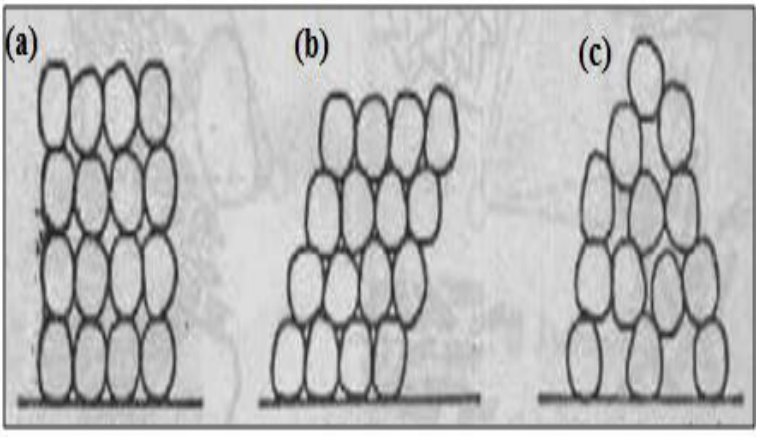


Figure 1.3. Arrangement of particles in granular soils: (a) Loose soil, (b) Dense soil, (c) Cellular soil

1.3.2.B. Fine-grained soils

These soils are produced as a result of the physicochemical weathering of coarse-grained rocks and soils (e.g., silt and clay). The arrangement of this type of soil occurs due to the forces of attraction and repulsion between the particles (Figure 1.4).

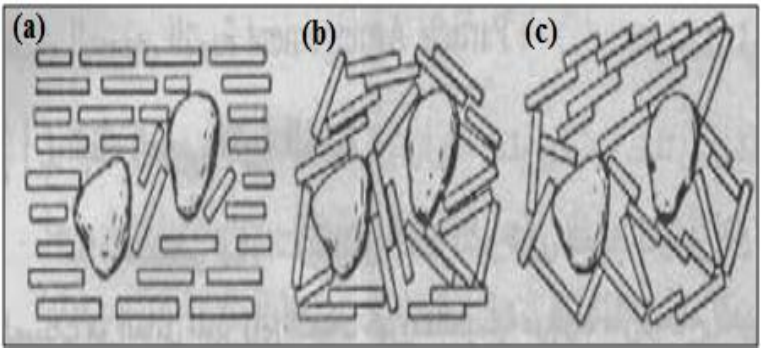


Figure 1.4. Arrangement of solid particles in fine soils: (a) Dispersed or oriented soil, (b) Moderately flocculating soil, (c) Strongly flocculating soil

Silt is composed of fine rock particles with an equivalent diameter ranging from 0.002 mm to 0.08 mm, and whose shape can be observed under a magnifying glass or optical microscope.

Clay is made up of crystalline particles that come from the chemical decomposition of rock constituents. These are, for the most part, silicates of aluminum, magnesium or iron whose atoms are arranged to form very regular geometric figures. Each clay mineral is formed by the stacking of microscopic crystals (sheets). These sheets are themselves made up of crystalline units that are called

fundamental structures. These are juxtaposed in a single plane, and this is why the sheets have a very large surface area compared to the thickness.

There are three main families of clay minerals (Figure 1.5):

- kaolinite: this clay is dangerous for the engineer. Danger relates to its swelling behavior, which is actually less expansive than montmorillonite.
- montmorillonite: susceptible to significant swelling or shrinkage depending on variations in water content.
- illite.

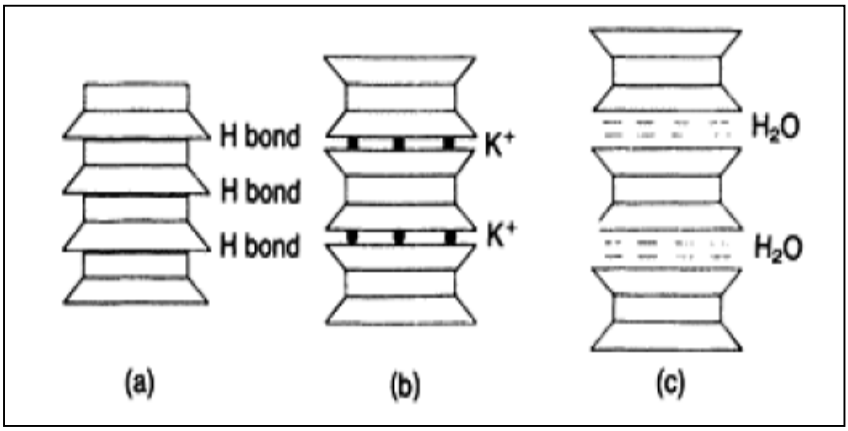


Figure 1.5. Clay minerals: (a) Kaolinite, (b) Illite, (c) Montmorillonite

1.3.2.C. Organic soils

Organic soils come from organic matter. They are the product of the accumulation and gradual decomposition of plant and animal materials over the years. Generally, a soil is said to be organic if it contains a high percentage of organic matter.

1.3.3 Cohesive and non-cohesive soils

If a soil is allowed to dry after being initially wet and the grains of soil stick together, a certain force is needed to separate them; the soil is coherent (Silts and clays).

If, on the other hand, the grains are distinguishable after drying, they do not remain largely fixed, the soil is said to be non-cohesive (sands and gravels).

- ***Noticed:*** *The behavior of cohesive soils is quite complex than non-cohesive soils.*

1.4 PHYSICAL PROPERTIES OF SOILS

1.4.1 Soil state diagram

Soil generally consists of solid grains and voids. The voids are usually filled with water, air, or water and air in varying proportions (Figure 1.6).

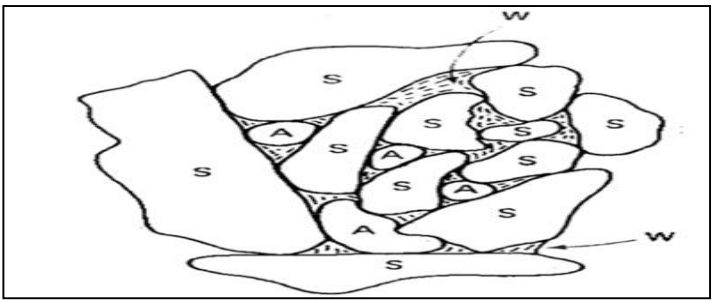


Figure 1.6. Soil element containing solid grains (S), void (A) and water (W)

The three phases of soil (solid phase, liquid phase and gaseous phase) can be schematically represented on the phase diagrams (volumes in left, weights in right) (Figure 1.7). The total volume is often taken as the unit (1 m^3 or 1 cm^3).

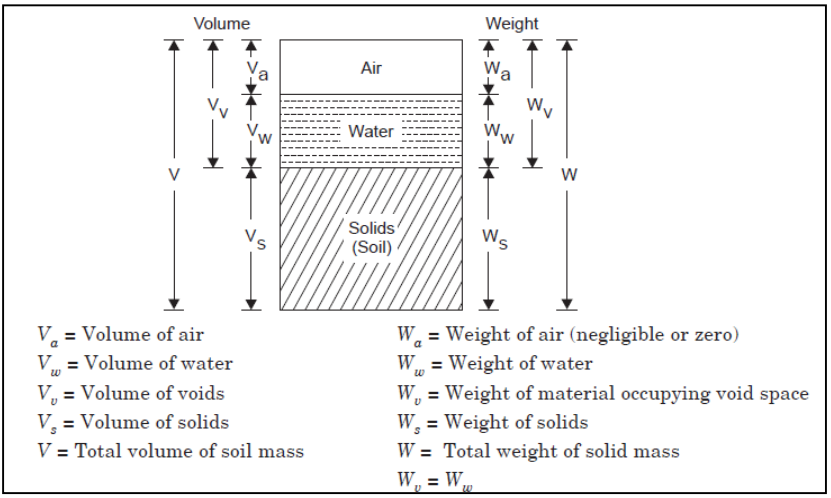


Figure 1.7. Soil state diagram (volumes and weights)

With the following relationships:

$$W = W_s + W_w \quad ; \quad V_v = V_w + V_a \quad ; \quad V = V_s + V_v = V_w + V_a$$

1.4.2 Basic terminology

We define the volumetric weights (dimensional parameters) and we define a certain number of dimensionless characteristics (state parameters) which express the relationships between weights and volumes which are very useful in geotechnics.

1.4.2.A. Dimensional parameters (volume weights)

- ***The Total Unit Weight (or apparent unit weight or wet unit weight):*** This is denoted by the letter symbol $\boldsymbol{\gamma}$ **or** $\boldsymbol{\gamma_h}$:

$$\gamma = \gamma_h = \frac{W}{V} \qquad (1.3)$$

For sand: γ =17 to 20 kN/m^3; for clay: γ =16 to 22 kN/m^3.

- ***The Dry Unit Weight*:** This is denoted by the letter symbol $\boldsymbol{\gamma_d}$ and is given by:

$$\gamma_d = \frac{W_s}{V} \qquad (1.4)$$

For sand: γ_d=14 to 18 kN/m^3; for clay: γ_d=10 to 20 kN/m^3.

- The Saturated Unit Weight: When all the voids are filled with water. This is denoted by $\boldsymbol{\gamma_{sat}}$ and given by:

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + \gamma_w \times V_v}{V} \qquad (1.5)$$

γ_w is the unit weight of water.

For sand and clay: γ_{sat}=19 to 22 kN/m^3.

- ***The Unit Weight of Solids***: This is noted by γ_s and given as follow:

$$\gamma_s = \frac{W_s}{V_s} \qquad (1.6)$$

For sand: γ_s=26.7 kN/m^3; for clay: γ_s=27.2 kN/m^3.

- ***The Submerged Unit Weight***: It is taken into account when the ground is completely submerged. It takes into account the presence of water which fills all the voids and the Archimedes' thrust. This is noted by $\grave{\gamma}$ and given by:

$$\grave{\gamma} = \gamma_{sat} - \gamma_w \qquad (1.7)$$

For sand and clay: $\grave{\gamma}$=9 à 12 kN/m^3

➢ ***Noticed:*** *The submerged unit weight is approximately given by the relation:*

$$\grave{\gamma} = 0.62 \times \gamma_d \qquad (1.8)$$

- ***The relative density***: The notion of density is often introduced in relation to water:

Wet density: $Dh = \frac{\gamma_h}{\gamma_w}$

Dry density: $Dd = \frac{\gamma_d}{\gamma_w}$

1.4.2.B. Dimensionless parameters (geometric parameters)

- ***Specific Gravity of Solids***, denoted **Gs**:It defines the number of times a unit volume of solid particles is heavier than water:

$$Gs = \frac{\gamma_s}{\gamma_w} \qquad (1.9)$$

Values of Gs are often between 2.67 and 2.72 except for mechanically strong soils.

➢ ***Noticed****: At the standard temperature (4)°*

$$\gamma_w = \frac{1g}{cm^3} \approx 9.81 \frac{kN}{m^3} \approx 10\ kn/m^3$$

- ***Water (Moisture) Content***, denoted **w** and is commonly expressed as a percentage:

$$w = \frac{W_w}{W_s} \qquad (1.10)$$

For sand: **w** =1 to 15%; for clay: **w** =10 to 20%.

➢ ***Noticed:*** *w can exceed 100% and even reach several hundred percent.*

- ***Porosity,*** denoted **n** : Porosity allows us to know the importance of voids that is to say to know if the soil is in a loose or tight state.

$$n = \frac{V_v}{V} \times 100 \qquad (1.11)$$

- ***The Void Ratio,*** denoted **e** **:** The sense of void ratio is analogous to that of porosity.

$$e = \frac{V_v}{V_s} \qquad (1.12)$$

For sand: **n** = 0.25 to 0.50 and **e** = 0.50 to 1; for clay: **n** = 0.20 to 0.80 and **e** = 0.30 to 1.

➢ ***Noticed:*** **n** *is always less than 1 (100%),* **e** *can be greater than 1.*

➢ ***Noticed:*** *To characterize the natural state of soil,* **e** *is typically used in soil mechanics.*

- ***The Degree of Saturation***, denoted by **Sr** :

$$Sr = \frac{V_w}{V_v} \times 100 \qquad (1.13)$$

➢ ***Noticed:*** $(0 \leq Sr \leq 1)$; **Sr** *= 0% (dry soil)<* **Sr** *= 100% (saturated soil).*

- ***The Air Content,*** denoted **A :**

$$A = \frac{Va}{V} \times 100 \qquad (1.14)$$

The parameters defined above are independent and other soil phase diagrams may be shown as in Figure 1.9 where important relationships between these different parameters can be drawn.

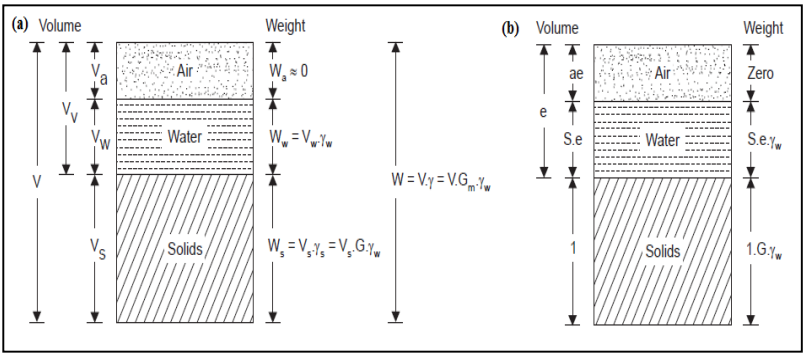


Figure 1.9. Soil phase diagrams: (a) diagram showing additional equivalents on the weight side, (b) unit phase diagram)

1.4.2. Density index

Density Index ID, indicates the relative compactness of the soil mass. In a dense condition, the void ratio is low whereas in a loose condition, the void ratio is high. Thus, the in-place void ratio may be determined and compared, with the void ratio in the loosest condition and that in the densest state (Figure 1.10).

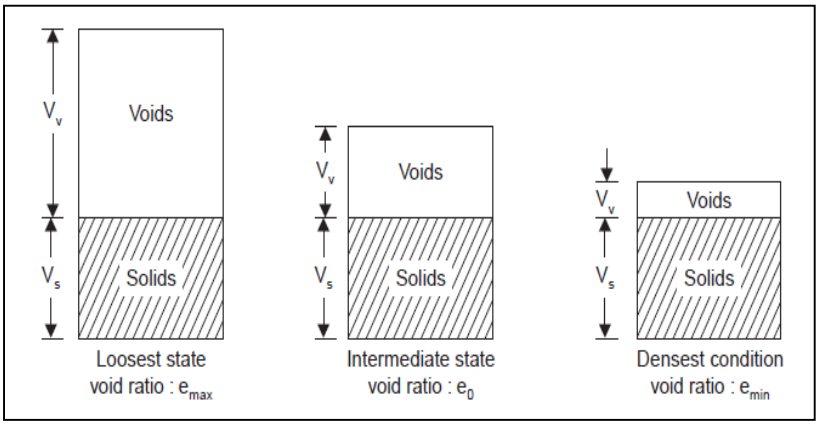


Figure 1.10. Relative states of packing of a coarse-grained soil

The density index may be defined as follows:

$$ID = \frac{e_{max} - e}{e_{max} - e_{min}} \qquad (1.15)$$

e_{max} : in the loosest state.

e_{min} : in the densest state.

e_0 : in the natural state.

e_{max} and e_{min} are referred to as the limiting void ratios of the soil.

Sometimes ID is expressed as a percentage also.

1.5 SOIL IDENTIFICATION TESTS

Soil behavior depends on a large number of factors, the two most important of which are: grain diameter and the tightness of the grains against each other. Grain sizes vary greatly, and several classifications are used based on laboratory analyses, such as grain size distribution analysis and Atterberg limits.

***Noticed**: A coarse-grained soil has little cohesion and greater internal friction than the grains is tightly packed.*

1.5.1 Particle size distribution analysis

Grain size analysis is a typical laboratory test performed in the field of soil mechanics. The analysis is performed via two techniques. Sieve particle size analysis is capable to determining particle sizes ranging from 0.08 mm to 100 mm while particles smaller than 0.08 mm can be distributed using the sedimentation test (Figure 1.11).

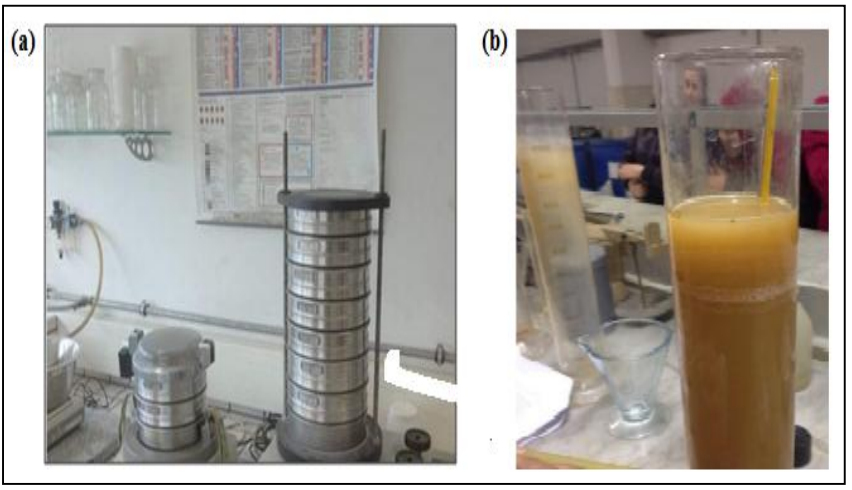


Figure 1.11. Particle size analysis by: (a) Sieving, (b) Sedimentation

1.5.1.A. Sieve analysis

The test is carried out using a series of sieves having different mesh sizes. The sieves are placing with increasingly smaller mesh sizes on top of each other and passing the soil sample through the "tower". The sieve separates the largest from the smallest particles, distributing the soil sample into two quantities. Grains with a diameter larger than the size of the openings are retained by the sieve (rejected or retained), while grains with a smaller diameter pass through the sieve (sieve or passing) (N FP94-056 Standard).

- ***Noticed**: Sieve sizes range from 0.08 to 125 mm. The sieve sizes and number of sieves are chosen based on the nature of the sample and the desired accuracy.*

1.5.1.B. Sedimentation analysis (Wet analysis)

Sedimentation is the settling by gravity of the grains of a soil sample placed under overpressure in a viscous liquid. Sedimentation analysis is used to define grains with a diameter of less than 0.08 mm (80 micrometers) (NF P94-057 Standard). In this case, it normally complements the sieving analysis.

- ***Noticed**: Sedimentation analysis is recommended for fine soils such as silt and clay.*

1.5.1.C. Sieve analysis results

The results of a sieve analysis are given in the form of a semi-logarithmic curve where the sieve size is on a horizontal 'logarithmic' scale, and the percentage by weight of the size smaller than a particular sieve-size is on a vertical 'arithmetic' scale (Figure 1.12).

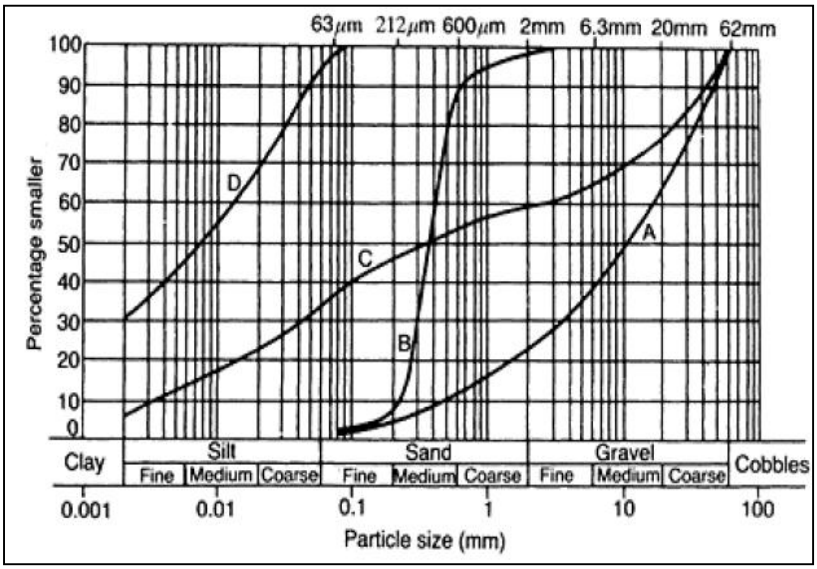


Figure 1.12. Particle size distribution curve (example)

Particle size distribution curve allow us to identify the types of soil that make up the sample being analyzed. If we examine a particle size diagram of a sample consisting of gravel, sand, silt and clay, we find the respective proportions of each type of soil expressed as percentages. When we know these proportions, it becomes possible to assign a name to the soil. For example, if the soil is composed of

27% gravel, 38% sand, 29% silt and 6% clay for example, it is called gravelly silty sand with traces of clay.

From the results of a sieve analysis, two coefficients can be obtained.

- ***The coefficient of uniformity (Cu):*** It is defined by:

$$Cu = \frac{D_{60}}{D_{10}} \qquad (1.16)$$

Where:

D_{10}: Represents the diameter such that 10% of the grains in a size smaller than that of (10%). It is called effective size.

D_{60} : are defined as D_{10} (60% finer size).

➢ ***Noticed:*** *In general, if* $Cu < 2$ *: The soil is said to be very uniform, if* $Cu \geq 4$ (for gravels) or $Cu \geq 6$ (for sands) *: it is said to be non-uniform or well-graded.*

On the average: for sands Cu = 10 to 20, for silts Cu = 2 to 4, and for clays Cu = 10 to 100.

D_{10} and Cu can be obtained from the particle size distribution curve as shown in Figure 1.13.

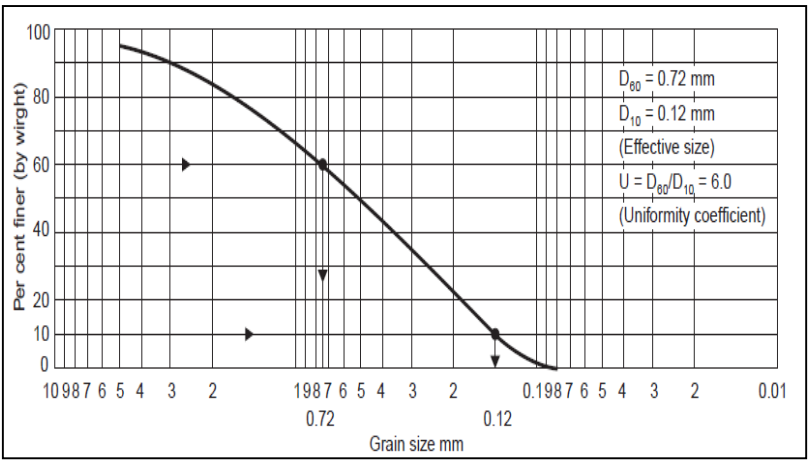


Figure 1.13. Effective size and uniformity coefficient from particle size distribution curve

- ***The coefficient of curvature (Cc):*** Defined by:

$$Cc = \frac{(D_{30})^2}{D_{60} \times D_{10}} \qquad (1.17)$$

Where D_{30} : 30% finer size

When $(1 \leq Cc \leq 3)$, the soil is said to be well-graded, that is to say that is well spread out, without predominance of a particular fraction.

➢ ***Noticed:*** *Well-graded soils are naturally dense deposits with high bearing capacity. They can be easily compacted into embankments and to form stable slopes.*

1.5.2 Consistency of fine soils (Atterbeg limits)

Fine soils such as clays have the property of absorbing a significant amount of water; on the contrary, of drying out in the presence of heat. For the fine fraction (gravel excluded), the cohesions due to the presence of water: perfectly dry, the material

would be coherent (shrinkage limit). Above a certain content (plasticity limit), it can be needed into a sausage, ball or thread. For a higher content (liquidity limit), it forms a viscous liquid, which does not retain the shape given to it. Formally, we distinguished the following stages of consistency: liquid, plastic, semi-solid, and solid (Figure 1.14).

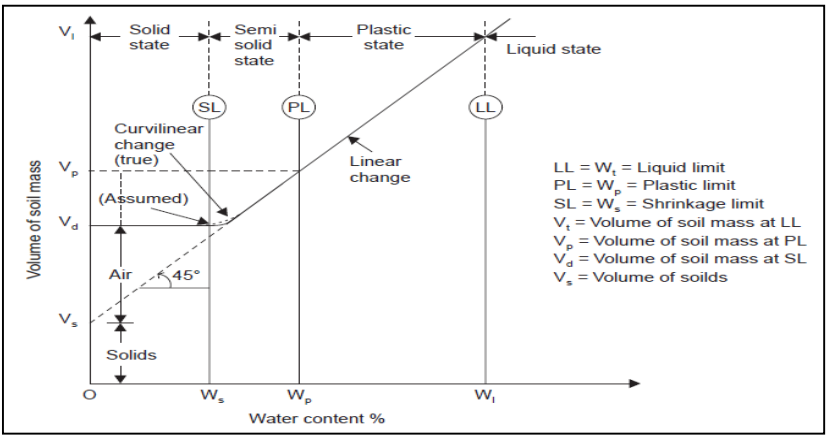


Figure 1.14. Variation of volume of soil mass with variation of water content

- ***Noticed**: Consistency has an influence on deformation strength and only concerns cohesive soils.*
- ***Noticed**: Consistency depends mainly on the distance between the particles in a soil.*

1.5.2.A. Soil consistency states

- ***The solid state:*** When the soil is in the solid state, its particles are in contact with each other, and the adsorbed water films are very

thin and touch each other: there is no free water between the particles. Drying out the soil does not produce any shrinkage. The soil exhibits very high shear strength and, under the effect of a load, deformations are small before it ruptures.

- ***The semi-solid state***: A soil in the semi-solid state has low water content, and the cohesive bonds between its particles are very strong. The adsorbed water films, although still thin, separate the particles slightly, so that drying out of the soil would cause shrinkage.
- ***The plastic state:*** When the soil is in a plastic state, its water content is greater and its particles are further apart from each other. The films of adsorbed water are much thicker, but they still touch each other; the cohesion of the soil is weaker than in the semi-solid state; it is a soil that can be shaped by hand.
- Liquid state: Soil behaves like a liquid. Its shear resistance is zero and it spreads when poured. The soil grains are practically separated by the water. The soil can then behave like a viscous liquid.

1.5.2.B. Consistency limits and indices

- ***Liquid limit (wl):*** Characterize the transition between a plastic state and a liquid state. It is the water content by weight above which the soil flows like a viscous liquid under the influence of its

own weight. It can reach 100% in the case of certain clays, but in most cases it does not exceed 100%.

- ***Plastic limit (wp) :*** Describe the transition between a solid state and a plastic state. It is the water content below which the soil no longer behaves as a plastic material. It varies from 0% to 100%, but generally below 40%.
- ***Shrinkage limit (ws) :*** Express the water content at which a fine-grained soil no longer changes volume upon drying, where any water loss is compensated by the entry of air into the pores. This limit separates the solid state from the semi-solid state.

Although arbitrary, the plasticity and liquid limits are the most practical means available for predicting the behavior of a soil as a function of its moisture content. They are used to classify fine soils. For determining the swelling and shrinkage capacity, it is practical to use the shrinkage limit and employed in certain regions where the soil undergoes large volume variations when the water content changes.

- ***Plasticity index (Ip):*** Which is expressed as a percentage and equivalent to the difference between the liquid limit and the plasticity limit.

$$\mathrm{Ip} = \mathrm{wl} - \mathrm{wp}(\%) \qquad (1.18)$$

(Ip) makes it possible to determine a zone in which this soil will be considered to be in a plastic state (Figure 1.12).

Plasticity characteristics of soils according to their plasticity indices are shown in Table 1.2.

Table 1.2. Plasticity characteristics (Burmister, 1947)

Plasticity index	*Plasticity*
0	Non-plastic
1 to 5	Slight
5 to 10	Low
10 to 20	Medium
20 to 40	High
> 40	Very high

- ***Consistency index (Ic):*** This is a derived indicator which characterize the water status of a soil:

$$Ic = \frac{wl - w}{Ip} \ (\%) \qquad (1.19)$$

Where w: the water content in its natural state and which does not contain elements larger than 400 µm.

$Ic = 1$, the soil is in semi-solid state and is stiff.

$Ic = 0$, the natural water content is greater than wl, and the soil behaves like a liquid.

- ***Liquidity index (Il):*** Allows to quickly determining whether a soil is in a liquid, plastic, semi-solid or solid state.

$$Il = \frac{w - wp}{wl - wp} = \frac{w - wp}{Ip} \qquad (1.20)$$

Soil is classified on the basis of CI or LI values as follows (Table 1.3):

Table 1.3. Consistency classification

CI	*LI*	Consistency
1.00 to 0.75	0.00 to 0.25	Stiff
0.75 to 0.50	0.25 to 0.50	Medium-soft
0.50 to 0.25	0.50 to 0.75	Soft
0.25 to 0.00	0.75 to 1.00	Very soft

In general:

- If : $\mathrm{Ic} > 1$ or $\mathrm{Il} < 0$, the soil is in a dry (hard) state le sol;
- If : $0 < Ic < 1$ or $0 < Il < 1$, describe the plastic state ;
- If : $\mathrm{Ic} < 0$ or $\mathrm{Il} > 1$, describe the liquid state.

1.6 SOIL CLASSIFICATION

1.6.1 Principle of soil classifications

Soil classification systems aim to classify soils into groups with the same or very similar geotechnical characteristics. Therefore, many soil classifications have been carried out in each country based on basic tests (sieve analysis and consistency limits).

1.6.2 Particle size or textural classifications

Generally, there are two categories which classification systems can be grouped.

—— Textural classification, based on the particle size distribution of the percentage of gravel, sand, silt, and clay fractions present in a given soil. This classification is based in the Massachusetts Institute of Technology (MIT) classification system and the triangular classification.

—— The other category is based on the geotechnical behavior of the soil and takes into account the particle size distribution and plasticity (i.e., the liquid limit and the plasticity index). A number of grain-size classifications have been evolved (ex. P.R.A and M.I.T in the U.S.A) (Figure 1.15).

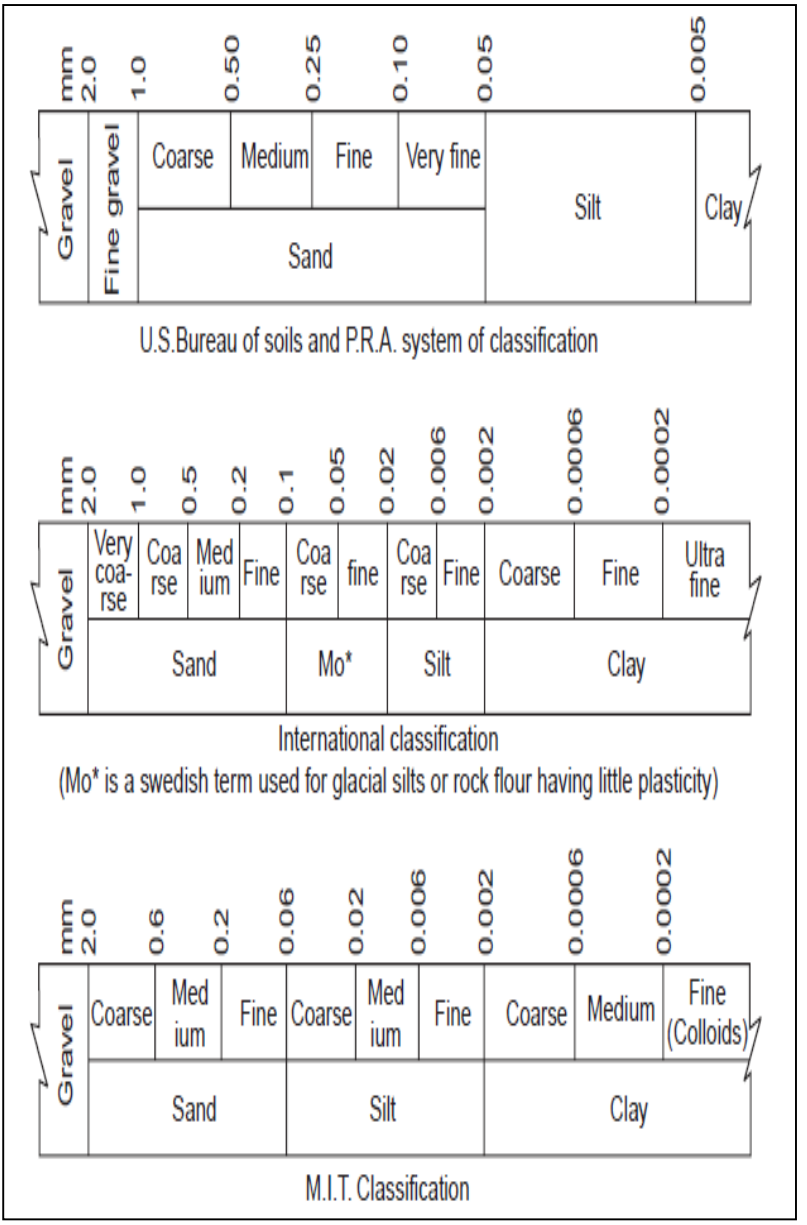


Figure 1.15. Particle size classifications

1.6.2 Textural (triangular) classification

Textural classification depicted by a 'Triangular chart' (Figure 1.17), which ignores the fraction coarser than sand:

The triangular soil classification is relatively simple; it is based entirely on particle size, but it is not applicable to ordinary sands and gravels. In a given soil sample, it is assumed that there are varying proportions of sand, silt, and clay, where three poles are distinguished:

- Sand: elements greater than 0.05 mm.
- Silt: elements between 0.05 and 0.005 mm.
- Clay: elements less than 0.005 mm.

Any soil with the three constituents: sand, silt and clay can be represented by one point on the Triangular chart. The use of triangular classification chart can be best illustrated by an example. If the particle size distribution of the soil shows 25% sand, 25% silt and 50% clay, its textural classification can be determined by proceeding as indicated by the arrows, this soil falls into the sand zone, represented by the point 'S', obtained by the dotted lines, as shown by the arrows (Figure 1.16).

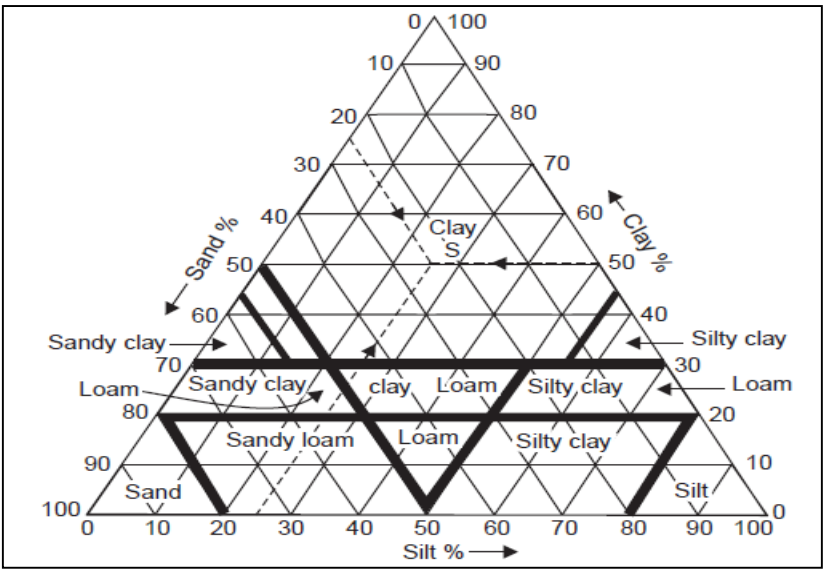


Figure 1.16. Example of a textural classification

1.6.3 Unified Soil Classification System

In 1972, Casagrande defined a diagram called the "Casagrande plasticity chart" which allows the classification of fine soils. It was later revised for universal use and redesignated as the "Unified Soil Classification" in 1957. In this system, soils are classified into three broad categories:

1. Coarse-grained soils with up to 50% passing (No. 75-μ IS Sieve);
2. Fine-grained soils with more than 50% passing;
3. Organic soils

The Casagrande diagram is divided into six sections by the oblique line A drawn so that $\mathrm{Ip} = 0.73(\mathrm{wl} - 20)$ and by two vertical lines drawn so that $\mathrm{wl} = 30\%$ and $\mathrm{wl} = 50\%$. Each

section of the diagram corresponds to a group of soils with well-defined mechanical properties (Figure 1.17).

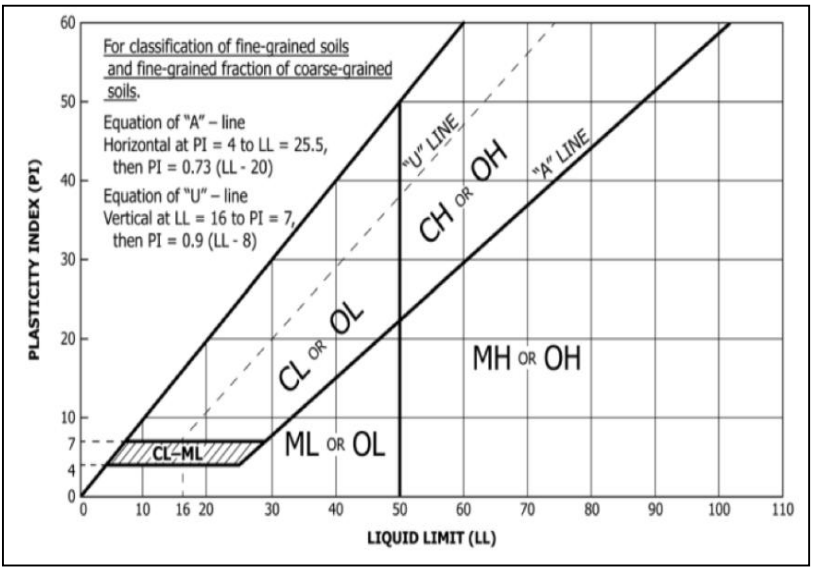


Figure 1.18. Casagrande Plasticity chart (unified soil classification)

1.6.3 LCPC classification of soils

The "Classification des Laboratoires des Ponts et Chaussées" (LCPC), adapted in France in 1965; it is almost identical to the "The Unified Classification System" (USCS).

To better describe soils containing organic matter, the "modified LCPC" classification (1974/1980) is used. The "GTR" classification of soils for road earthworks is used in France for earthwork studies, and is often used in geotechnical route study reports.

The LCPC classification is based essentially on sieve analysis and on the plasticity characteristics of the fine fraction, supplemented by very simple tests (color, odor, effects of water, etc.). The LCPC classification system results in 15 soil types, each assigned a two-letter symbol, taken from the three sets (Table 1.5).

Table 1.4. Soil groups according to the LCPC classification

Soil element	Soil sieve size	Soil plasticity
G: Gravel. Gravel is the main fraction	**b**: Well graduated **m**: Poorly graduated	**t**: Very plastic **p**: Little plastic
S: Sand. Sand is the main fraction		
L: Silt or loamy		
HAS: Clay or clayey		
T: Peat		
O: Organic. Soil contains organic matter		

The modified LCPC classification (1974/1980) relates exclusively to the description of organic soils (organic matter content greater than 3%). For these soils, the organic matter content CMO is determined, which determines three classes:

- $3\% < C_{MO} < 10\%$: Weakly organic soils (fo) (mud);
- $10\% < C_{MO} < 30\%$: Medium organic soils (MO) (peaty soil);
- $C_{MO} > 30\%$: Very organic soils (to) (Peat).

1.7 EXERCISES

Exercise 1.1

Establish the relationship between Sr, w, G and e.

Exercise 1.2

A sample of clay has a wet weight of 1.553 N and volume of 95.3 cm3. After drying in an oven, its weight 1.087 N. Determine e and Sr of the clay sample (Gs=2.75).

Exercise 1.3

A sample of has a volume of 785 cm3 and weight of15.80 N. If w=36%, determine γ and γ_d of the soil. If Gs= 2.6, compute the void ratio.

Exercise 1.4

A sand sample has porosity n=35% and Gs=2.73. What is its dry density and void ratio?

Exercise 1.5

The results of sieve on soil sample were summarized in the table. 2.3 g passed through the 63 um. Plot the particle size distribution curve and determine the uniformity coefficient of soil.

Sieve (mm)	Mass retained (g)
10	0.0
6.3	5.5
2	25.7
1	23.1
0.6	22.0
0.3	17.3
0.15	12.7
0.063	6.9

CHAPTER 2 :

SOIL COMPACTION

2.1 INTRODUCTION

Compaction improves soil properties, increases strength and bearing capacity, and reduces compressibility and permeability. In earliest times, when road builders were unfamiliar with soil mechanics, it was known that soil compaction produced higher-quality roads. Even today, soil compaction remains one of the most common construction activities in civil engineering, with applications in various geotechnical constructions and structures such as pavements (roadworks), earth dams, and road embankments (slopes near roads).

Compaction can be achieved through several methods:

- Addition of chemical materials (chemical treatment);
- Underfloor heating (heat treatment);
- Application of an electric current (electrical process);
- Groundwater lowering to reduce pore water pressures;
- Increasing soil density (a mechanical process) is the main result of compaction.

2.2 COMPACTION THEORY

2.2.1 Definition of compaction

In geotechnics, compaction of soil is the process in which mechanical energy is applied to soil, causing densification when air is expatriated from the pores between the soil grains. Therefore, the

water content W of the soil is not altered. The specific gravity of the soil, on the other hand, γ_h will be increased, and with it, the dry specific gravity, because:

$$\gamma_d = \frac{\gamma_h}{1 + w} \qquad (2.1)$$

Compaction tightens the soil texture, which reduces deformation and settlement, increases load-bearing capacity, and ensures the stability of slopes and embankments. When a load is applied that causes densification due to the movement of water between the soil grains, consolidation, not compaction, has occurred.

Normally, compaction is the result of heavy machinery compressing the soil, but it can also occur due to the passage of, for example, animal hooves.

2.2.2 Parameters influencing compaction

Proctor, an American engineer, demonstrated that compaction is a function of four parameters: water content, dry soil density (weight), compaction energy, and soil type (particle size distribution, mineralogy, presence of clay, etc.). Compaction results are presented as a curve (compaction curve). This curve is drawn from the water content (W) and dry unit weight of the soil (γ_d) or dry density (the ratio of the dry unit weight of the soil to the unit weight of water).

2.2.2.A. Moisture content—Dry density relationship

For a given soil and compaction energy, the degree of compaction is assessed by the dry unit weight of the compacted soil. When water is added to the soil during compaction, it acts as a lubricant between the soils particles. These particles slide over one another, resulting in a denser and more compact mass. Following compaction, the unit weight of the soil increases each time more water is added.

After certain moisture content is reached, further increases in moisture content tend to decrease the dry weight. This decrease in dry weight occurs because water begins to occupy the spaces that would otherwise be occupied by the solid particles. By constructing the variation of dry unit weight or dry density as a function of water content using a series of experimental points, a "Proctor curve" is generally obtained. The shape of this curve allows us to define a maximum value of dry density (γ_{dmax}), which corresponds to optimal water content value denoted (w_{opt}) (Figure 2.1).

In common practice, the "dry" side refers to water contents below the optimum, and the "wet" side to those above the optimum.

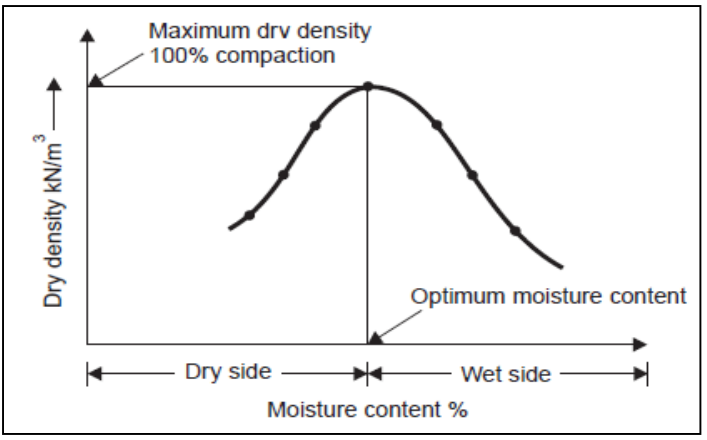


Figure 2.1. Moisture content versus dry density at a particular compactive effort

2.2.2.B. Influence of soil type

The compaction curves are very flat for sands whose compaction is little influenced by water content (Figure 2.2). The optimum water content for sands is less than that of clays (Table 2.1).

➢ ***Noticed:*** *Sands are the best backfill materials.*

For the same compaction energy, well-graded soils will have a higher and less optimal Proctor unit weight than poorly graded soils.

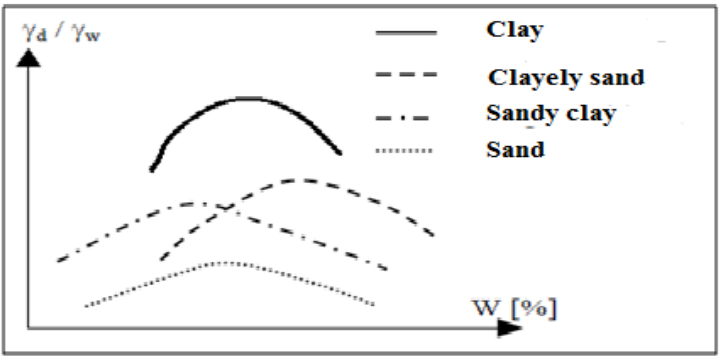


Figure 2.2. Influence of soil type

Table 2.1. Typical values of maximum dry density and optimum moisture content

Soil type	Maximum dry density γ_{dmax}(kN/m^3)	Probable value of the optimal water content (W_{opt}) (%)
Sand	19	11
Sandy clay	18.1	14
Silty clay	16.3	21
Clay	15.2	28

2.2.2.C. Influence of compaction energy (Effect of compactive effort)

Increasing the compaction energy increases the dry unit weight and decreases the Proctor optimum (Figure 2.3).

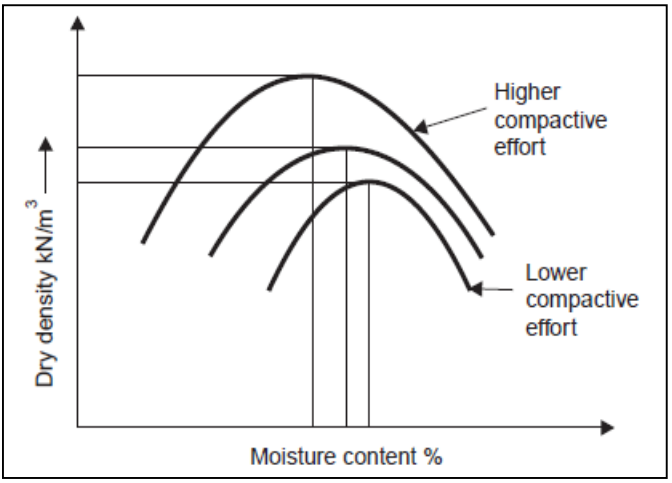


Figure 2.3. Effect of compactive effort on compaction characteristics

2.3 LABORATORY COMPACTION TESTS

2.3.1 Purpose and principle of the Proctor test

Both standardized tests: the standard Proctor test (ASTM D1140, NF P94-093) and the modified Proctor test (ASTM D1557), denoted CBR (ASTM D 1557) aim to determine the optimum water content w_{opt} for a given fill soil and precise compaction conditions, which leads to the best possible compaction or maximum bearing capacity.

In standard practice, the CBR (California Bearing Ratio) and the modified Proctor test use the same mould but are fundamentally different tests. The modified Proctor test (ASTM D1557) uses the CBR-sized mould but determines γdmax and wopt, while the CBR test measures bearing strength.

The test consists of compacting soil samples in a standardized mould, using various moisture content values. For each test, the dry density is determined. The results are then plotted on a graph, and the curve that best connects the obtained points passes through a maximum corresponding to the Proctor optimum. To perform the test in the laboratory, a standardized apparatus is used (Figure 2.4).

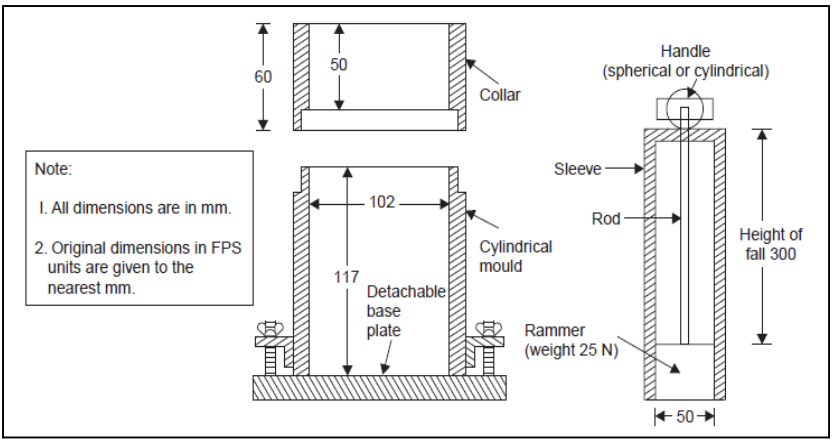


Figure 2.4. Apparatus for standard Proctor test

2.3.2 Types of moulds

Depending on the fineness of the soil grains, two types of molds of different dimensions are used for these tests: standard Proctor mould and modified Proctor mold.

2.3.2.A. Standar Proctor mould

It is a cylindrical mould with a volume of approximately $9.44*10^{-4}$ m^3=948 ^{cm3}, mold diameter ϕ_{mould} =101.6 mm and its height Hmould=116.4 mm) (Figure 2.4). The standard Proctor mould is used when the material is sufficiently thin (no elements larger than 5 mm; $\Phi \leq 5$ mm).

In the standard Proctor test, a sample of dry soil is mixed with water and compacted in the cylindrical mould by a 2.49 kg (a hammer) falling freely from a height of 305 mm. The soil is

compacted in three layers, each of which is subjected to 25 blows from the rammer.

2.3.2.B. Modified Proctor mould (CBR)

It is a cylindrical mould with 2755 cm^3 as approximately volume and 152 mm as diameter and its height is152 mm). The modified Proctor mold, also known as CBR "California Bearing Ratio", is used for materials with dimensions greater than 5 mm and less than 20 mm (5 mm π Φ π 20 mm). The modified Proctor test uses the same principle as the standard Proctor test. A 4,540 kg rammer is dropped freely from a height of 457 mm. The soil is compacted in five layers with 25 blows per layer in the mould.

2.3.3 Compaction energy

With each of these moulds, two types of tests can be performed (the choice depends on the compaction energy): the standard Proctor test (for most projects such as building and/or dam compaction work) and the modified Proctor test (for projects involving heavy loads such as road compaction and runway construction). According to standard NF EN 13286-2, the equivalencies between results obtained with new and/or alternative equipment are based on specific energy and it is given by the following equation:

$$E_{comp} = m_h \times g \times \frac{h_d}{V} \times N_b \times N_l \qquad (2.2)$$

- **Example for the standard Proctor test in a Proctor mould**:

The mass of the hammer is mh=2.490 kg, the acceleration of gravity g=9.8, the height of fall hd=30.5 cm, the volume of the mold V=9.48 m^3, the number of blows Nb=25 blows, the number of layers N_l=3 layers.

$$E_{comp} = 2.49 \times 9.8 \times \frac{0.305}{9.44 \times 10^{-4}} \times 25 \times 3 \times 10^{-3}$$
$$= 591 \frac{kJ}{m^3} \approx 0.59 \frac{kJ}{dm^3}$$

Table 2.2. Conditions of the normal Proctor test are modified

Proctor Test	Hammer mass (kg)	Drop height (cm)	Number of strokes per layer	Number of layers	Compaction energy (kJ/m^3)
Normal	2.490	30.50	25 (Proctor mould)	3	587
			55 (CBR mould)	3	533
Modified	4.540	45.70	25 (Proctor mold)	5	2680
			55 (CBR mold)	5	2435

Table 2.2 summarizes the conditions of each laboratory compaction test according to the mold selected (NF P 94-093 standard.

2.3.4 Proctor compaction test results

The Proctor compaction test is repeated several times in succession (5-6 times) on moistened samples brought to different water contents (increasing by 2%). Several points are thus defined on a curve where hyperbolic "compaction curve" is plotted, representing a maximum whose abscissa is the optimum water content (W_{opt}) and the ordinate is the maximum dry unit weight or maximum dry density (γ_{dmax}).

On the same graph, theoretical curves can be plotted showing the relationship between dry unit weight and water content for different degrees of saturation. These curves are called saturation curves. Two Proctor curves are shown, corresponding to two different energies (Figure 2.5).

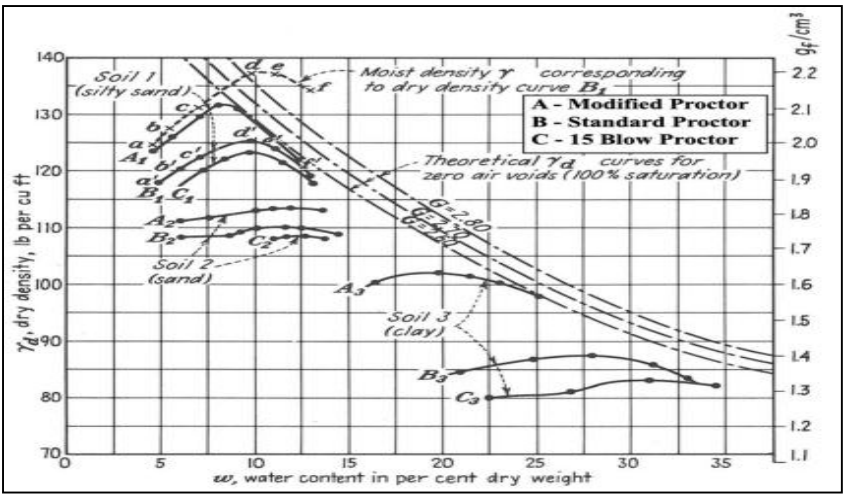


Figure 2.5. Normal and modified Proctor curves with saturation curve

We can obviously plot the saturation curve, where represents the air content. The percentage of air in the soil is expressed by the ratio of the volume of air to the total volume ($A = \frac{V_a}{V}$) and not by the volume of voids.

In practice, experimental observations show that the maximum degree of saturation obtained by compaction does not exceed 80 to 90% on average for fine soils. This "blockage" actually corresponds to a very abrupt change in the soil's permeability to air; indeed, when the percentage of air voids falls below 5 or 10%, the soil becomes practically impermeable to air, and the residual air remains trapped in the macropores.

To plot the hyperbolic compaction curve and the saturation curve, the following formulas can often be used:

- For the different values of S_r , we use:

$$\gamma_d = \frac{G_s \times \gamma_w}{1 + (w \times \frac{G_s}{S_r})} \qquad (2.3)$$

Where G_s is the specific weight of the solid grains.

- For saturation, we use: $(S_r = 100\% = 1,\ A = 0\%)$

$$\gamma_d = \frac{G_s \times \gamma_w}{1 + (w \times G_s)} \qquad (2.4)$$

- To plot the saturation curve, we use the following equation:

$$\gamma_d = \frac{\gamma_w \times (1 - A)}{\left(\frac{1}{G_s}\right) + w} \qquad (2.5)$$

Where A (%) is the air content.

2.4 IN-SITU COMPACTION

2.4.1 Common in-situ compaction methods

In common in-situ compaction processes, light and heavy equipment (Figure 2.6) are used, depending on the specific soil to be compacted and the nature of the work to be carried out.

2.4.1.A. Vibration

For powdery and granular soils, effective compaction is achieved by vibration using a manual vibrating plate and a self-propelled vibrating roller.

2.4.1.B. Compressed air drums

For compacting thin layers:

- Explosion ladies (frog) for small areas of cohesive or non-cohesive terrain.
- 2-3 ton rammers mounted on a rolling crane are used for all terrains but are only useful for small areas.

2.4.1.C. Smooth rollers

For cohesive, non-clayey soils, they provide a contact pressure of around 300-400 kN/m_2. This type of machine ensures 100% coverage.

2.4.1.D. Tire rollers

For compacting loose soils we use the tire rollers consist of a heavily loaded chassis with several rows of tires arranged side by side (4 to 6 tires per row). The contact pressure under the tires varies between 600 and 7000 kN/m^2, meaning they exert a force of 2000 kN. This type of machine provides coverage of up to 80%.

2.4.1.E. Sheep's foot rollers

For cohesive soils, in particular, they are essential for clay soils. This type of machine transmits energy of 1500-7500 kN/m^2.

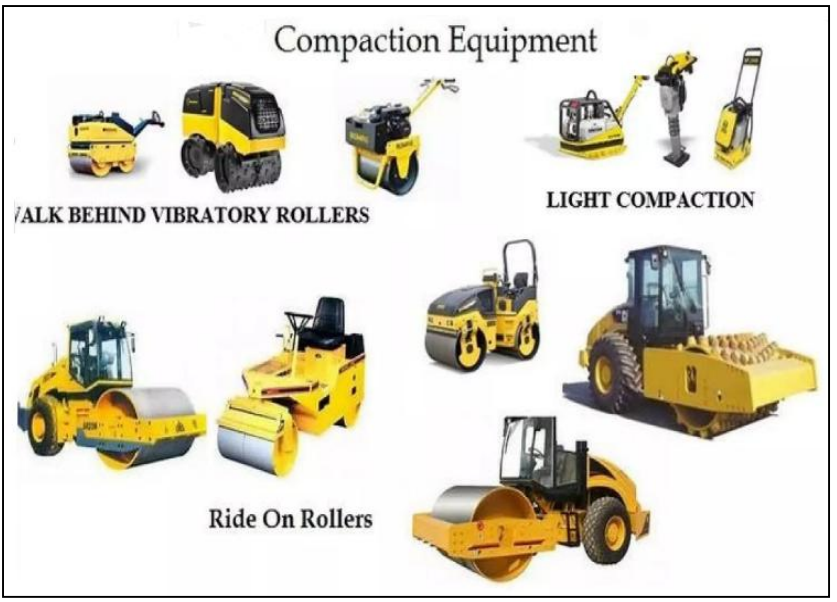


Figure 2.6. Some light and heavy compaction equipment

2.4.2 Special compaction processes

In the case of thick layers, dynamic processes are used such as:

2.4.2.A. Compaction by explosives

- Point explosives: for powdery soils where compaction is achieved by creating a compression shock wave.
- Linear explosives: for cohesive soils where compaction is achieved by placing sand piles.

2.4.2.B. Vibroflotation compaction

The process involves the generation of alternating stresses and deformations, resulting in a rearrangement of the grains.

- Vibrating tubes: used for highly permeable materials.

- Stone columns: the columns are made of compacted granular materials and they are used in cohesive soils (Figure 2.7).

2.4.2.C. Dynamic consolidation

It is suitable for all types of soil. It involves transmitting high-energy shocks to the surface of the soil being treated (free fall of a mass of 10 to 30 tonnes, exceptionally 140 tonnes from a height of 15 to 30 m).

- ***Noticed:*** *Regardless of the equipment used, on-site compaction must be carried out in thin layers of 20 to 30 cm (road works) or 10 to 15 cm (building works).*

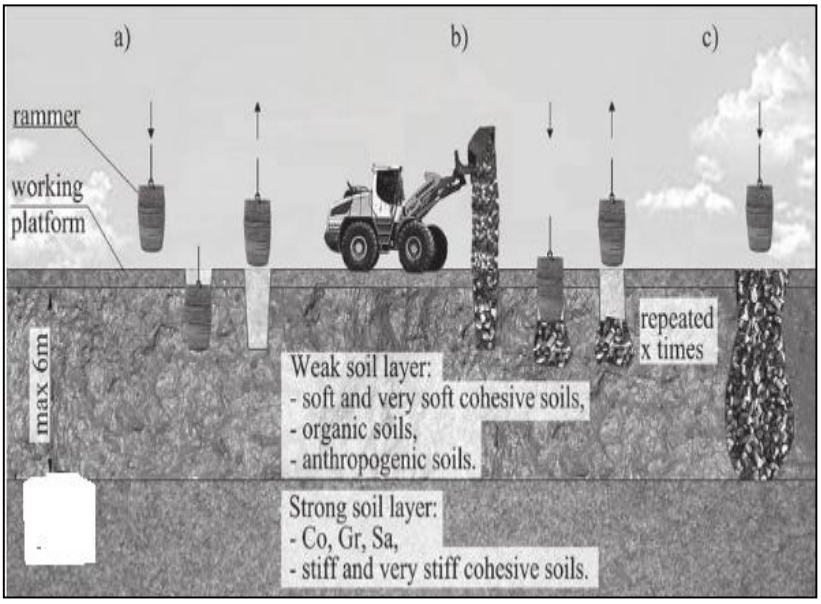


Figure 2.7. Compaction process using stone columns

2.4.3 Factors affecting on-site compaction

Despite the extreme diversity of compaction equipment, it is possible to identify several factors that primarily influence the effectiveness of a machine on a given soil and that have a significant impact on the quality of compaction and the desired final value.

These factors include, of course, those specific to the compacted soil (nature, water content, etc.), but also those characterizing the machine and the compaction sequence (number of passes, speed, contact pressure, vibration frequency and intensity, etc.).

The surface area on which the pressure is applied is also a factor not to be underestimated (Figure 2.8).



Figure 2.8. Use of the main compaction equipment (road works)

2.4.3.A. The thickness of the layer to be compacted

As the pressure applied to the surface of the layer decreases with depth, the degree of compaction also decreases with depth.

2.4.3.B. The number of roller passes

On the other hand, the degree of compaction expressed by (γ_{dmax}) (optimal) increases with the number of passes of the compactor (roller). For a given machine and set quality parameters, there is an optimal number of passes, depending on the machine speed, the layer thickness, and the nature of the material, to achieve maximum compaction. More stringent quality requirements involves higher number of optimal passes generally, 3 to 8 passes are needed to compact a 30 cm thick layer of soil. This number can easily reach 12 depending on the soil type, moisture content, and compactor weight. If the desired compaction is not achieved after 12 passes under optimal moisture conditions, it is concluded that the compaction operations have not been successful and that the compactor used is probably unsuitable.

2.4.3.C. Speed of the vehicle

For a given machine and specified quality requirements, there is an optimal speed, determined by the layer thickness and the nature of the material that achieves maximum compaction. The more stringent the quality requirements, the lower the optimal translation

speed. It is recommended to limit the speed of most compactors to 8 km/h. In the case of vibratory compactors, the optimal speed is around 5 km/h so that the vibrations can effectively act throughout the entire thickness of the layer.

2.4.4 Degree of compaction

2.4.4.A. Compaction control

By comparing the unit weight of the dry soil on site (in-situ) ($\gamma_{\text{din-situ}}$) with the maximum dry unit weight in the laboratory ($\gamma_{\text{dmaxproctor}}$) (Optimum Proctor) we establish the degree of compaction or percentage of compaction DC using the following equation:

$$\text{DC} = \frac{\gamma_{\text{din-situ}}}{\gamma_{\text{dmaxproctor}}} \qquad (2.6)$$

The degree of compaction is one of the criteria used to accept or reject compaction. This degree, expressed as a percentage, approaches 100% when the $\gamma_{dworks\ \text{site}}$ value approaches the laboratory value. In most specifications for compaction work, the contractor is advised to achieve 90 to 95% of the maximum value γ_{dmax} determined in the laboratory by the standard Proctor test or the modified Proctor test.

- ***Noticed :*** *The specifications require:* $90\% < DC < 100\%$
- ***Noticed:*** *The higher* DC *value according the greater soil compaction and more effective compaction.*

The degree of soil compaction can be estimated from the results of dry unit weight determined by the Proctor test in the laboratory (Table 2.3).

Table 2.3. Degree of compactness taken into account for construction sites according to Proctor results

Dry unit weight value γ_{dmax} (kN/m³)	Degree of compaction, DC (%)
$\gamma_{dmax} > 19$	$DC = 90\%$
$17 < \gamma_{dmax} < 19$	$90\% < DC < 95\%$
$16 < \gamma_{dmax} < 17$	$95\% < DC < 100\%$
$\gamma_{dmax} < 16$	$DC = 100\%$

2.4.4.B. State of compaction of granular soils

The physical parameter indicating the state of a granular soil with respect to the proportion of voids present in it is the relative density index, denoted ID, which is defined by the following relationship:

$$ID = \frac{e_{max} - e}{e_{max} - e_{min}} \qquad (2.7)$$

Where:

e_{min} : is the index of voids corresponding to the most compact state.

e_{max}: is the index of voids corresponding to the loosest state.

e: is the index of voids in the soil in place.

For granular soils, the values of e_{max} and e_{min} are presented in Table 2.4.

- ***Noticed:*** *The density index gives an idea of the compactness of a given soil, for the loosest state ($ID = e_0 = e_{max}$) and for the densest state ($ID = e_1 = e_{min}$).*

Depending on the soil conditions, the relative density index falls within the ranges indicated in Table 2.5. The parameter that illustrates the compactness of a soil deposit can also be expressed by the soil unit weights according to the following formula:

$$\mathrm{ID} = \left(\frac{\gamma_d - \gamma_{dmin}}{\gamma_{dmax} - \gamma_{dmin}}\right) \times \left(\frac{\gamma_{dmax}}{\gamma_d}\right) \qquad (2.8)$$

γ_{dmax} :The dry unit weight at its densest state corresponds to e_{min};

γ_{dmin} :The dry unit weight in the loosest state corresponds to e_{max};

γ_d: The dry unit weight in its natural state (without compaction) corresponds to e.

There is an empirical relationship between ID and DC (only for granular, cohesionless soils):

$$\mathrm{ID} = \left(\frac{\mathrm{DC} - 80}{0.2}\right)(\%) \qquad (2.9)$$

Table 2.4. Extreme values of the soil void index

Soil types	Index of voids	
	e_{max}	e_{min}
Spheres of the same diameter	0.92	0.35
Clean, uniform sand	1.00	0.40
Uniform inorganic silt	1.10	0.40
Silty sand	0.90	0.30
Fine to coarse sand	0.95	0.20
Mica sand	1.20	0.40
Silty sand and gravel	0.85	0.14

Table 2.5. Values of the relative density index as a function of soil condition

Relative density index(%)	Condition (description)
15-35	Coward
35-65	Medium compact
65-85	Compact
85-100	Very compact (dense)

2.4.5 Determination of field dry unit weight in-situ

There are several standard procedures for determining dry unit weight ($\gamma_{on\ site}$) in the field (Figure 2.9):

- Sand cone method (ASTM standard D-1556);

- Membrane densitometer (NF P94-061-2 standard);
- Nuclear density gauge (the AASHTO T 310 standard).



Figure 2.9. Some methods for determining dry unit weight in-situ

2.4.5.A. Sand cone method (ASTM D-1556)

This involves a glass or plastic jar with a metal cone attached to its opening. The jar is filled with dry sand, and the entire container is weighed (weight 1). In the field, a hole is dug in the compacted soil, and the recovered material is weighed (weight 2), at a pre-determined moisture content. The dry weight (weight 3) is then determined using the following relationship:

$$\text{Weight } 3 = \left(\frac{\text{Weight } 2}{1 + \frac{w(\%)}{100}} \right) \qquad (2.10)$$

Next, the cone is placed over the hole and the sand is allowed to flow freely until the hole is full. The volume of sand that filled the hole is then determined.

$$\text{Weight } 5 = \text{ Weight } 1 - \text{Weight } 4$$

Weight 1: weight of the jar filled with sand and the cone,

Weight 4: weight of the jar and the sand remaining after filling the hole,

Weight 5: weight of the sand that filled the hole and the cone.

The volume of soil excavated from the hole equals the volume of sand that filled the same hole.

$$V = \left(\frac{\text{Weight } 5 - \text{Weight C}}{\gamma_{d(\text{sand})}}\right) \qquad (2.11)$$

Weight C: weight of the sand that fills the cone.

The weight C and the weight of the sand $\gamma_{d(\text{sable})}$ are known.

$$\gamma_d = \left(\frac{\text{The weight of the soil recovered from the hole after drying (Weight 3)}}{\text{Volume of the hole}}\right) = \frac{\text{Weight 3}}{V}$$

2.4.5.B Membrane densitometer (NF P94-061-2 standard)

The aim is to measure the apparent masses or bulk weights of soils in place before bulking (wet,) γ_h or γ_d after settling or compaction.The test consists of digging a cavity, collecting and weighing all of the extracted material, and then measuring the volume of the cavity using a membrane densitometer.The device is equipped with a piston which, under the action of the operator, forces a volume of water into a flexible, watertight membrane that conforms to the shape of the cavity. A graduated rod allows the volume to be read directly.

2.4.5.C. Nuclear density gauge(AASHTO Standard T 310)

In this method, a nuclear probe is used which emits radiation and the reflections are used to determine the dry unit weight of compacted soils.

2.5 EXERCISES

Exercise 2.1

An earth embankment is compacted at a water content of 18% to a bulk density of 19.2 kN/m^3. If the specific gravity of the sand is 2.7, find the void ratio and the degree of saturation of the compacted embankment.

Exercise 2.2

A moist soil sample compacted into a mould of 1000 cm^3 capacity and weight 35 N, weight 53.5 N with the mould. A representative sample of soil taken from it has an initial weight of 0.187 N and even dry weight of 0.1691 N.

- Determine: (a) water content, (b) wet density, (c) dry density, (d) void ratio and (e) degree of saturation of sample.
- If the soil sample is so compressed as to have all air expelled, what will be the new volume and new dry density?

Exercise 2.3

A piece of clay taken from a sampling tube has a wet weight of 1.553 N and volume of 95.3 cm^3. After drying in an oven for 24 hours at 105°C, its weight 1.087 N. Assuming the specific gravity of the soil particles as 2.75, determine the void ratio and degree of saturation of the clay sample.

Exercise 2.4

The following data have been obtained in a standard laboratory Proctor compaction test on glacial till:

Water content %	5.02	8.81	11.25	13.05	14.40	19.25
Weight of container and compacted soil (N)	35.80	37.30	39.32	40.00	40.07	39.07

The specific gravity of the soil particles is 2.77. The container is 9.44 cm^3 in volume and its weight is 19.78 N. Plot the compaction curve and determines the optimum moisture content.

Also compute the void ratio and degree of saturation at optimum condition

CHAPTER 3 :

PERMEABILITY AND WATER FLOW

3.1 INTRODUCTION

Soils are permeable materials due to the existence of interconnected voids through which water can flow from high-energy points to low-energy points. A good understanding of soil permeability is necessary to estimate the amount of infiltration under dams, for example, to facilitate underground construction.

One of the main physical parameters of a soil that controls the infiltration rate is the permeability coefficient (hydraulic conductivity). This coefficient can be measured by laboratory tests, field tests, or empirical formulas based primarily on the results of particle size analysis.

The study of water flow through permeable soils is important, even essential, in soil mechanics. Its objectives are:

- ✓ Evaluate the flow rate under dams and hydraulic structures;
- ✓ Determine the shear strength of the soil when there is water flow;
- ✓ To study the problems related to water pumping for underground construction and to perform stability analyses of earth dams and retaining soil structures.

3.2 BASIC CONCEPTS

3.2.1 Concept of the table cloth

All soils are impermeable. Water is free to infiltrate and circulate through the pores of the solid particles, and in some cases accumulate there to form more or less saturated zones. If, in a certain area, water is in a state of low or unsaturated interconnection, it is called a water table.

Two types (categories) of aquifers are distinguished: free aquifer and confined aquifer.

3.2.1.A. The captive aquifer

Groundwater is isolated between two impermeable (fixed) layers, and held under pressure.

3.2.1.B. The loose tablecloth

The level of the free water table can rise or fall depending on climatic conditions.

3.2.1.C. The table cloth

This is the first aquifer located below the topographic surface. This aquifer is of greater interest to builders (geotechnical engineers), as it determines the construction aspects and the cost of the operation (cost of the structure).

3.2.2 Interstitial water (free water and capillary water)

Interstitial water is the dominant factor in soil behavior. Interstitial water exists as free water when the soil is saturated and bathed in a water table.

This water obeys only gravity, flows through the soil, and can be extracted using simple techniques. Subject to the laws of hydraulic flow, it transmits hydrostatic pressure, and under the influence of pressure differences, it can circulate freely.

Interstitial water is in the form of capillary water above the water table. This water is held in equilibrium in the pores under, on the one hand, the action of gravity, and on the other hand, those of the tension forces which develop at the water-air interface.

Depending on the nature of the interstitial water, soils can be classified into two categories:

- ***Unsaturated soils:*** presence of capillary water.
- ***Saturated soils:*** presence of open water.

3.3 LAW OF WATER FLOW IN SOILS

3.3.1 Bernoulli's energy equation

When water flows through a saturated soil mass, there is some resistance to flow due to the presence of solid matter. The flow conditions are shown in tubes equipped with piezometers, above which lies free water (Figure 3.1.). Representing the head variations

with height alongside the tube, shows that the total head is constant in the water. However, when observing the piezometers embedded in the soil, the total head decreases downwards (in the direction of flow). At each elevation Z, the total head remains the sum of the pressure, velocity and elevation heads.

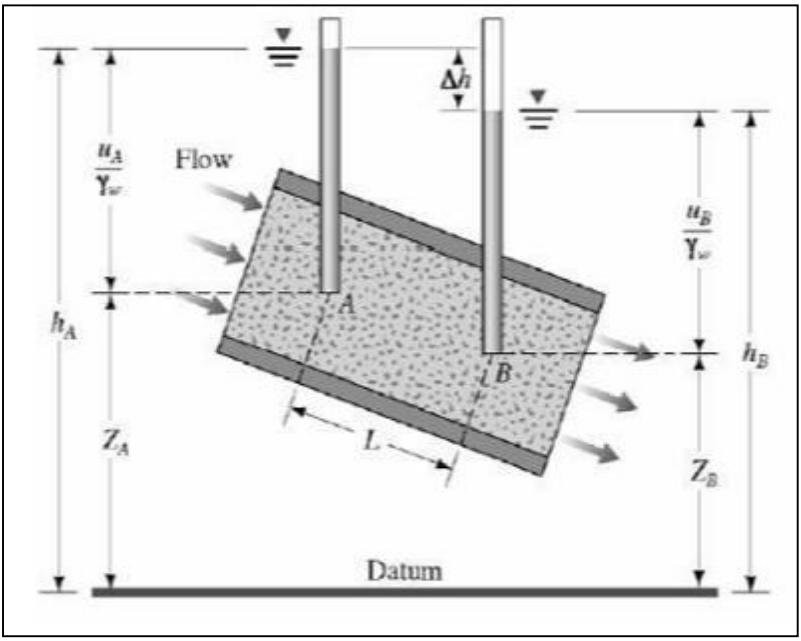


Figure 3.1. The flow of water through a sample of soil of length L

According to Bernoulli's equation, the total hydraulic energy (head) at any point in the water under constant flow conditions can be expressed as follows:

$$\text{Total head} = \text{Pressure head} + \text{Velocity head} + \text{Elevation head}$$

So, the total available hydraulic energy between A and B can be expressed in terms of distance from the reference level:

$$h_A = Z_A + \frac{U_A}{\gamma_w} + \frac{{V_A}^2}{2g} \quad ; \quad h_B = Z_B + \frac{U_B}{\gamma_w} + \frac{{V_B}^2}{2g} \qquad (3.1)$$

Where:

h_A,h_B: The total energy at points A and B.

Z_A, Z_B: The altitudes of the points considered (elevation load or position load) relative to the reference level.

U_A,U_B Water pressures (pressure loads).

γ_w : Specific gravity of water.

$\frac{U_A}{\gamma_w}$ and $\frac{U_B}{\gamma_w}$: Piezometric heights (pressure head).

V_A and V_B : Flow velocity of A and B.

g : Acceleration due to gravity.

$\frac{V_A^2}{2g}$ and $\frac{V_B^2}{2g}$: Called dynamic heights.

In soils, voids are small and the quantities $\frac{V_A^2}{2g}$ and $\frac{V_B^2}{2g}$ can be neglected, so the formulas for hydraulic heads at points A and B are written as:

$$h_A = Z_A + \frac{U_A}{\gamma_w} \quad ; \quad h_B = Z_B + \frac{U_B}{\gamma_w} \qquad (3.2)$$

According to Bernoulli's theorem, the flow between any two points depends only on the total difference in height. Thus:

- If :$h_A = h_B = Cste \Rightarrow$No flow, the water table is in equilibrium.
- If :$h_A > h_B \Rightarrow$There is a flow from A to B.

In the second case, the pressure loss Δh ($\Delta h = h_A - h_B$) is not constant, it is the difference in reference level and corresponds to the energy lost due to the friction of the water on the tube walls or grains.

- ***Noticed:*** *Pressure loss is both the driver and the consequence of the flow.*

3.3.2 Hydraulic gradient

Assuming water flows from A to B in a homogeneous and isotropic soil (Figure 3.2), the head loss is uniform and along the direction AB. its value (magnitude) is given by:

$$i = \frac{h}{l} = \frac{\Delta h}{l} = \frac{h_A - h_B}{l} \qquad (3.3)$$

Where:

i: The hydraulic gradient (head loss per unit length in the direction of flow).

l: This is the distance between A and B (the flow length over which the pressure loss occurred).

An equipotential surface is defined as a surface over which the total hydraulic head is constant: there is no flow along an equipotential surface; the hydraulic gradient vector is normal to the equipotential surface.

A surface over which the water pressure is constant is called an isopiestic surface; in a static aquifer, isopiestic surfaces are horizontal.

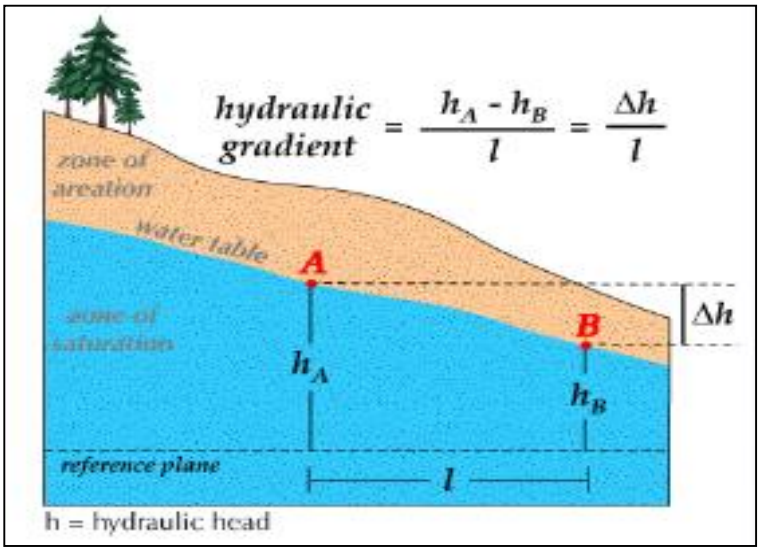


Figure 3.2. Hydraulic gradient definition

3.3.3 Flow characteristics (Darcy's law)

Water percolating through soil grains does not move along linear paths; it bypasses "obstacles." However, if we look at a sufficient scale, we can define an average path (Figure 3.3). Furthermore, the flows only occur through voids, whereas we will be led to consider a quantity of water flowing through a given total cross-section S, results in an apparent average velocity (Qv).

The relationship between flow velocity (flow rate) and hydraulic head was established experimentally in 1856 by Darcy in the case of tubes filled with sand (Figure 3.4).

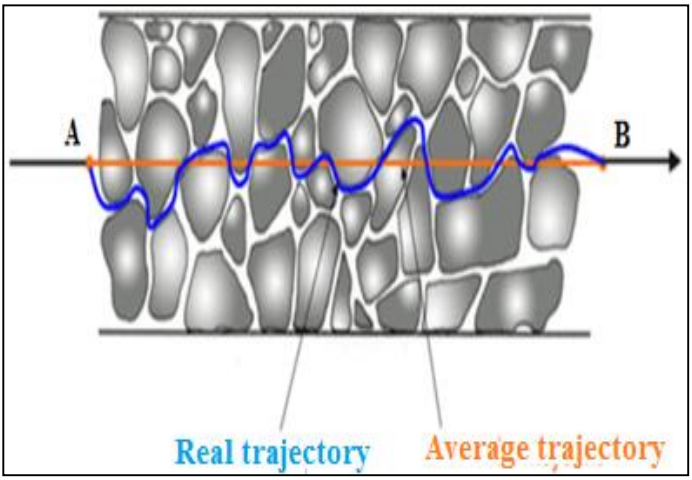


Figure 3.3. Trajectory of water in soil

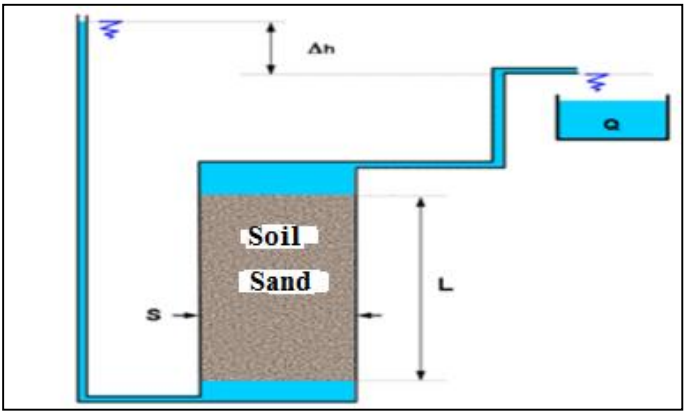


Figure 3.4. Experimental precess corresponding to Darcy's experiments

Darcy showed that, considering given sand, the flow rate in a tube of constant cross-section was proportional to the ratio where is the difference in hydraulic head between the inlet and outlet and the length of the sample considered, i.e.: $\mathrm{i} = \frac{\Delta \mathrm{h}}{\mathrm{l}} \Delta \mathrm{hl}$

$$q = Cste \times S \times i$$

This leads to the elementary form of "Darcy's Law":

$$q = k \times S \times i \quad or \quad v = \frac{q}{s} = k \times i \qquad (3.4)$$

Where:

$q = \frac{Q}{t}$: The flow rate.

t : Flow time.

S : Surface area of the cross-section of the sample.

v : Flow velocity.

i: The hydraulic gradient.

k : Permeability coefficient.

- ***Noticed:*** *Darcy's law is valid only for laminar flows. Laminar flows are established for low speeds: for these flows, the average trajectories of the fluid particles do not intersect.*

If the speed increases, the trajectories of the fluid particles intersect several times; the flow is said to be turbulent.

- ***Noticed:*** *In steady state, time does not come into play.*

3.4 SOIL PERMEABILITY

34.1 Soil permeability coefficient

Permeability is the property of soils that allows water to pass through them at a certain rate. Darcy's law introduces the coefficient of permeability (k), defined as the ratio of flow rate per unit area of

soil under a unit hydraulic gradient; it has the dimensions of velocity: $\left[\frac{m}{s}\right]$, $\left[\frac{cm}{s}\right]$. This quantity, which is used by soil mechanics engineers, depends on several factors such as:

- ✓ The shape and size of soil particles; generally, the smaller the particles, the smaller the pores, and the lower the permeability coefficient.
- ✓ The distribution of soil particles and porous spaces,
- ✓ The void ratio e, permeability, increases with increasing void ratio.
- ✓ The degree of saturation, the permeability, increases with increasing saturation.
- ✓ The structure and the mineralogical nature of the soil particles,
- ✓ The direction of stratification.

Relationships exist between the physical characteristics of the medium and permeability.

For example, the following relationship is interesting from a theoretical point of view but rarely used:

$$k = \frac{e^3}{1+e} \qquad (3.5)$$

The permeability coefficient (k) varies widely for different soils (Table 3.1).

Table 3.1. Indicative values of soil permeability coefficient $\mathbf{k}$ in cm/s

Type of soil	Permeability coefficient $\mathbf{k}$ $\left[\frac{\mathbf{m}}{\mathbf{s}}\right]$	Degree of permeability
Medium to large gravel	$10^{-3} < k < 10^{-1}$	Very high
Small pebbles, sand	$10^{-5} < k < 10^{-3}$	Quite high
Very fine sand, silty sand	$10^{-7} < k < 10^{-5}$	Weak
Compact silt, silty clay	$10^{-9} < k < 10^{-7}$	Very low
Clay	$10^{-12} < k < 10^{-12}$	Virtually waterproof

In the case of uniform and loosely compacted sands, Hazen proposed the following relationship:

$$k = 10^{-2} \times D_{10}{}^{2} \ (D_{10} \text{ en mm et k en } {}^{cm}/_{s})$$

➢ ***Noticed:*** *Permeable soil* $k \uparrow$*:; impermeable soils* $k \downarrow$*:.*

3.4.2 Determination of the permeability coefficient in the laboratory

Two types of equipment are commonly used in the laboratory to determine the soil permeability coefficient (k) and are: the constant head permeameter and the variable head permeameter.

3.4.2.A. Constant-head permeability test

The constant-head permeameter is used for highly permeable soils with a permeability coefficient greater than 10-7m/s (this is the case for sands and gravels). A soil sample is placed in a cylindrical tube of cross-section filled with sand to a given height (Figure 3.5). It is equipped with two piezometers which allow the pressure applied over a path of length to be measured, causing water to flow from top to bottom through the soil.

The quantity of water percolating through the sample during a given time is determined; this gives the unit flow rate : $q = {}^{Q}/_{t}$.

The permeability coefficient k can be calculated using Darcy's equation:

$$k = \left(\frac{Q}{A \times t}\right) \times \left(\frac{l}{h}\right) = \left(\frac{q}{A \times t}\right) \times \left(\frac{1}{i}\right) \qquad (3.6)$$

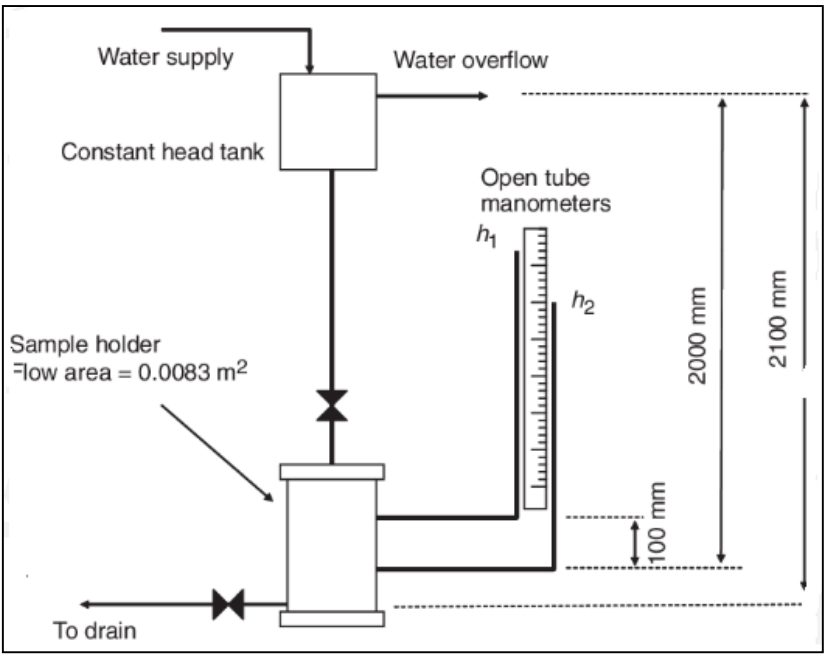


Figure 3.5. Schematic of constant-head permeability test instrument (example)

3.4.2.B. Variable-head (falling-head) permeability test

This test is used in the case of soils with low permeability ($k < 10^{-5} m/s$) and for which the constant head test would be too long given the low flow rate (case of clays for example) .The experimental setup consists of a soil sample of a given cross-section (A) and height (l) placed in a container filled with water to ensure a constant outlet level (Figure 3.6.a). The water is supplied through a small-diameter tube (a). During this test, the water flow through the soil sample is caused by a pressure drop that decreases over time.

During the experiment, the water level in the tube drops. The experiment starts with an initial charge h_0, and the heights h_t are measured over time to plot ($\log h_0/h_t$) a curve as a function of time t (Figure 3.6.b).

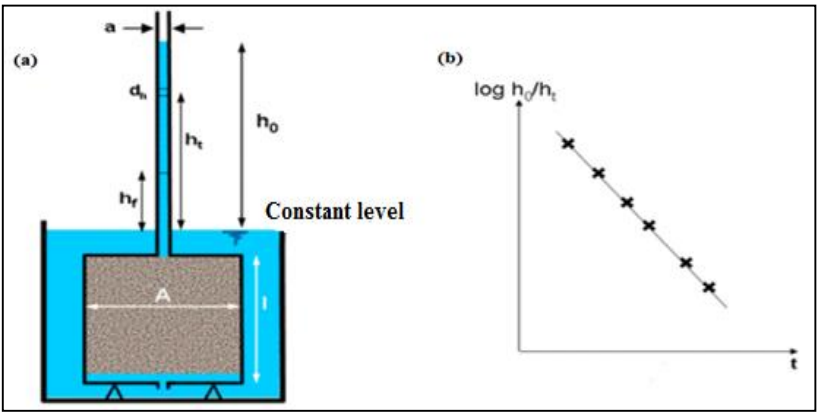


Figure 3.6. Variable head-permeameter: (a) Schematic of the experimental test, (b) Relationship between log (ho/ht) and time

To obtain the permeability coefficient, we consider a time t, with a time interval dt during which the level drops by dh amount in the tube. The instantaneous velocity in the tube is:

$$v = -\frac{dh}{dt}$$

The instantaneous flow rates in the tube and in the sample are respectively:

$$q_1 = a \times v$$
$$= -\frac{dh}{dt}$$
$$\times a \; ; \quad (-: \text{Because at every moment, h decreases})$$

$q_2 = A \times v = A \times k \times i$; $(i = \frac{h}{l})$ (h: Pressure drop from top to bottom)

$q_1 = q_2$, SO :

$$-\frac{dh}{dt} \times a = A \times k \times \frac{h}{l} \Rightarrow kdt = -\frac{a}{A} \times l \times \frac{dh}{h}$$

By integration, we obtain:

$$\int_0^t kdt = -\frac{a}{A} \times l \int_{h_1}^{h_2} \frac{dh}{h} \Rightarrow k = \frac{a \times l}{A \times t} \ln \frac{h_1}{h_2}$$

$$k = 2{,}3 \frac{a \times l}{A \times t} \log \frac{h_1}{h_2} \qquad (3.7)$$

Where h_1 and h_2 : The difference in charge over time t_1 and t_2 respectively.

- ***Noticed****: Precautions must be taken to ensure that the degree of saturation is equal to 100% ($Sr = 100\%$) (for both permeameters).*
- ***Noticed:*** *In general, and if possible, on site measurement methods are preferable.*

3.4.3 Permeability of stratified soils

In the case of stratified soils, each layer has its own permeability which influences the overall permeability of the soil mass. The permeable soil of total thickness H is composed of (n) successive layers of thickness $(h_1, h_2, \ldots . h_n)$ and permeability coefficient $(k_1, k_2, \ldots . k_n)$ respectively (Figure 3.7).

The average permeability value or the equivalent permeability can be determined in the horizontal kh_e or vertical kv_e direction, depending on the imposed hydraulic conditions.

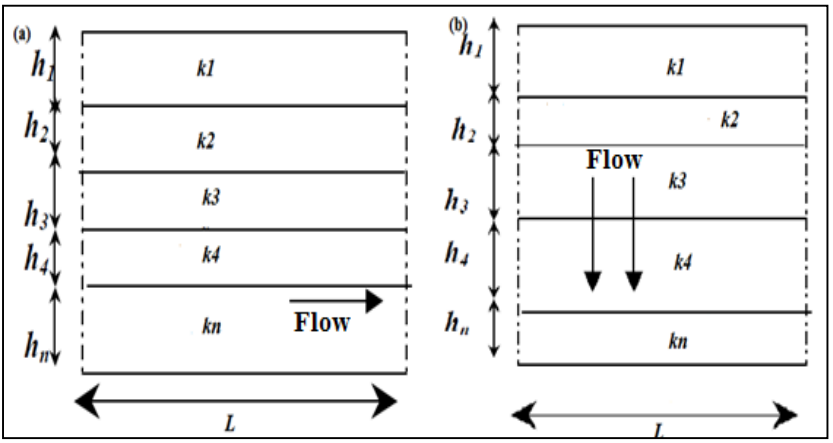


Figure 3.7. Permeability in stratified soils: (a) Horizontal flow, (b) Vertical flow

3.4.3.A. Case of horizontal flow

If the flow is horizontal, the flow velocities will be different in each layer and proportional to the permeability of each layer.

In this case, the hydraulic gradient is the same across each layer i.

$$I = i_1 = i_2 = i_3 = \cdots = i_n.$$

The equivalent permeability coefficient in the horizontal direction kh_e is equal to:

$$kh_e = \frac{\sum_{i=1}^{n} k_i \cdot h_i}{\sum_{i=1}^{n} h_i} = \frac{\sum_{i=1}^{n} k_i \cdot h_i}{H} \qquad (3.8)$$

3.4.3.B. Case of a vertical flow

If the flow is vertical, the flow velocities (flow rates) will be identical in each layer, but the head loss (energy expenditure) will be greater in the layers with lower permeability.

$$Q_1 = q_2 = q_3 = \cdots = q_n$$

The equivalent permeability coefficient in the vertical direction kv_e is equal to:

$$kv_e = \frac{\sum_{i=1}^{n} h_i}{\sum_{i=1}^{n} \frac{h_i}{k_i}} \qquad (3.9)$$

- ***Noticed**: In all cases we have: $kh_e \gg kv_e$; remember that $kh_e = 10kv_e$ is an order of magnitude commonly encountered in a stratified deposit.*

3.4.4 Determination of soil permeability in-situ

In-situ testing is often recommended for non-cohesive soils. Numerous useful methods exist for determining soil permeability in situ. The average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing well pumping tests, which are generally recommended for major projects such as dams, bridges, etc.

3.4.4.A. Case of an unconfined (loose tablecloth) aquifer (without leakage)

Consider a case where the top layer is permeable, and its hydraulic conductivity is to be determined, resting on an impermeable layer (Figure 3.8). During the test, water is pumped at a constant rate from a test well equipped with a perforated casing. Several observation wells at different radial distances are constructed around the test well. Continuous observations of the water level in the test well and observation wells are made after pumping begins until a steady state is reached. Steady state is established when the water level in the test and observation wells becomes constant.

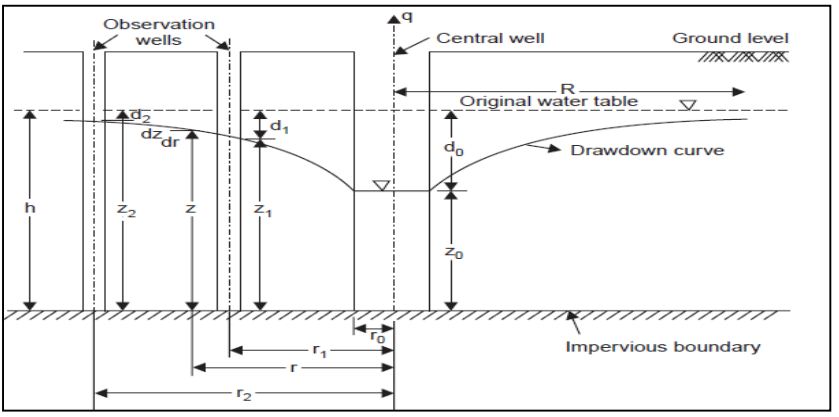


Figure 3.8. Unconfined well pumping test

The expression for the groundwater flow rate in the well, which is equal to the pumping discharge rate, can be written as:

$$q = k \times \left(\frac{dh}{dr}\right) \times 2\pi \times r \times h \Rightarrow \frac{dr}{r} = \frac{2\pi \times k}{q} \times (hdh)$$

So:

$$\int_{r_1}^{r_2} \frac{dr}{r} = \left(\frac{2\pi \times k}{q}\right) \int_{h_1}^{h_2} hdh \Rightarrow k$$

$$= \frac{2.303 \times q \times \log_{10}\left(\frac{r_1}{r_2}\right)}{\pi \times (Z_1^2 - Z_2^2)} \quad (3.10)$$

From field measurements, if q, r_1, r_2, Z_1, Z_2 are known, the hydraulic conductivity can be calculated from the simple relationship.

3.4.4.B. Case of a confined (captive) aquifer

The average hydraulic conductivity of a confined aquifer can also be determined by performing a pumping test from a well (artesian well) with a perforated casing that penetrates to the full depth of the aquifer. The piezometric level is observed in a number of observation wells at different radial distances (Figure 3.9). Pumping is continued at a uniform speed q until a steady state is reached.

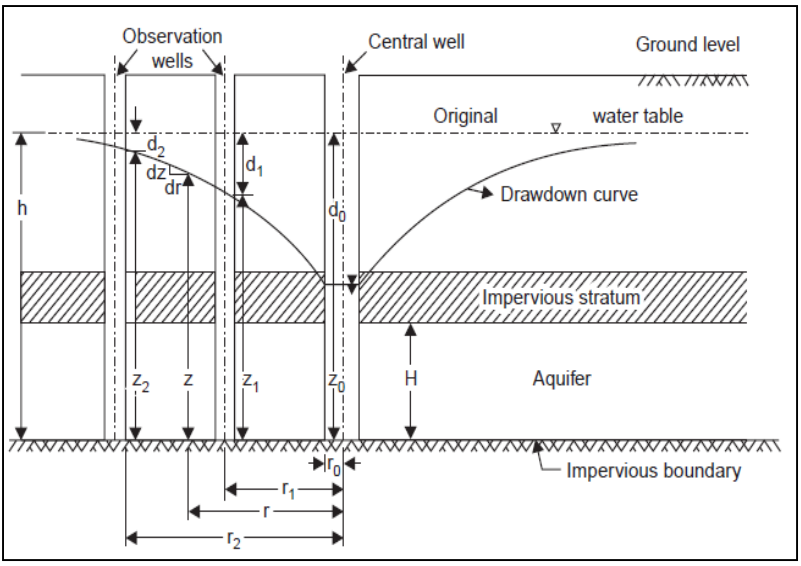


Figure 3.9. Well pumping test for a confined aquifer

Since water can only enter the test well from the aquifer of thickness$H$, the constant discharge state is:

$$q = k \times \left(\frac{dh}{dr}\right) \times 2\pi \times r \times H \Rightarrow \frac{dr}{r} = \frac{2\pi \times k \times H}{q} \times dh$$

So:

$$\int_{r_1}^{r_2} \frac{dr}{r} = \left(\frac{2\pi \times k \times H}{q}\right) \int_{h_1}^{h_2} dh \Rightarrow k$$

$$= \frac{q \times \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727 \times H \times (Z_1 - Z_2)} \quad (3.11)$$

➢ ***Noticed:*** *An aquifer is an underground geological formation capable of storing and supplying water.*

3.5 WATER FLOW

3.5.1 Mechanical effects of water flow

Consider the experimental setup: container (A) contains a layer of sand held at its base by a permeable wire mesh (Figure 3.10a). The sand layer has a thickness of H, and the water level in the container is at a height H_1, measured above H. Container A is connected to a container (B) that can be moved up or down.

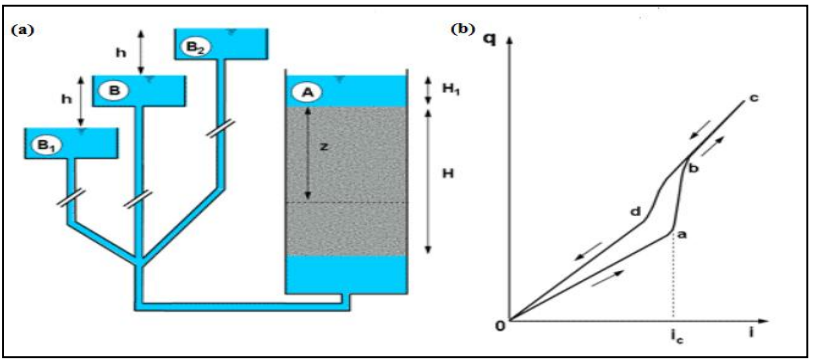


Figure 3.10. Mechanical effect of water on soil: (a) Experimental setup used to demonstrate flow forces in soil, (b) Gradient-discharge relationship

Initially, the water level is the same in A and B: the water is at rest. When container (B) is moved downwards (B1) by a certain distance h, a downward vertical flow is established in (A), and conversely, when container (B) is moved upwards (B2), an upward vertical flow occurs in A. These flows will cause a change in the hydraulic gradient.

In the case of upward vertical flow, it is of course observed that initially up to point a, the flow rate is proportional to the gradient (Oa) (Figure 3.10.b). If we have initially put in place a compacted sand, we observe at point a that the gradient has reached the value of the critical gradient i_c:

$$i_c = \frac{\gamma'}{\gamma_w} \qquad (3.12)$$

Where:

$\grave{\gamma}$:The volumetric weight of the dewatered soil. γ_w : The specific weight of water.

Beyond this point, we find a linear variation bc of the flow rate as a function of the gradient. If we now lower side B, the gradient decreases; initially, from c to b, we observe the same law variation. Beyond point b, the sand being disorganized, a slightly lower gradient value is required to obtain a straight section dO with a slope greater than Oa because the unit weight has decreased. When we reach point a, bubbling occurs on the surface of the sand. This phenomenon is called "piping/heave" or "internal erosion/piping" phenomenon and also called "renard's" (in French).

This phenomenon of the mechanical action of water flow on soil is widespread regardless of the gradient's orientation. Its effect is frequently encountered in problems of slope stability, excavation stability, or leaks through a dam, where it can lead to internal erosion.

Naturally, when this occurs at the bottom of an excavation, very significant damage can occur.

To combat this phenomenon, it is necessary to control the upward vertical gradients. In the area where a sand trapping event is likely to occur, an overload can be installed using a filter that helps maintain the sand in equilibrium.

- ***Noticed:*** *The value of the critical hydraulic gradient is very often close to 1 because the average of the saturated volumetric weights of soils is 20 kN/m³.*

3.5.2 Flow net

The purpose of constructing flow networks is to know, at every point in the soil mass, the hydraulic head, the velocity, and to determine the values of the flow forces or the flow rates passing through a given section.

In soil subjected to permanent flow, and if the soil has a constant volume, the combination of Darcy's law and the continuity condition allows us to write:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \qquad (3.13)$$

With :

$$V_x = -k\frac{\partial h}{\partial x} \; ; \; V_y = -k\frac{\partial h}{\partial y} \; ; \; V_z = -k\frac{\partial h}{\partial z}$$

Either :

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

The differential equation that the hydraulic head satisfies is Laplace's equation.

In many practical cases, we will consider a two-dimensional flow; this equation then reduces to:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (3.14)$$

At the boundaries, either flow conditions or hydraulic head conditions are imposed. The goal is to determine a function $h(x, y)$ satisfying Laplace's equation.

A drainage network consists of two types of lines:

- equipotential lines (h = constant).
- the power lines.

➢ *Noticed: The graphical method is an approximate solution, but sufficient in our educational case.*

➢ ***Noticed:*** *These two lines are orthogonal in the case of a homogeneous soil.*

We will ensure that the equipotential lines are drawn so that the head loss between two successive equipotential lines is constant. In the simple case of flow in a tube, the streamlines will be parallel to the tube walls and the equipotential lines will be perpendicular

(Figure 3.11). Since the soil is homogeneous, the head loss will be proportional to the distance traveled.

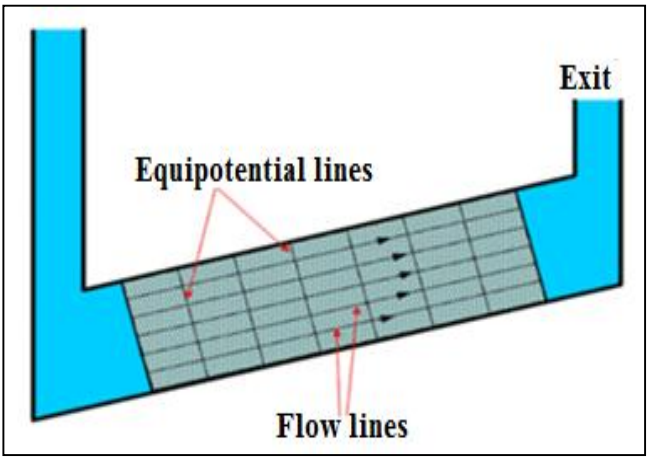


Figure 3.11. Equipotential and flow lines in a soil tube

3.5.3 Construction of a flow net

We will distinguish between confined flows, for which the water table (upper surface of the water in the soil) is known (e.g., flow under sheet piles), and unconfined flows, in which the water table is initially unknown and must be determined by construction or calculation (e.g., the case of flow through an earth dam).

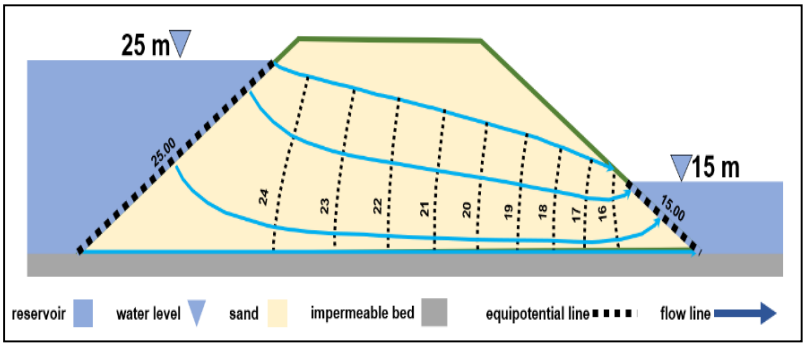


Figure 3.12. Constructing of flow net in the case of earth dam

The rules to be applied are as follows (Figure 3.12):

- ✓ equipotential lines and current lines are perpendicular;
- ✓ Shape of the basic elements: the intersection of the two lines is represented by a flow network of square elements (Figure 3.11); each element is a curvilinear square of dimension $\frac{b}{l} \cong 1$;
- ✓ a flow line cannot cross another flow line;
- ✓ an equipotential line cannot cross another equipotential line;
- ✓ impermeable boundaries: these are streamlines, example of a clay layer or the base of a concrete dam;
- ✓ Permeable boundaries: when a permeable soil is in contact with the outside water level, this boundary constitutes an equipotential line: for example, infiltration into the soil upstream of a dam or through an earth dam. It should be noted that this phreatic line (surface) is not known a priori: it can be constructed as will be seen in earth dams for example;
- ✓ Contact between two soils of different permeability: changes in direction of streamlines.

Figure 3.13 shows drainage net construction in the case of a partial watertight cut with soil of homogeneous and isotropic permeability (Figure 3.13). In this example, the flow is restricted to a layer of sand.

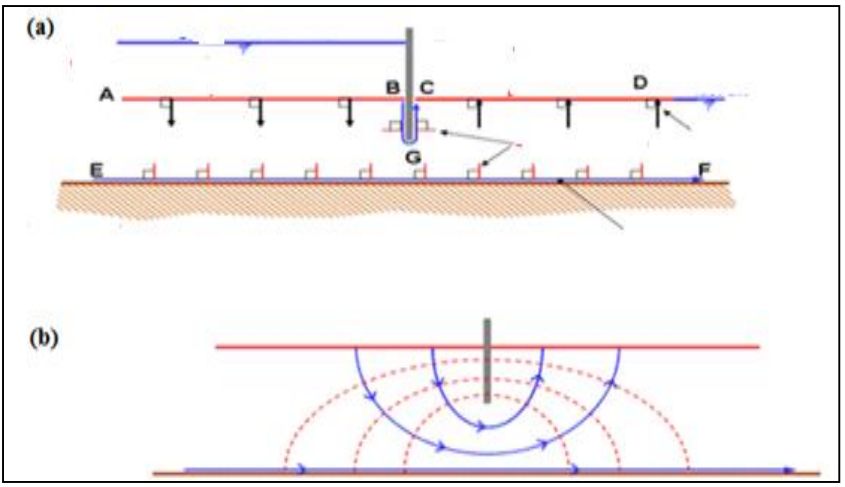


Figure 3.13. Example of graphical construction of a flow net: (a) Initial conditions, (b) Sketch of some streamlines and equipotential lines

The conditions imposed are as follows (Figure 3.13.a):

✓ Line AB : maximum equipotential ;

✓ CD line : minimum equipotential ;

✓ EF line: waterproof base: longest current line;

✓ BGC line: vertical break: shortest current line.

Based on these initial indications, we can note that the current lines are perpendicular to AB at the input and perpendicular to CD at the output. As for the equipotential lines, they are perpendicular to EF and BGC. From these initial observations, we can sketch the network and, respecting the construction rules defined above, complete it (Figure 3.13.b). Once the network is constructed, it can be used (with a two-dimensional net, we consider elements of unit

width). If we denote the total number of flow "tubes" and the number of equipotential lines crossed, we can calculate the total flow rate through the device:$n_f n_d$

$$Q = k \times H \times \frac{n_f}{n_d} \qquad (3.15)$$

Where:

k : Horizontal permeability coefficient.

H : Difference in total charge between the first and last equipotential lines.

$\frac{n_f}{n_d}$: Form factor.

Currently, and following the spectacular progress of numerical methods for solving the differential equations governing many problems in mechanics (including flow equations), the problems are solved by the finite element and difference methods using computer programs.

3.6 EXERCISES

Exercise 3.1

Establish the relationship between degree of saturation, soil moisture content, specific gravity of soil particles, and void ratio. The discharge of water collected from a constant head-permeameter in a period of 15 minutes is 500 ml.

The internal diameter of the permeameter is 5 cm and the measured difference in head between two gauging points 15 cm vertically apart is 40 cm. Calculate the coefficient of permeability.

If the dry weight of the 15 cm long sample is 4.86 N and the specific gravity of the solids is 2.65, calculate the seepage velocity.

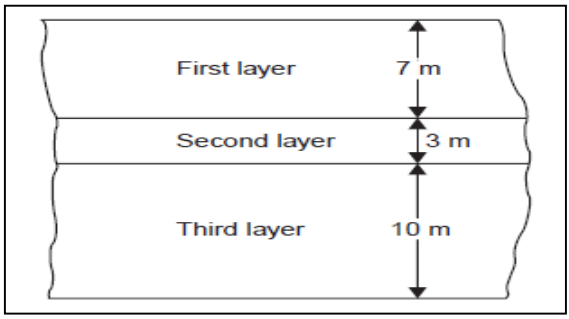
Exercise 3.2

A glass cylinder 5 cm internal diameter and with a screen at the bottom was used as a falling (variable) head-permeameter. The thickness of the sample was 10 cm. With the water level in the tube at the start of the test as 50 cm above the tail water, it dropped by 10 cm in one minute, the tail water level remaining unchanged.

Calculate the value of k for the sample of the soil. Comment on the nature of the soil.

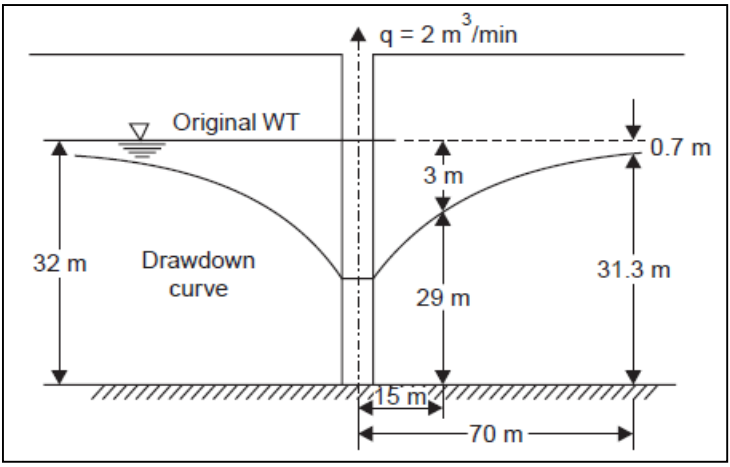
Exercise 3.3

A horizontal stratified soil consists of three layers each uniform initself. The permeabilities of these layers are 8×10^{-4} cm/s, $52\times10^{-}4$ cm/s, and 6×10^{-4} cm/s, and their thicknesses are 7, 3 and 10 m respectively. Find the effective average permeability of the deposit in the horizontal and vertical directions.



Exercise 3.4

An unconfined aquifer is known to be 32 m thick below the water table. A constant discharge of 2 cubic metres per minute is pumped out of the aquifer through a tube well till the water level in the tube well becomes steady. Two observation wells at distances of 15 m and 70 m from the tube well show falls of 3 m and 0.7 m respectively from their static water levels. Find the permeability of the aquifer.


q = 2 m3/min
Original WT
0.7 m
3 m
32 m
Drawdown
curve
31.3 m
29 m
15 m
70 m

CHAPTER 4 :

STRESSES AND DEFORMATIONS IN SOIL

4.1 INTRODUCTION

In soils multiphase systems, the voids between particles are continuous and are occupied by water and/or air, where it is obviously impossible to track the forces at every particles point contact. To analyze embankment stability, foundation bearing capacity, compressibility and lateral pressure on earth retaining structures, it is necessary to understand the nature of these forces acting in soil. Furthermore, it is essential to use stress notion to determine the pressure distribution within a soil profile. The development of these stresses leads to deformations in the soil mass.

4.2 CONCEPTS OF STRESS AND DEFORMATION

4.2.1 Notions of stress

A stress is an (internal) force per unit area exerted within an element (material, soil, etc...). Therefore:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \qquad (4.1)$$

Consider a solid Ω subjected to forces (concentrated or distributed) (Figure 4.1.a). The solid is cut into two sections S_1 and S_2 (Figure 4.1.b). The resultant of the internal forces gives rise to two stresses: σ (normal to the plane of the facet) an dτ (tangential to the facet dS). The total stress C at point M is given by:

$$C = \sigma + \tau \qquad (4.2)$$

The SI unit of stress is the Pascal (1Pa=1 N/m^2), being very small compared to the stresses usually encountered, the megapascal and the daN/mm^2 are traditionally used with the equivalence/

$1daN/mm^2$=1 MPa=10^6 Pa=$1N/mm^2$.

- ***Noticed:*** *stress is a calculation tool; we cannot observe it directly, however we can observe its effects: studies of deformations for example.*

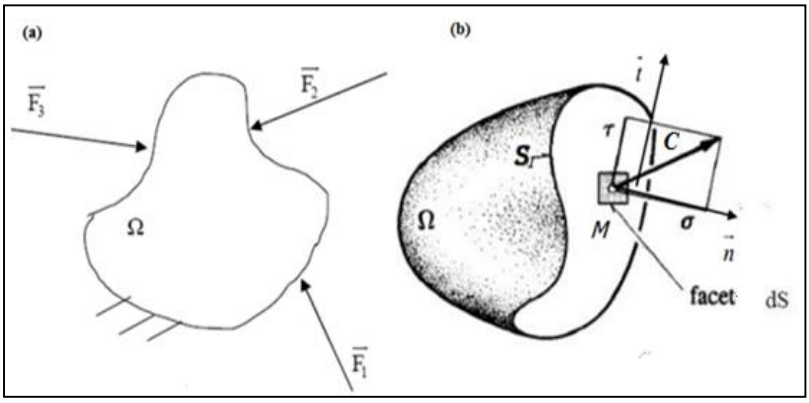


Figure 4.1. The concept of the stress

4.2.2 Stress tensor

Consider an infinitesimally small rectangular parallelepiped isolated around a point (Figure 4.2). It is in elastic equilibrium. Each face of the parallelepiped is subjected to a normal stress and two shear stresses. For example: on the face X (direction X), normal stress σ_X and two tangential stress τ_{xy} and τ_{xz} exerted.

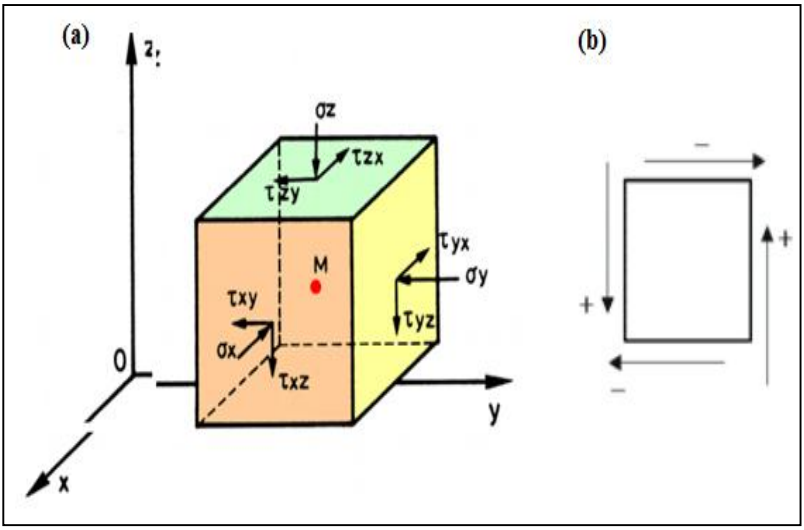


Figure 4.2. Stress parallelepiped: (a) Stress state at a point M, (b) Signs of shear stresses

The state of stress at a point M is characterized by a symmetric square matrix of order three (stress tensor), which is written as:

$$[\sigma]_{(M,x,y,z)} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \qquad (4.3)$$

Where :$\tau_{xy} = \tau_{yx}$,

$\tau_{xz=\tau_{zx}}$and $\tau_{yz} = \tau_{zy}$ (symmetric matrix)

Since the primary cause of stress in soils is Earth's gravity, creating downward vertical forces, it is intuitively understood that tensile stresses are generally rare or even absent.

According to the sign convention in classical continuum mechanics, in soil mechanics, it is was adopt the following signs:

— Positively, the normal stresses σ, corresponding to a compression (resulting in a shortening),

— Negatively, normal stresses σ, corresponding to tension (resulting in elongation),

— Shear stresses are positive when they induce on a soil element torques acting in the counterclockwise direction (Figure 4.2.b).

- ***Noticed:*** *All normal and tangential stresses of the element shown in figure 4.2 are positive (convention signs for soil mechanics).*
- ***Noticed:*** *The sign of the tangential constraint has no intrinsic meaning (the sign depends on the position of the observer).*

4.2.3 Principal stresses

By varying (rotating) the axes (x, y and z) at an angle θ, the normal and shear stresses vary. There is always a position of the axes such that there are only normal stresses on the facets perpendicular to these axes. For given values of the stress matrix, three values of θ will be given, separated by 90°. As a result, there are three planes perpendicular to each other on which the shear stress is zero. These planes are called principal planes (principal directions). The normal stresses acting on the principal planes are called principal stresses (Figure 4.3).

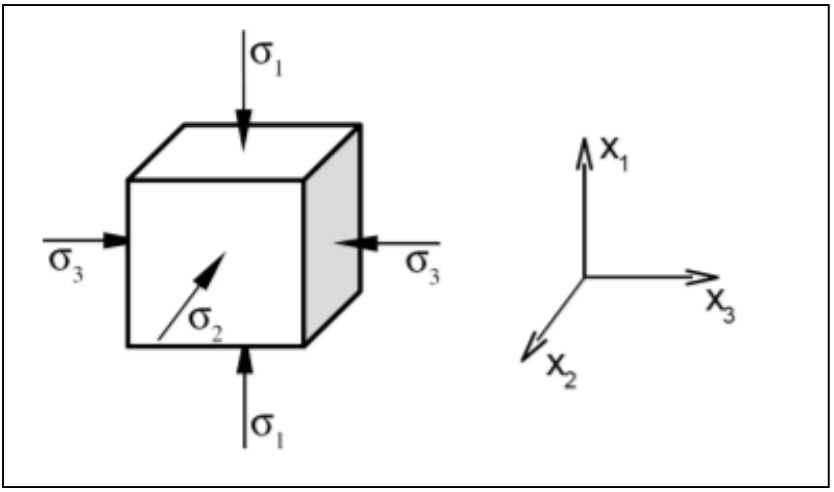


Figure 4.3. Principal stresses in three dimensions

Analytically, the principal stresses are the the eigenvalues of the stress matrix. Consider the three-dimensional state at a point M given by the matrix:

$$[\sigma]_{(M,x,y,z)} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

The eigenvalues of the matrix are the solutions of the following equation:

$$\mathrm{Det}(\sigma - \lambda I_d) = 0 \qquad (4.4)$$

Either :

$$\mathrm{Det}(\sigma - \lambda I_d) = \begin{pmatrix} \sigma_x - \lambda & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \lambda & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \lambda \end{pmatrix} = 0 \qquad (4.5)$$

The solution to the characteristic equation gives:

$$\lambda_1, \lambda_2 \text{and } \lambda_3 \Rightarrow \sigma_1, \sigma_2 \text{ and } \sigma_3 \text{ Principal stresses}$$

The largest principal constraint is called the major principal constraint (σ_1), the intermediate constraint is denoted (σ_2), and the smallest principal constraint is called the minor principal constraint (σ_3) ($\sigma_1 > \sigma_2 > \sigma_3$).

Isotropic soils are those that have similar properties at a given location in all planes and directions. The stress state at a given point is therefore entirely defined solely by the major and minor principal stresses σ_1 and σ_3 (therefore, $\sigma_2$they will no longer be mentioned hereafter).

To determine the values of the principal stresses and the directions of the principal planes, graphical method (Mohr's circle) is widely used.

4.2.4 Plane stress in a soil element

In soil mechanics, most cases are 2D (planar) problems. The planar stress state is the special case of a single volume facet where a normal stress and a shear stress act on each side (Figure 4.4.a ($\sigma_y > \sigma_x$).

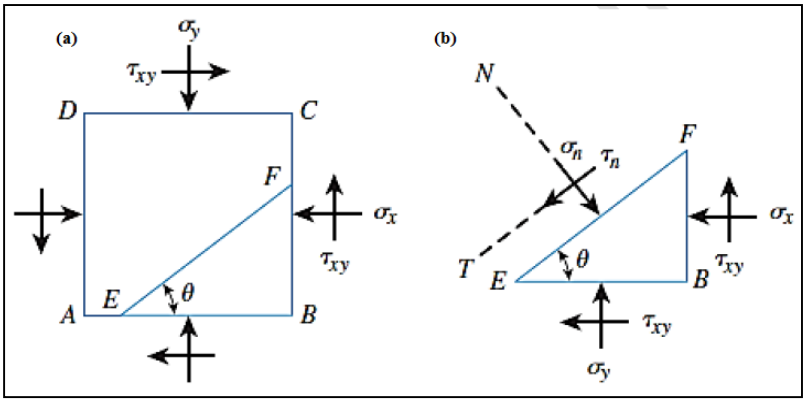


Figure 4.4. Stresses acting on a soil element: (a) Element under plane stress, (b) Diagrams of the element under normal and tangential stress

4.2.4.A. Plane stress equation

To determine the normal stress and shear stress on a plane EF which makes an angle θ with the plane AB, the free body diagram EFB is shown (Figure 4.4.b).

Let σ_n and τ_n the normal stress and the shear stress, respectively, acting on the plane EF. From the geometry, we write:

$$\overline{EB} = \overline{EF}\cos\theta \;\; et \;\; \overline{FB} = \overline{EF}\sin\theta$$

Adding the components of the forces acting on the element in the directions of N and T gives:

$$\sigma_n(\overline{EF}) = \sigma_x(\overline{EF})\sin^2\theta + \sigma_y(\overline{EF})\cos^2\theta + 2\tau_{xy}(\overline{EF})\sin\theta\cos\theta$$

$$\text{ou} \;\; \sigma_n = \sigma_x\sin^2\theta + \sigma_y\cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\text{ou} \;\; \sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Again :

$$\tau_n(\overline{EF}) = -\sigma_x(\overline{EF})\sin\theta\cos\theta + \sigma_y(\overline{EF})\sin\theta\cos\theta - \tau_{xy}(\overline{EF})\cos^2\theta + \tau_{xy}(\overline{EF})\sin^2\theta$$

$$\text{or } \tau_n = \sigma_y \sin\theta\cos\theta - \sigma_x \sin\theta\cos\theta - \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tau_n = \left(\frac{\sigma_y - \sigma_x}{2}\right) \times \sin 2\theta - \tau_{xy}\cos 2\theta$$

4.2.4.B. Principal stresses in the plan state

Consider the two-dimensional state given by the matrix:

$$[\sigma]_{(x,y)} = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \qquad (4.6)$$

The eigenvalues of this matrix are the solutions of the following characteristic equation:

$$\mathrm{Det}(\sigma - \lambda I_d) = 0$$

Either :

$$\mathrm{Det}\begin{pmatrix} \sigma_x - \lambda & \tau_{xy} \\ \tau_{yx} & \sigma_y - \lambda \end{pmatrix} = 0 \qquad (4.7)$$

The solution of the characteristic equation gives the principal constraints σ_1 and σ_3.

The values of the principal constraints are also given directly by the following formulas:

$$\sigma_{1.3} = \frac{\sigma_{x+}\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2} \qquad (4.8)$$

In one main direction, the stress matrix is given as follows:

$$[\sigma]_{(X_1,Y_1)}\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_3 \end{pmatrix} \qquad (4.9)$$

The principal stresses act on the prncipal planes. For example, the main stress σ_1 acts on a principal plane (direction) X_1 such that:

$$\theta = \left(\overrightarrow{x}, \overrightarrow{X_1}\right) ; \ \tan(\theta) = \frac{\sigma_1 - \sigma_x}{\tau_{xy}}; ; \ \tan(2\theta) = \frac{\tau_{xy}}{\sigma_y - \sigma_x}$$

➢ ***Noticed:*** *In a soil mass, the maximum stress s* σ_1 *often refers to vertical stress, while* σ_3 *refers to horizontal stress.*

4.2.4.C. Graphical method (Mohr's circle) in two dimensions

The normal stress and the shear stress acting on any plane can also be determined by drawing a Mohr's circle (Figure 4.5).

The sign conventions used in Mohr's circles are the same as previously used; the angle θ is positive when measured counterclockwise from the principal plane.

For plane AD, the normal stress is equal to $+\sigma_x$ and the shear stress is equal to $+\tau_{xy}$ (Figure 4.5). Considering the plane AB, the normal stress is equal to $+\sigma_y$ and the shear stress is $-\tau_{xy}$.

Points R and M represent, respectively, the stresses on planes AD and AB (Figure 4.5). O is the point of intersection of the stress normal axis with line RM. The circle MNQRS drawn with O as its center and OR as its radius is the Mohr circle for the considered stress conditions.

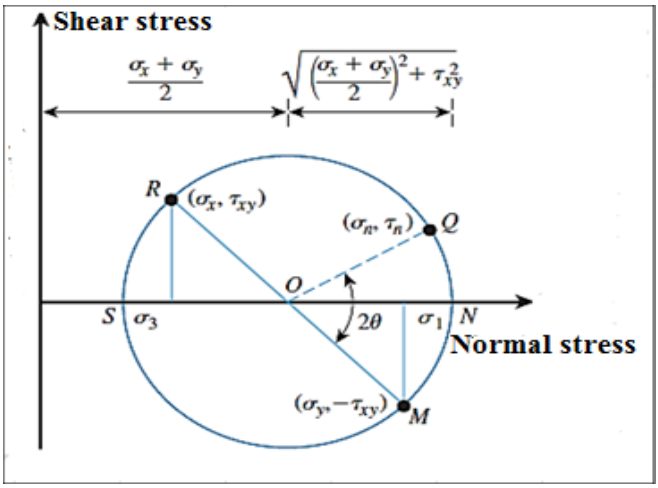


Figure 4.5. Representation of the plan stress state (Mohr's circle)

The radius of Mohr's circle (the maximum shear stress) is:

$$R = \tau_{max} = \sqrt{\left[\frac{\sigma_{y-}\sigma_x}{2}\right]^2 + \tau_{xy}^2} \qquad (4.10)$$

The stress on plane EF can be determined by moving an angle 2θ a twice the counterclockwise angle that plane EF makes with plane AB from point M along the circumference of Mohr's circle to point Q. The x-coordinate and y-coordinate of point Q, respectively, give the normal stress σ_n and shear stress $+\tau_n$ on plane EF. The x-coordinate of point N is equal to σ_1, and the x-coordinate of point S is σ_3.

In a specific case, if planes AB and AD were major and minor principal planes, the normal stress and shear stress on plane EF could be found by substituting: $\tau_{xy} = 0$.

The particular stress state for the case $\sigma_y = \sigma_1$ and $\sigma_x = \sigma_3$ is plotted (Figure 4.6).

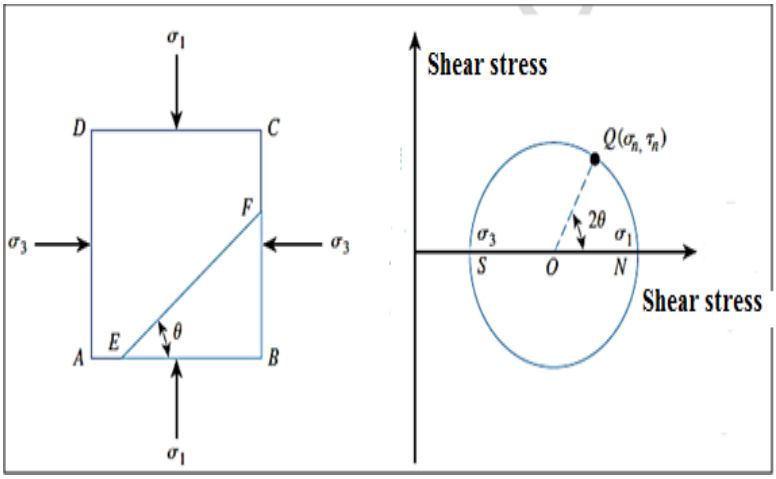


Figure 4.6. Special state $\sigma_y = \sigma_1$ and $\sigma_x = \sigma_3$: (a) Stresses acting on a soil element, (b) Mohr's circle corresponds

4.2.5 State of deformation

In general, when materials (soils in particular) are subjected to stress, deformations occur. These soil deformations affect structures and can cause problems that jeopardize the proper use, or even the safety, of the structures. These issues are encountered, for example, in the case of building foundations or civil engineering structures, but also for embankments and earthworks in general.

In general, soil deformations can be attributed either to deformations of the individual components or to deformations of the soil structure. However, apart from air, the other soil components (grains and water) are generally considered practically

undeformable. Consequently, in the case of saturated soils, this means that deformations can only occur through the release or release of pore water.

4.2.5.A. Strain tensor

In an element subjected to forces, the normal stresses σ_x, σ_y and σ_z cause, respectively, variations in length (relative shortenings) in the directions x, y and z denoted $\varepsilon_x, \varepsilon_y$ and ε_z. The shear stresses τ_{xy} and τ_{xz} cause sliding (rotations or changes in angle) of the facets of the element, denoted: Υ_{xy} and Υ_{xz}. By analogy with the stress tensor, the strain tensor at a point M is written as follows:

$$[\varepsilon]_{(M,x,y,z)} = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\Upsilon_{xy} & \frac{1}{2}\Upsilon_{xz} \\ \frac{1}{2}\Upsilon_{yx} & \varepsilon_y & \frac{1}{2}\Upsilon_{yz} \\ \frac{1}{2}\Upsilon_{zx} & \frac{1}{2}\Upsilon_{zy} & \varepsilon_z \end{pmatrix} \qquad (4.11)$$

Where: $\Upsilon_{xy} = \Upsilon_{yx}$ and $\Upsilon_{xz} = \Upsilon_{zx}$ (equilibrium condition).

In the plan case adopted in soil mechanics, the tensor at a point M is written as follows:

$$[\varepsilon]_{(M,x,y)} = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\Upsilon_{xy} \\ \frac{1}{2}\Upsilon_{yx} & \varepsilon_y \end{pmatrix} \qquad (4.12)$$

4.2.5.B. Deformation theory

The theory of deformation is entirely analogous to the theory of stress (they have both principal directions and principal deformations). The analogy is defined by:

$\varepsilon \rightarrow \sigma$ and $\frac{1}{2}\Upsilon \rightarrow \tau$

Therefore, we can determine the principal deformations ($\varepsilon_z > \varepsilon_x$) by:

$$\varepsilon_{1,3} = \frac{\varepsilon_z + \varepsilon_x}{2} \pm \sqrt{\left(\frac{\varepsilon_z - \varepsilon_x}{2}\right)^2 + \left(\frac{\Upsilon_{zx}}{2}\right)^2} \qquad (4.13)$$

And the orientation of plane deformations by:

$$\tan(2\theta) = \frac{\Upsilon_{zx}}{\varepsilon_z - \varepsilon_x} \qquad (4.14)$$

And the maximum angular deformations by:

$$\frac{\Upsilon_{max}}{2} = \sqrt{\left(\frac{\varepsilon_z - \varepsilon_x}{2}\right)^2 + \left(\frac{\Upsilon_{zx}}{2}\right)^2} \qquad (4.15)$$

4.3 STRESS-DEFORMATION RELATIONSHIPS

In soil mechanics, a constitutive law is a mathematical relationship established between the stress tensor and the strain tensor. Here, we will limit our analysis to small deformations. Materials assumed to be continuous, homogeneous, and isotropic. We will also assume that stresses are proportional to strains. This is referred to as the elastic domain or linear soil behavior.

4.3.1 Uniaxial compression state (linear Hooke's law)

$$[\sigma]_{(M,x,y,z)} = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{pmatrix} \qquad (4.16)$$

Under the effect of compressive stress sσ_x, a unit shortening $\varepsilon_x$is assumed. Assuming that we remain within the linear elastic range, there is then a proportional relationship between σ_x and ε_x, namely: $\sigma_x = -E \times \varepsilon_x$ (Hooke's law).

Where E: is a characteristic of the soil called Young's modulus, usually expressed in GPa.

The minus sign was introduced to take into account the sign convention for stresses in soil mechanics.

Under the effect of unit reduction ε_x, increases in lengths in directions orthogonal to x, i.e., the directions y and z , product. We assume that: $\varepsilon_y = -\nu \times \varepsilon_x$.

Where : ν is the second elastic constant of the material, called Poisson's ratio (ν is dimensionless). Poisson's ratio reflects the compressibility of the material. It can be shown that an incompressible material has a Poisson's ratio of 0.5 and a perfectly compressible material has a Poisson's ratio of 0, thus:

$$0 \leq \nu \leq 0.5$$

We can apply the principle of superposition, we find that:

$$\varepsilon_x = -\frac{1}{E} \times [\sigma_x - \nu(\sigma_y + \sigma_z)] \qquad (4.16a)$$

$$\varepsilon_y = -\frac{1}{E} \times \left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \qquad (4.16b)$$

$$\varepsilon_z = -\frac{1}{E} \times \left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \qquad (4.16c)$$

In the plan case, we find:

$$\varepsilon_x = -\frac{1}{E} \times \left[\sigma_x - \nu(\sigma_y)\right] \qquad (4.17a)$$

$$\varepsilon_y = -\frac{1}{E} \times \left[\sigma_y - \nu(\sigma_x)\right] \qquad (4.17b)$$

These latter relationships express Hooke's law for a soil compression state.

4.3.2 Shear state (Hook's law in shear)

Assuming that we remain within the linear elastic domain, we assume a proportional relationship between the shear stress τ_{xy} and the sliding (rotation) Υ_{xy}:

$$\tau_{xy} = G \times \Upsilon_{xy} \qquad (4.18)$$

Where : G is The shear modulus.

This last relationship expresses Hooke's law for a shear state.

4.3.3 Variation in soil volume

The volume of a small elementary soil can increase if its length increases, or its width increases, or its height increases. The sum of the deformations in the three coordinate directions called the total volumetric deformation ε_{vol}. Therefore:

$$\varepsilon_{vol} = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \qquad (4.19)$$

Where: ε_x, ε_x, and ε_z are the principal deformations.

The negative value of ε_{vol} describes the compression of the soil.

Isotropic stress σ_0 is the average normal stress, given by the following relationship:

$$\sigma_0 = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \qquad (4.20)$$

In soil, volume changes are primarily determined by changes in isotropic stress. This means that volumetric deformation is a function of isotropic stress only. The compressibility modulus K provides the relationship between volumetric deformation ε_{vol} and isotropic stressσ_0 :

$$\sigma_0 = -K \times \varepsilon_{vol} \qquad (4.21)$$

4.3.4 Relationship between deformation parameters

The isotropic soil is completely defined by E and ν because they are linked by the following relationship:

$$G = \frac{E}{2(1+\nu)} \qquad (4.22)$$

And it is easy to show that:

$$K = \frac{E}{3(1-2\nu)} \qquad (4.23)$$

- ***Noticed:*** *For the description of compression and distortion, the parameters K and G are more suitable than E and ν.*

In the case of elasticity, we can define the moduli according to the applied load in a plan state (X, Z) (Figure 4.7).

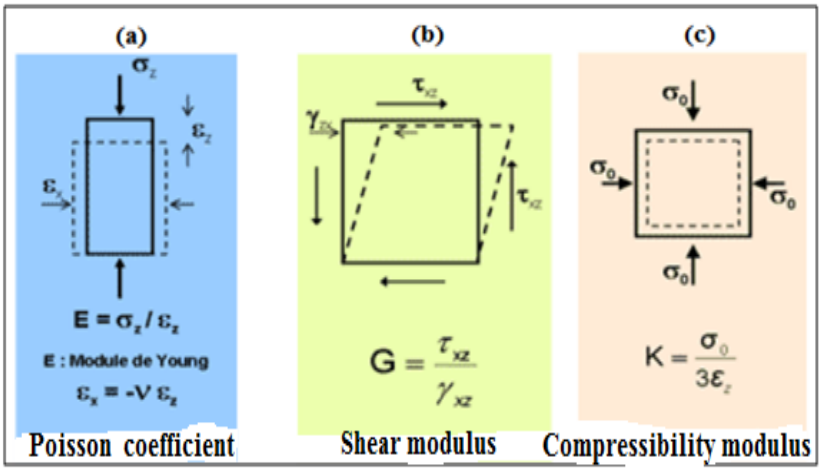


Figure 4.7. Definition of soil parameters: (a) Uniaxial compression, (b) Pure or simple shear, (c) Isotropic compression

4.4 TOTAL AND EFFECTIVE STRESSES

Liquid and solid phases are characterized by different constitutive laws. It is the solid phase (or granular skeleton) that is responsible for soil deformation and its mechanical resistance to shear. The liquid phase (water) is incompressible and exhibits no resistance to shear.

To be able to describe the behavior (or response) of a soil subjected to mechanical stress, it is necessary to distinguish the part of the mechanical stress taken up by the solid skeleton from that supported by the water (Figure 4.8).

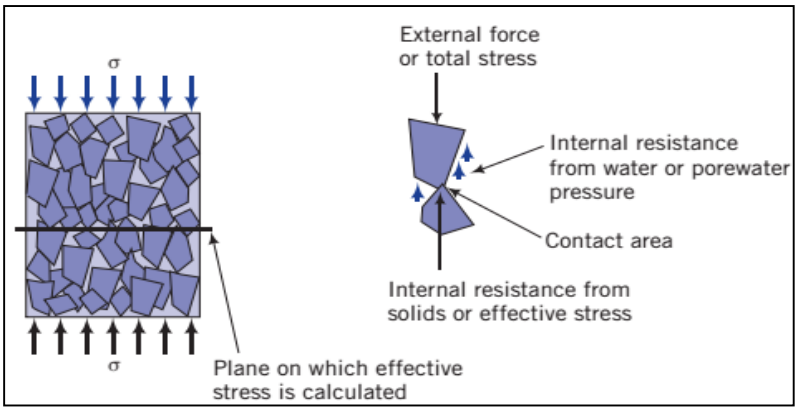


Figure 4.8. Stresses in a soil mass

The idea of separating the stresses for each phase originated with Terzaghi (in the early 1920s), who postulated the existence of stresses, called effective stresses, which govern the mechanical response of the solid phase.

4.4.1 Total stress

In a saturated soil mass, if we consider the soil as a whole (without distinguishing between the solid and liquid phases), the stresses correspond to the total stresses. However, total stresse defined in this way do not allow us to fully study the soil's behavior (compressibility, shear, etc.). Indeed, the two phases of the soil (solid particles and water) do not obey the same constitutive laws.

The notations for the component of total stress to the facet under consideration are:

σ : Normal stress;

τ : Tangential stress.

4.4.2 Effective stress

The most important concept in soil mechanics is the principle of effective stress where the compressibility and shear strength of a soil depend largely on the effective stress. Thus, the concept of effective stress is important for solving geotechnical problems. Effective stress is expressed as the difference between total stress and pore water pressure (it is determined by calculation; it cannot be measured directly). The notations for the effective stress component are:

σ' : Normal stress on the facet under consideration;

τ': Tangential constraint to the facet under consideration.

The effective constraint and the total constraint are linked by the following relationships:

$$\sigma' = \sigma - u \qquad (4.24a)$$

$$\tau' = \tau \qquad (4.24b)$$

Where: u is the water pore pressure (hydrostatic pressure).

- ***Noticed:*** *Effective stresses concept of is only valid for normal stresses, because shear stresses* τ *are not transferred by water and, therefore, are always effective (*$\tau' = \tau$*).*
- ***Noticed:*** *In dry soils, the effective stresses (*$\sigma' = \sigma; \quad u = 0$*).*

4.5 GEOSTATIC VERTICAL STRESS

4.5.1 Superposition principle

The total stress depends on the soil's loading mode, both by its own weight and by external effects. In the linear elastic behavior, the effect produced by the simultaneous action of several forces is equal to the sum of the effects produced by each force acting separately.

Thus, the stress exerted at depth on a horizontal facet is equal to the sum of the natural stress σ_{v0} due to the weight of the overlying soil and the stress due to surcharges $\Delta\sigma_z$ (Figure 4.9). So:$\sigma_z = \sigma_{v0} + \Delta\sigma_z$

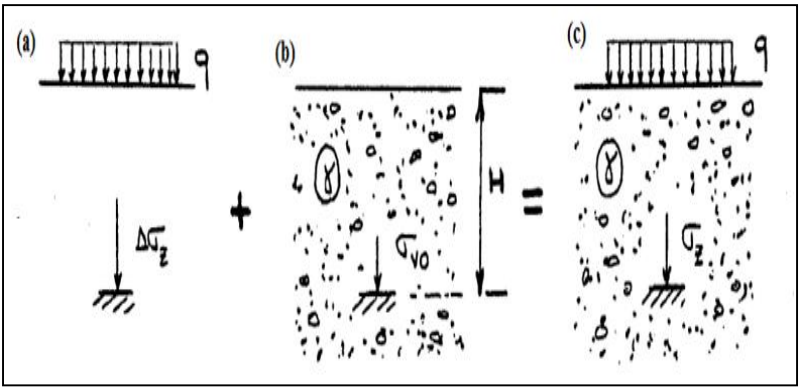


Figure 4.9. Principle of stress superposition: (a) Loaded mass ($\boldsymbol{\gamma = 0}$ and $\boldsymbol{q \neq}$), (b) Massive mass ($\boldsymbol{0\gamma \neq 0}$ and $\boldsymbol{q = 0}$), (c) Massive and loaded mass ($\boldsymbol{0\gamma \neq 0}$ and $\boldsymbol{q \neq 0}$)

4.5.2 Geostatic (natural) vertical stress

When the soil surface is horizontal and the nature of the soil varies little in the horizontal direction and before any loading and at a given point, the stress is called geostatic or natural stress, denoted σ_{v0}. It generally corresponds to the weight of the earth that rises above this point.

4.5.2.A. Case of a homogeneous soil mass

Consider a mass of dry or moist (unsaturated) homogeneous soil of unit weight γ, with a horizontal free surface and in equilibrium under the action of its own weight (Figure 4.10.a).At any depth, vertical geostatic stresses can be calculated by considering the weight of the soil above that depth. Therefore:

- If the weight is constant with depth (dry or wet soil):

$\sigma_{v0} = \gamma \times z$ (for z= h, $\sigma_z = \gamma \times h$)

In this first case (dry or wet soil), the stress transmitted by the granular skeleton (water, if present) is "attached" to the grains by capillary tension and increases their weight. This is called inter granular or effective stress. It is denoted σ' ($\sigma_{v0} = \sigma'$).

- If the water table is at the surface of the ground (saturated soil):

$\sigma_{v0} = \gamma_{sat} \times z$ (for z= h, $\sigma_z = \gamma_{sat} \times h$)

In this second case (saturated soil) (Figure 4.11), the water is in hydrostatic equilibrium and exerts a pressure u on the facet equal to:

$$u = \gamma_w \times h \qquad (4.25)$$

The total stress includes both the action of water and the action of the granular particle. The effective stress transmitted by the granular particle:

$$\sigma' = \sigma_{v0} - u = (\gamma_{sat} \times h) - (\gamma_w \times h) = (\gamma_{sat} - \gamma_w) \times h = \gamma' \times h$$

4.5.2.B. Case of a multilayered soil

In the case of several horizontal soil layers with varying thickness h_i and unit weight γ_i (Figure 4.10.b), the stress at M is expressed by the following:

$$\sigma_{vM} = \sum_{1=1}^{i=n} \gamma_i \times h_i \qquad (4.26)$$

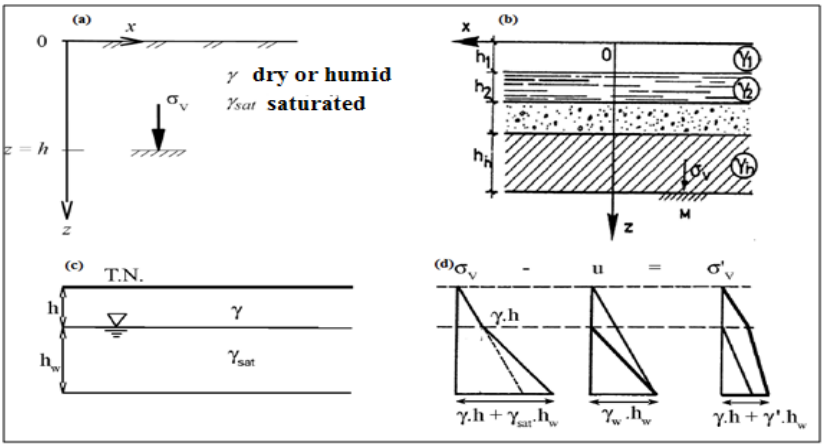


Figure 4.10. Geostatic vertical stress: (a) Case of homogeneous soil, (b) Case of stratified soil, (c) Case of presence of a water table, (d) Stress diagram in the presence of water

4.5.2.C. Presence of the water table

At the base of the soil section (Figure 4.10.c):

-The total vertical stress: $\sigma_{vM} = \gamma \times h + \gamma_{sat} \times h_w$

-The water pressure: $u_M = \gamma_w \times h_w$

So, the effective stress:

$$\sigma' = \sigma_{vM} - u_M = (\gamma \times h) + (\gamma_{sat} \times h_w) - (\gamma_w \times h_w) = (\gamma \times h) + (\gamma' \times h_w)$$

Stress diagrams for the presence of a water table are shown (Figure 4.10.d). As the water table level h rises and h_w falls, the effective stress increases .This phenomenon can cause soil settlement and disrupt structures $(\gamma > \gamma')$ (Figure 4.11).

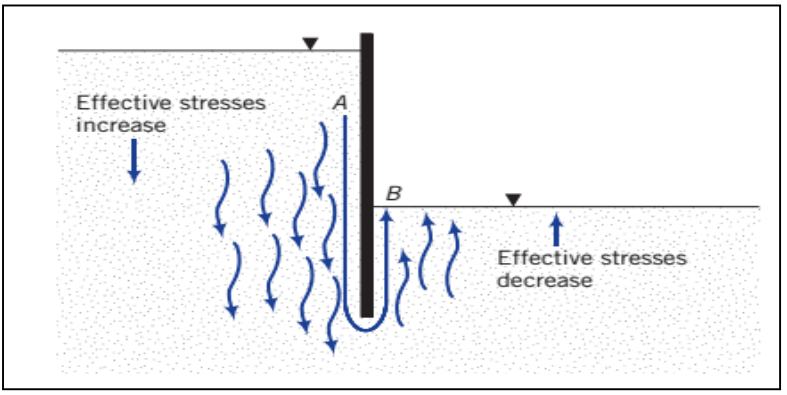


Figure 4.11. Effects of flow (seepage) on the effective stresses near a retaining wall

4.6 EXERCISES

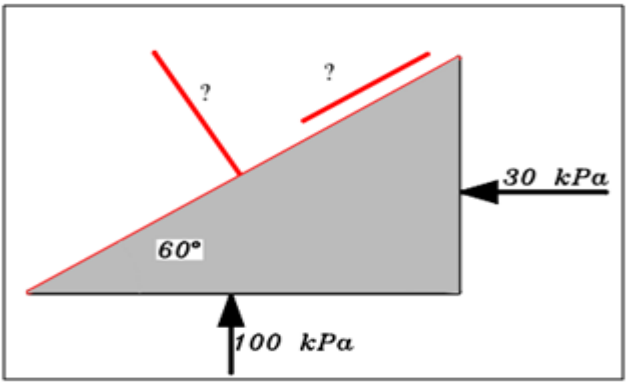
Exercise 4.1

Consider the state of stress for the element given bellow.

1- Draw Mohr's circle.

2- Find the stresses that develop on the plane inclined at α=60° from the base of the element.

3- Determine the magnitude and directions of the maximum shear stress.

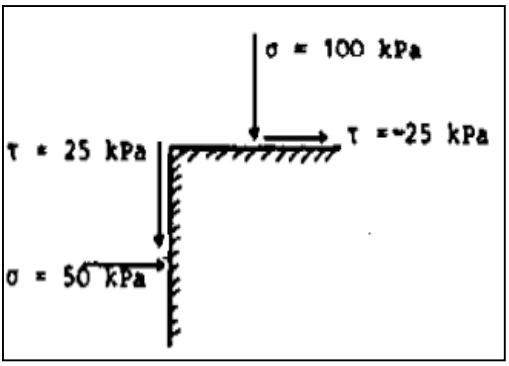


Exercise 4.2

The state of stress is given at a point in the ground, as illustrated bellow.

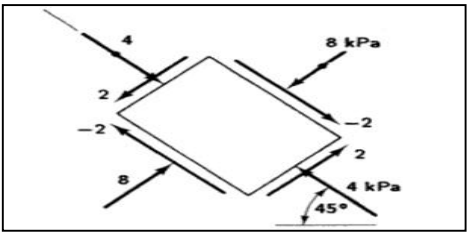
1- Find the major and minor principal stresses and their orientation (analytically).

2- Find the major and minor principal stresses and their orientation (graphically).



Exercise 4.3

The same exercise as the previous one, with the following case.

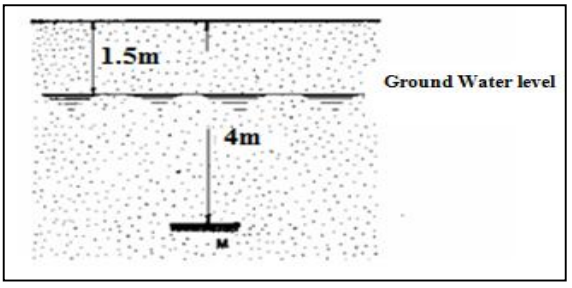


Exercise 4.4

Determine the total and effective stress value due to the self-weight of the soil and the pore water pressure at point M located at a depth of 4m below the ground surface.

The wet unit weight: $\gamma h = 21\ kN/m^3$.

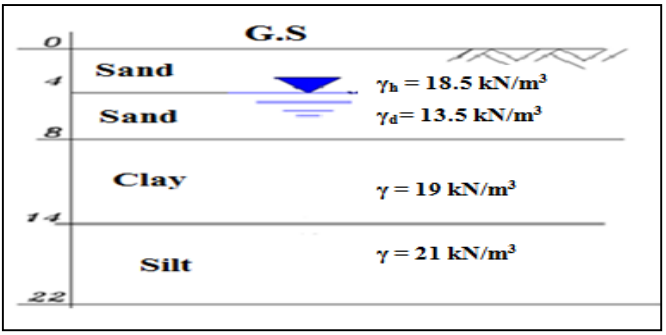
The dry unit weight: $\gamma_d = 19\ kN/m^3$.



Exercise 4.5

For the soil profile shown bellow.

Draw the variation of total vertical stresses, effective vertical stresses, and pore water pressure with depth (from 0 to 22m).



Exercise 4.6

Consider a point M located at a depth of 6m below the ground surface, the ground is homogeneous and subjected to an upward flow (upward seepage).

- The wet unit weight: $\gamma_h = 19\ kN/m^3$
- The hydraulic gradient: i = 0.6

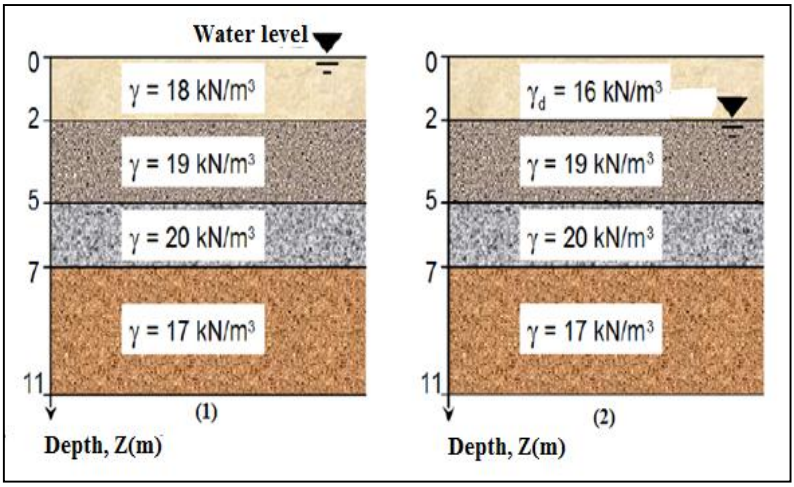
1- What is the effective stress at point M?

2- For what value of hydraulic gradient does the effective stress become zero?

Exercise 4.7

Consider a soil mass composed of three soil layers.

1- Draw the diagrams of stresses in the two cases illustrated in the figure. Comment on the results.


Water level
0
2
5
7
11
γ = 18 kN/m³
γ = 19 kN/m³
γ = 20 kN/m³
γ = 17 kN/m³
(1)
Depth, Z(m)
0
2
5
7
11
γd = 16 kN/m³
γ = 19 kN/m³
γ = 20 kN/m³
γ = 17 kN/m³
(2)
Depth, Z(m)

CHAPTER 5 :

APPLIED LOADS IN SOIL

5.1 INTRODUCTION

To analyze problems (such as soil compressibility, foundation bearing capacity, embankment stability, and lateral pressure on earth retaining structures), it is necessary to calculate and understand the distribution of stresses imposed on a soil mass by external loads. This type of calculation generally requires the application of principles from the theory of elasticity, even though soil in nature is not perfectly homogeneous, elastic, and isotropic.

However, soil stresses are of two types: geostatic stresses due to the soil's own weight and stresses due to surface loading. The development of these stresses leads to deformations in the soil mass.

5.2 STRESSES INDUCED BY APPLIED LOADS

Determining the distribution of stresses induced by applied loads (house, building, bridge pier, road...) (Figure 5.1) is essential to verifying two major problems encountered in the act of building on soils: mechanical resistance (foundation stability) and deformation (settlement) of the soils.

The net increase in stress in the soil ($\Delta\sigma_z$) calculation depends on the load per unit area to which the foundation is subjected, the depth below the foundation at which the stress estimate is desired, and other factors. The vertical stress estimate is based on the theory of elasticity. Loads can include point loads,

distributed loads, etc. The calculation of the vertical stress due to external loadings was carried out according to the type of loading.

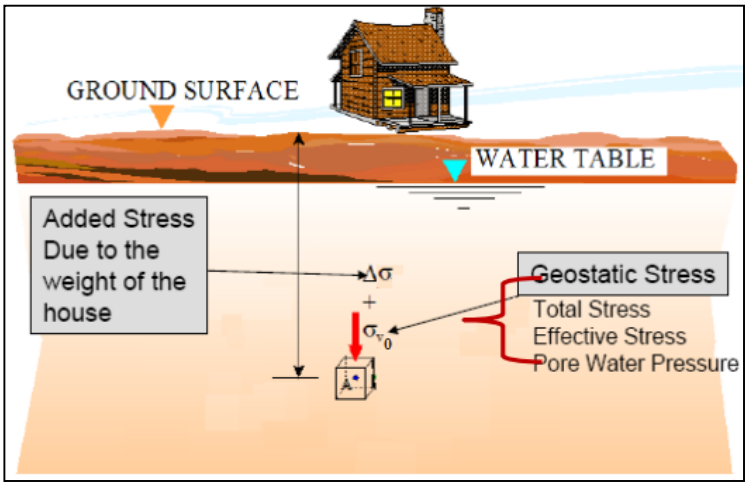


Figure 5.1. Stresses acting in soil (in general)

5.2.1 Case of a point load

Boussinesq (1885) provided the solution for the distribution of stresses caused by the application of a point load to the surface of a homogeneous, elastic, isotropic, and semi-infinite medium, using the mathematical theory of elasticity. A semi-infinite medium is bounded by a horizontal boundary plane, which is the ground surface for the medium.

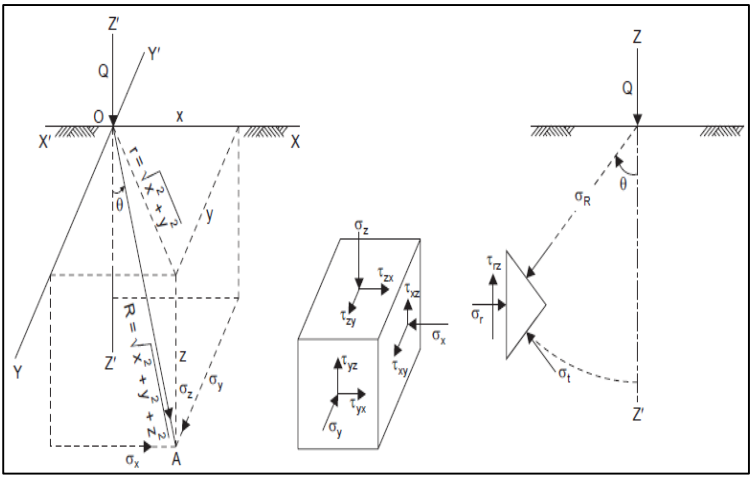


Figure 5.2. Stresses generated by a point load

The notation for the stress components and the coordinate system is shown in Figure 5.2. The stresses acting at point A on the normal planes are shown in the cube above.

The equations for the vertical stress of Boussinesq are:

$$\Delta\sigma_z = \frac{3Q}{2\pi} \times \frac{1}{z^2} \times \cos^5\theta \qquad (5.1)$$

Where:

$$\Delta\sigma_z = \frac{3Q}{2\pi} \times \frac{z^3}{R^5} = \frac{3Q}{2\pi z^2} \times \left[\frac{1}{[(r/z)^2+1]^{5/2}}\right] \qquad (5.2)$$

This last equation can be written in the following form:

$$\Delta\sigma_z = \frac{Q}{z^2} \times \left\{\frac{3}{2\pi} \frac{1}{[(r/z)^2+1]^{5/2}}\right\} = \frac{Q}{z^2}\, I \qquad (5.3)$$

With I: the influence factor given by Boussinesq:

$$I = I_B = \left\{\frac{3}{2\pi} \times \frac{1}{[(r/z)^2+1]^{5/2}}\right\} \qquad (5.4)$$

To better understand the distribution of stresses in the soil induced by point load Q, we can consider the distribution of vertical stresses $\Delta\sigma_z$ along horizontal planes ($z = Cste$). The highest stresses are located directly above the point of application of the point force (Figure 5.2). Then, for several assumed values of r, we can calculate r/z and subsequently find the value of I. The value of $\Delta\sigma z$ in this case is then calculated.

For r = 0, $\Delta\sigma z$ is the maximum with a value of 0.4775 Q/z²; for r = 2z, it is only about 1.8% of the maximum, and for r = 3z, it is only 0.3% of the maximum. The stress distribution is shown in Figure 5.3.

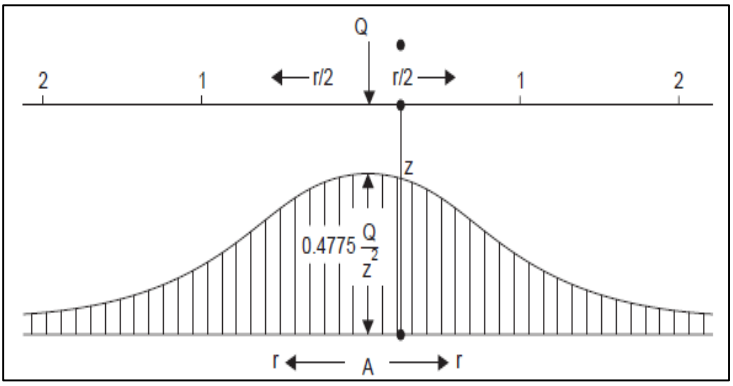


Figure 5.3. Distribution of vertical stresses on a horizontal plane at depth z

Theoretically, the stress approaches zero at infinity, although in practice it reaches a negligible value at a short, finite distance. The maximum pressure gradient is relatively high at low altitudes and decreases with increasing depth.

In 1938, Westrgraad proposed the following formula for the influence factor I:

$$I = I_W = \left\{\frac{1}{\pi} \times \frac{1}{[2(r/z)^2 + 1]^{3/2}}\right\} \qquad (5.5)$$

The influence coefficient I can be derived from the abacuses (Figure 5.4) or from a table (Table 5.1) as a function of r and z (Table 5.1).

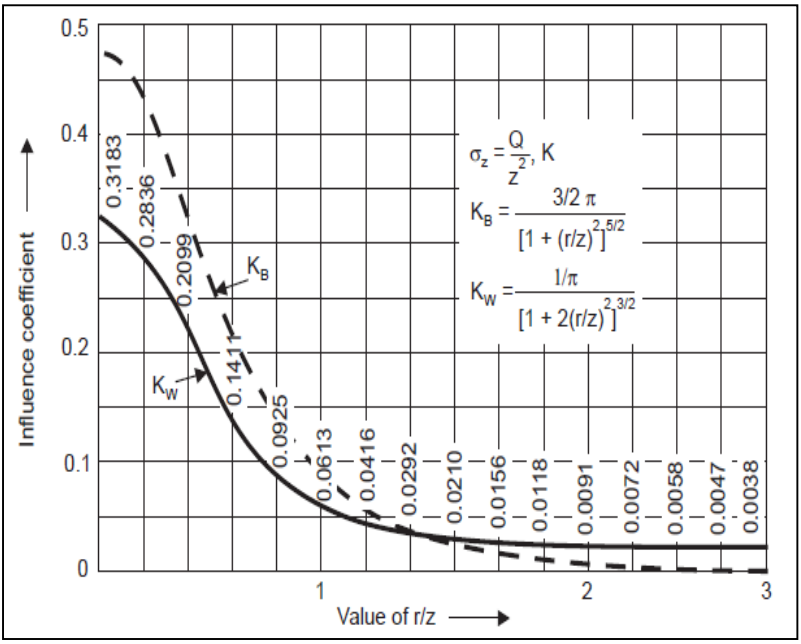


Figure 5.4. Influence coefficient for vertical stress due to concentrated load : Boussinesq influence coefficient I_B (K_B), Westergraad influence coefficient I_W (Kw)

➢ ***Noticed:*** *($\Delta\sigma_z$) is independent of all soil parameters and in particular E and ν.*

➢ ***Noticed:*** *For:* $r/z = 0$ *, the value of* $I_W \approx 0.32$ *which is less than* $I_B \approx 0.48$ *by about 33%, then the Boussinesq solution gives conservative results at shallow depths.*

Table 5.1. Table giving some values of I_B as a function of r/z

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

5.2.2 Case of linear loads

Consider a uniformly distributed load of intensity q′, applied along a straight line and of infinite extent, acting on the surface of a semi-infinite elastic medium (Figure 5.5).

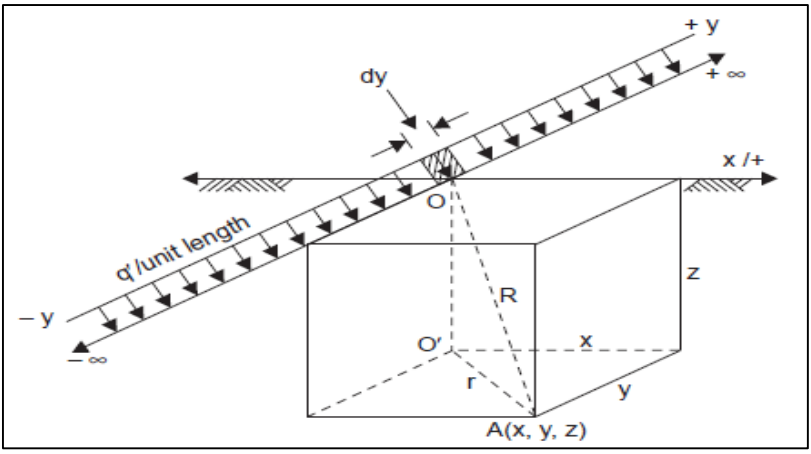


Figure 5.5. Linear load acting on a semi-infinite elastic soil surface

The elementary stress d ($\Delta\sigma z$) due to a load applied over a very small length dy is given by the following Bouusinesq formula:

$$d\,\Delta\sigma_z = \frac{3(q'dy)z^3}{2\pi r^5} \qquad (5.6)$$

The vertical stress $\Delta\sigma z$ at point A for different forms of distribution of infinite length surcharges can be obtained by integrating the Boussinesq equation with respect to the variable y in the limits - ∞ and + ∞.

$$\Delta\sigma_z = \frac{2q'z^3}{\pi(x^2+z^2)^2} = \frac{2q'}{\pi z} \times \frac{1}{[1+(x/z)^2]^2} \qquad (5.7)$$

Equation 5.6 can be written in another form:

$$\Delta\sigma_z = \frac{q'}{z}.I \qquad (5.8)$$

With:

$$I = \frac{2/\pi}{[1+(x/z)^2]^2} \qquad (5.9)$$

5.2.3 Uniformly rectangular charge of an infinite strip

Consider a uniform charge of intensity q per unit area acting on a strip of infinite length and constant width B (= 2b) as shown in Figure 5.6. Figure 5.7 shows some pressure bulbs for a strip loading case.

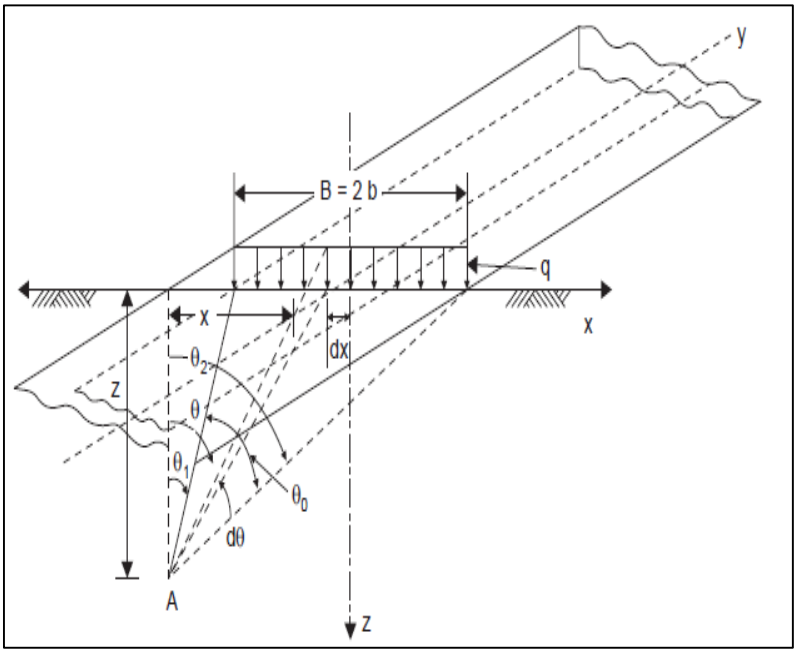


Figure 5.6. Load from a strip of infinite length acting on a surface of a semi-infinite elastic medium

$\Delta\sigma_z$ at point A is:

$$\Delta\sigma_z = \frac{q}{\pi}[(\theta_2-\theta_1)+\cos(\theta_2+\theta_1)\times\sin(\theta_2-\theta_1)] \quad (5.10)$$

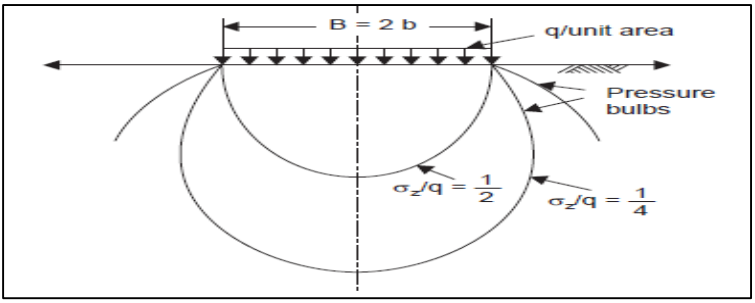


Figure 5.7. Pressure bulbs for a load strip of infinite length

5.2.4 Case of a uniform load on regular area

The most common shapes of a loaded area in foundation engineering practice are the rectangle (e.g., buildings) and the circle (e.g., reservoir).

Consider a distributed load of intensity q acting over an area (S) (Figure 5.8.a). Integrating Boussinesq's formula over this area (S) allows us to determine the vertical stress ($\Delta\sigma_z$) for different load distribution configurations. In practice, the stress $\Delta\sigma_z$ resulting from the action of a uniformly distributed vertical load of intensity q is given by the following relation:

$$\Delta\sigma_z = I \times q \qquad (5.11)$$

The influence coefficient I is a dimensionless number, less than or equal to 1. This coefficient is provided by charts established for given loading geometries.

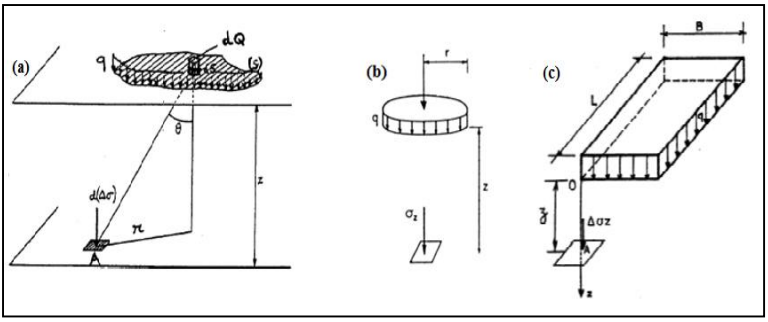


Figure 5.8. Stress under a uniformly distributed load: (a) Consideration of the load, (b) Case of a circular uniform load, (c) Case of a rectangular uniform load

5.2.4.A. Uniform load on circular area

Along the axis of a uniform circular charge of radius R, at depth z (Figure 5.10.b), we have:

$$\Delta\sigma_z = q \times I$$

With :

$$I = 1 - \left[\frac{1}{1 + (R/z)^2}\right]^{3/2} \qquad (5.12)$$

The coefficient I is sometimes presented in the form of a chart as a function of R/z (Figure 5.9).

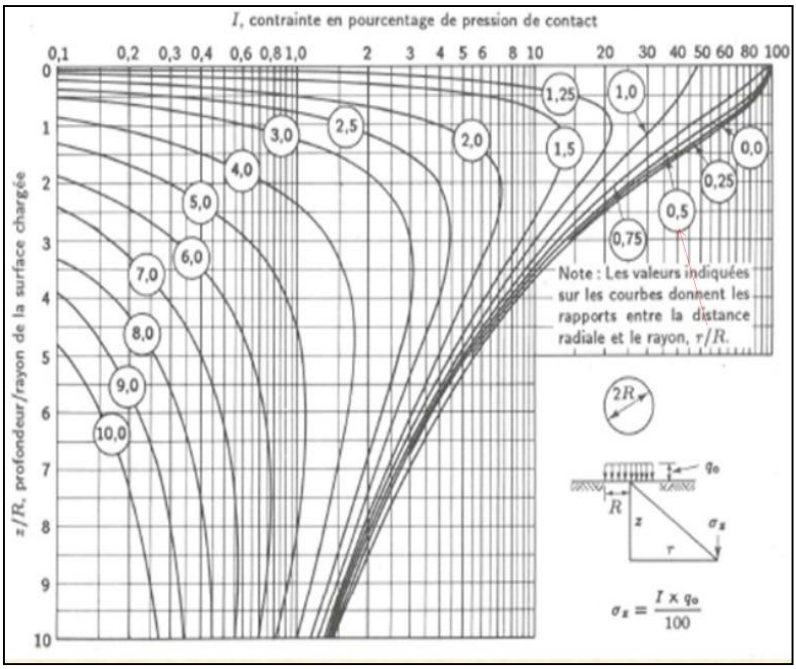


Figure 5.9. Determination of the influence coefficient for a uniform load on circular area

5.2.4.B. Uniform load on rectangular area

By applying the principle of integration, we can obtain the vertical stress at a point located at a certain depth below the center or a corner of a uniformly loaded rectangular area. The calculation is based based either on the solution of Boussinesq, or on that of Westergaard for a point load (Figure 5.8.c).

➢ *Boussinesq's solution*:

Boussinesq's solution can also be used to calculate the increase in vertical stress under a flexible rectangular load area (Figure 5.9).

The loaded area is located at ground level, has a length L and a width B. The uniformly distributed load per unit area is equal to q.

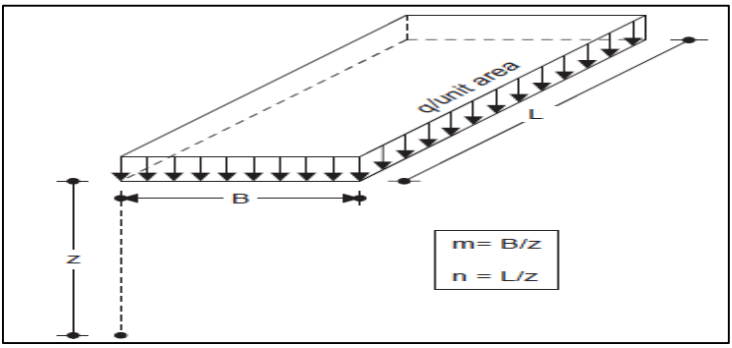


Figure 5.9. Vertical stress at the corner of a uniformly loaded rectangular area

The increase in vertical stress ($\Delta\sigma z$) at point A, which is located at depth z below the **corner of the rectangular area**, can be calculated using the following formula:

$$\Delta\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1} \times \left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \qquad (5.13a)$$

With: m=B/z and n= L/z

Equation 5.13 can be written in the following form:

$$\Delta\sigma_z = q \,.\, I \qquad (5.13b)$$

Where:

$$I = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1} \times \left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \qquad (5.14)$$

I: Influence factor.

Based on this equation, Fadum (1941) prepared a chart to calculate influence values for sets of values of m and n, as shown in Figure 5.10.

For a foundation having L length and B width, the graphic gives the coefficient I as a function of L/z for different values of B/z. L and B are interchangeable in Fadum's chart.

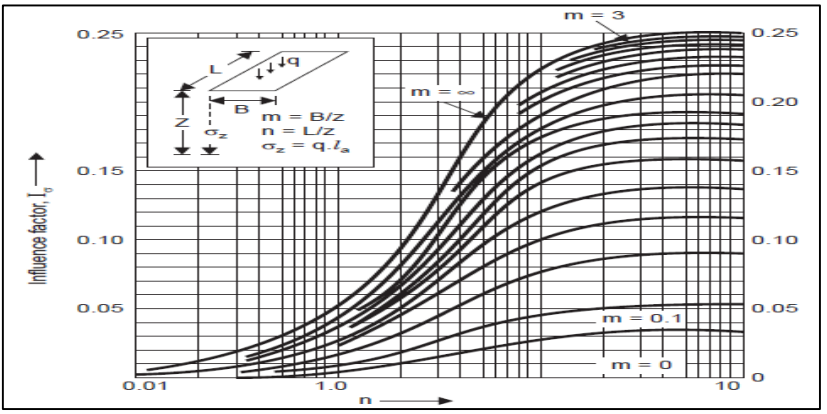


Figure 5.10. Influence factor for the vertical stress under a corner of a uniformly loaded rectangular area (Fadum, 1941)

Steinbrenner (1934) gave another scheme for this purpose, which is shown in Figure 5.11, by plotting the influence values I on the horizontal axis and 1/m (= z/B) on the vertical axis, for different values of L/B (= n/m).

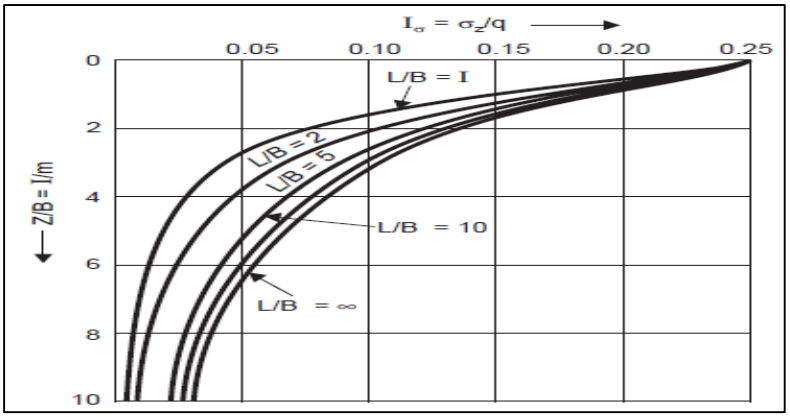
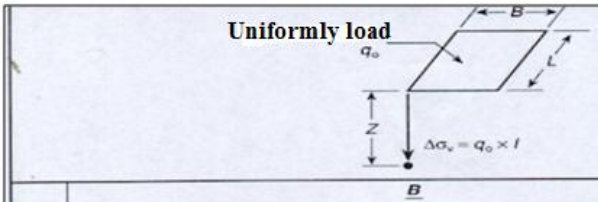


Figure 5.11. Influence factor for the vertical stress under a corner of a uniformly loaded rectangular area (Steinbrenner, 1934)

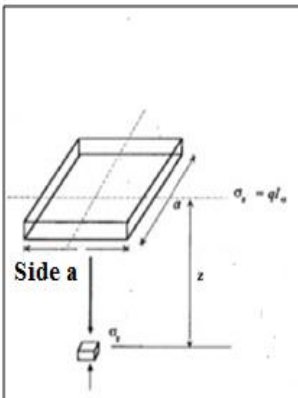
For faster reading, tables giving the coefficient I for a uniform rectangular load (Table 5.2) and for the particular case of a square load of side a (Table 5.3) can be used.

Table 5.2. Stress influence value for a uniformly distributed load rectangular area



L/Z	B/Z												
	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1	2	3	∞
0,1	0,005	0,009	0,013	0,017	0,020	0,022	0,024	0,026	0,027	0,028	0,031	0,032	0,032
0,2	0,009	0,018	0,025	0,033	0,039	0,043	0,048	0,050	0,053	0,055	0,060	0,061	0,062
0,3	0,013	0,025	0,037	0,048	0,056	0,065	0,068	0,073	0,077	0,081	0,088	0,089	0,090
0,4	0,017	0,033	0,048	0,060	0,071	0,080	0,087	0,093	0,098	0,101	0,113	0,114	0,115
0,5	0,020	0,039	0,056	0,071	0,085	0,097	0,106	0,111	0,118	0,120	0,136	0,138	0,139
0,6	0,022	0,043	0,065	0,080	0,097	0,107	0,120	0,125	0,136	0,139	0,153	0,156	0,157
0,7	0,024	0,048	0,068	0,087	0,106	0,120	0,132	0,140	0,148	0,154	0,169	0,172	0,173
0,8	0,026	0,050	0,073	0,093	0,111	0,125	0,140	0,146	0,156	0,164	0,181	0,183	0,185
0,9	0,027	0,053	0,077	0,099	0,118	0,136	0,148	0,156	0,166	0,172	0,191	0,195	0,196
1	0,028	0,055	0,081	0,101	0,120	0,139	0,154	0,164	0,172	0,179	0,200	0,203	0,205
2	0,031	0,060	0,088	0,113	0,136	0,153	0,169	0,181	0,191	0,200	0,232	0,237	0,240
3	0,032	0,061	0,089	0,114	0,138	0,156	0,172	0,183	0,195	0,203	0,237	0,244	0,247
∞	0,032	0,062	0,090	0,115	0,139	0,157	0,173	0,185	0,196	0,205	0,240	0,247	0,250
										0,22			

Table 5.3. Influence factor for a uniformly distributed square load area



a/z	I_σ	
	Boussinesq	Westergaard
∞	1.0000	1.0000
20	0.9992	0.9365
16	0.9984	0.9199
12	0.9968	0.8944
10	0.9944	0.8734
8	0.9892	0.8435
6	0.9756	0.7926
5	0.9604	0.7525
4	0.9300	0.6971
3.6	0.9096	0.6659
3.2	0.8812	0.6309
2.8	0.8408	0.5863
2.4	0.7832	0.5328
2.0	0.7008	0.4647
1.8	0.6476	0.4246
1.6	0.5844	0.3794
1.4	0.5108	0.3291
1.2	0.4276	0.2858
1.0	0.3360	0.2165
0.8	0.2410	0.1560
0.6	0.1494	0.0999
0.4	0.0716	0.0477
0.2	0.0188	0.0127
00	0.0000	0.0000

➢ *Westergaard's solution :*

Westergaard assumed that the soil consists of very thin horizontal sheets of infinite rigidity, which prevent the development of any lateral stress. The increase in vertical stress at a point located under a corner of a uniformly loaded rectangular area can be obtained by integrating the stress due to a point load under similar conditions:

$$\Delta\sigma_z = \frac{q}{2\pi} \times \left[\cot^{-1} \sqrt{\left(\frac{1-2\nu}{2-2\nu}\right)\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\left(\frac{1-2\nu}{2-2\nu}\right)^2 \cdot \frac{1}{m^2 n^2}} \right] \quad (5.15)$$

Where :

ν: Poisson's ratio

m=L/z and n= B/z

- ***Noticed:*** *If m and n are less than one, the values of the influencing factors represent approximately two-thirds of I in Boussinesq's theory. In this case, discernment is needed to determine which theory is most appropriate for the specific conditions of the soil environment.*

5.2.4.C. Case of a trapezoidal load

In several practical cases, it is necessary to determine the increase in vertical stress in a soil mass due to the load of the embankments. Figure 5.12 shows a cross-section of an embankment of height H. For this two-dimensional loading condition, the increase in vertical stress can be expressed as follows:

$$\Delta\sigma_z = \frac{q}{\pi} \times \left[\left(\frac{a+b}{b} \right) (\alpha_1 + \alpha_2) - \frac{a}{b} (\alpha_2) \right] \qquad (5.16)$$

where q = γH

γ: unit weight of the backfill soil.

H: embankment height

$$\alpha_1 \text{ (rads)} = \tan^{-1}\left(\frac{a+b}{z}\right) - \tan^{-1}\left(\frac{a}{z}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{a}{z}\right)$$

Equation 5.15 can be written in the following form:

$$\Delta\sigma_z = q\, I$$

Where I is the factor of influence.

Osterberg (1957) established a chart to calculate the factor of influence for an embankment (trapezoidal load) of infinite length (Figure 5.13). The point considered is under an angle of the loaded area.

In the case of a triangular load (in the shape of a slope) of length b, Fadum's chart gives the coefficient I in the case of a triangular load distributed over a rectangle of sides a and b as a function of the parameters a/z and b/z (Figure 5.14).

➢ ***Noticed:*** *The point in question is under an angle of the loaded area.*

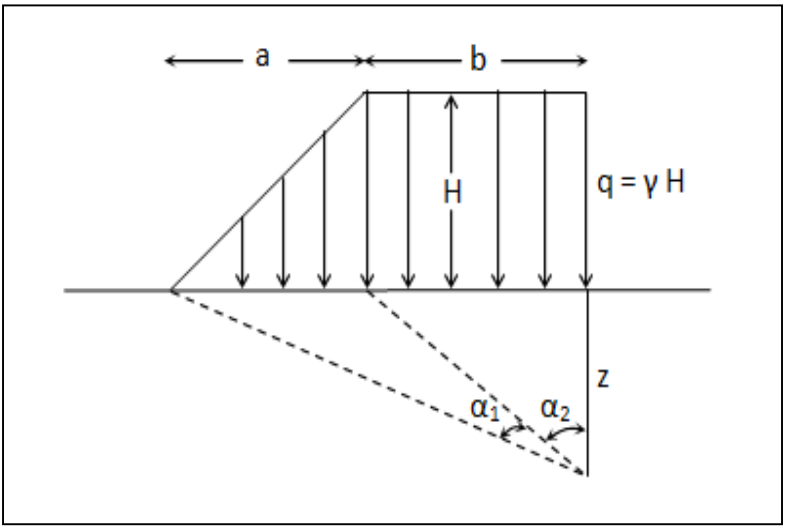


Figure 5.12. Load on a trapezoidal shape

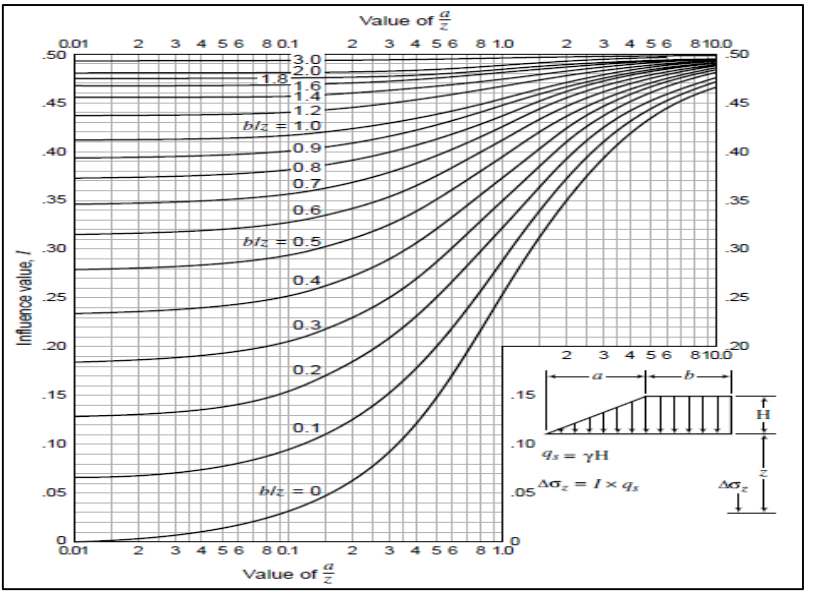


Figure 5.13. Osterberg chart for determining the vertical stress due to the load of embankments (trapezoidal load) of infinite length

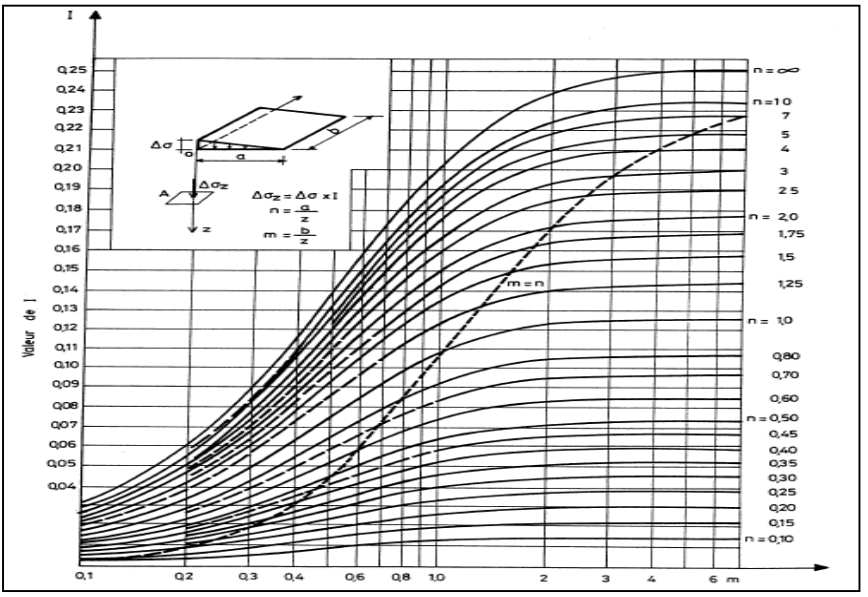


Figure 5.14. Fadum's chart for determining the vertical stress in this case of a triangular (slope-shaped) load of length b

5.2.5 Uniform load on an irregular surface

Newmark (1942) developed a simple graphical procedure for calculating the vertical stress within a soil subjected to a uniformly distributed vertical load. The chart he designed for this purpose is called the "influence chart" (Figure 5.15). It is applicable to a semi-infinite, homogeneous, isotropic, and elastic soil mass (and not to a stratified soil).

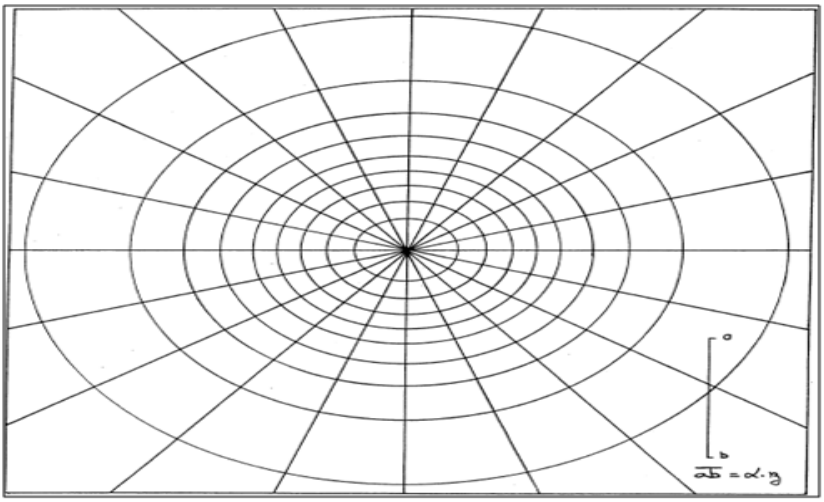


Figure 5.15. Newmark's chart for irregular surfaces

5.2.6 Superposition principle

In the case of more complex load distributions or non-uniform intensity, the principle of superposition will be used. The load distribution is broken down into elementary shapes for which the influence coefficients are easily determined using the charts or expressions given previously.

We consider, for example, the case of a load where rectangular surfaces numbered 1 and 2 are each subjected to different surcharges q_1 and q_2 (Figure 5.16.a). The calculation of the influence factor at point A will be carried out as follows:

$$\Delta\sigma_z = \Delta\sigma_{Z1} + \Delta\sigma_{Z2} \qquad (5.17)$$

So :

$$\Delta\sigma_Z = (I_1 \times q_1) + (I_1 \times q_2) \qquad (5.18)$$

Where I_1 is the coefficient of influence of the load q_1 at A (without taking into account the load q_2), and the coefficient of influence I_2 of the load q_2 at A (without taking into account).

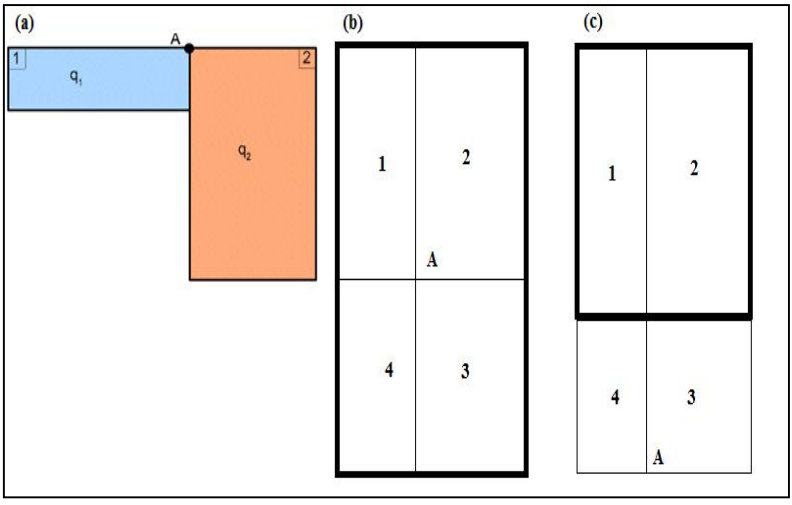


Figure 5.16. Superposition principle: (a) Case of a non-uniform loading intensity, (b) Case of a point within the loaded zone, (c) Case of a point outside the loaded zone

If the load is of uniform intensity over a rectangular surface, the total influence coefficient is obtained by algebraically summing the influence coefficients of each of the rectangles.

- The vertical line passing through A crosses the loaded area (Figure 5.16.b):

$$I = I_1 + I_2 + I_3 + I_4 \qquad (5.19)$$

- The vertical line passing through A does not cross the loaded area (Figure 5.16.c):

$$I = I_1 + I_2 - I_3 - I_4 \qquad (5.20)$$

5.3 EXERCISES

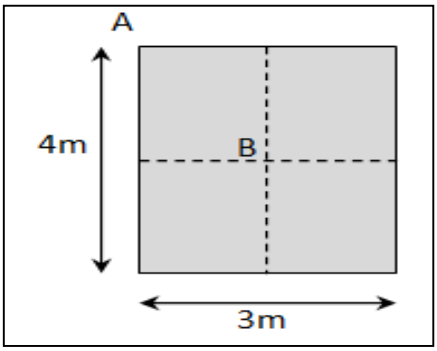
Exercise 5.1

Consider a concentrated load P = 5 kN applied to a soil surface.

1. Calculate the increase in stress $\Delta\sigma$ as a function of the depth Z (take Z= 0, 2m, 4m, 6m, 8m) for the two cases: r = 0 and r = 2m.
2. Draw the diagram of $\Delta\sigma$ as a function of depth.

Exercise 5.2

Consider a rectangular foundation (3m×4m) on which a uniform load of intensity q=150 kN/m² is applied. Calculate the stress increment $\Delta\sigma$ produced at a depth of 2m at points A and B.

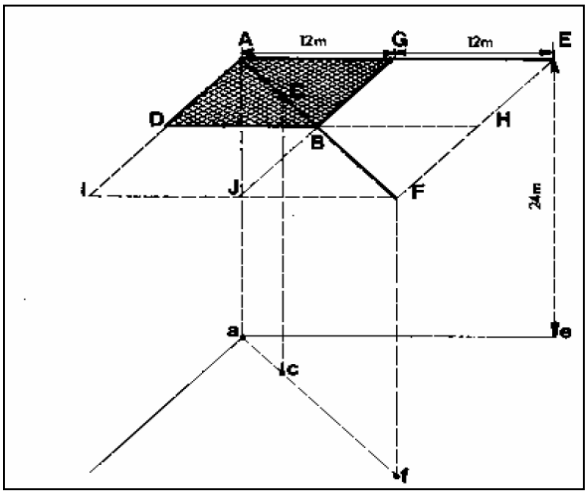


Exercise 5.3

A square raft foundation of 12 m side transmits a stress of 83 kPa.

1. Calculate the vertical stress increase due to the foundation (induced stress) at a depth of 24 m at points a, c, e, and f.
2. Compare the values of the vertical stresses obtained in question 1 with those found if the overload is a concentrated load at point C.

3. Calculate the total stress (geostatic + induced stresses) at point "a" if this point is located in saturated soil with a wet unit weight equal to γ_h=21kN/m^3.



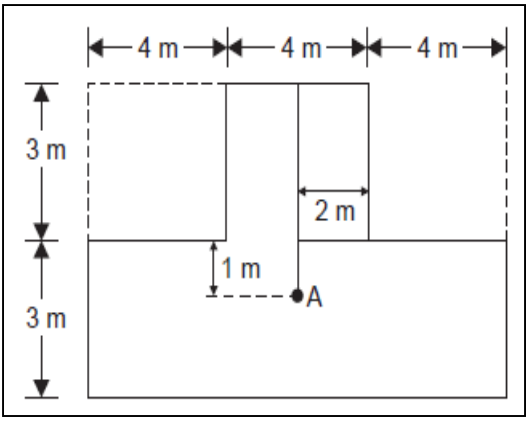
Exercise 5.4

A 4.5m square foundation exerts a uniform pressure of 200kN/m3 on a soil. Determine:

1. The vertical stress increment due to the foundation load to a depth of 10m below its center.
2. The vertical stress increment at a point 3m below the foundation and 4m from its center along one of the axes of symmetry.

Exercise 5.5

The plan of a foundation is shown in bellow. The uniform pressure on the soil is 45 kN/m2. Determine the vertical stress due to the foundation at a depth of 4 m below the point A.



CHAPTER 6 :

SETTLEMENT AND SOIL CONSOLIDATION

6.1 INTRODUCTION

Any structure on the ground compresses the soil. Soil compressibility is primarily due to a reduction in void volume. This occurs as a result of a more compact arrangement of soil grains due to the expulsion of water, air, or air from the pores.

Many civil engineering projects include placing loads onto the ground, which produce corresponding increases in the vertical effective stress. These increases are important because they induce vertical strains in the soil, and thus cause the ground surface to move downward. We call this downward movement settlement. The uniform settlements are generally not detrimental. On the other hand, differential settlements can cause serious problems.

Soil settlement must not be excessive and must not exceed acceptable limits. These requirements necessitate, firstly, understanding the stress distribution within the soil, and secondly, conducting a study of the soil's compressibility and consolidation properties. For settlements, there are two types: (1) instantaneous or immediate settlement which lasts a few days, this type occurs in saturated soils, (2) settlement due to consolidation which occurs over several years.

6.2 DEFINITION OF SOIL COMPRESSIBILITY

Soil is a material composed of solid grains and voids enclosed by the grains. The voids can be filled with air or another gas, water or another liquid, or a combination of these elements.

Under the typically loads encountered in geotechnical engineering practice, the solid particles and pore water can be considered incompressible. In this case, air compression and the expulsion of pore water occupying soil voids are the main sources of volume reduction. A partially saturated soil can experience a significant decrease in volume due to the expulsion of interstitial air before water. However, for saturated soils, the decrease in volume is solely due to the expulsion of interstitial water. Submerged clay layers exhibit a decrease in volume under external pressure through the expulsion of water over time; this soil property, whereby volume decreases under pressure, is called "compressibility."The compressibility of a soil depends on the structural arrangement of the soil particles and, in fine-grained soils, on the degree of bonding of adjacent particles together.

6.3 EVOLUTION OF SOIL SETTLEMENT

The velocity of soil settlement depends on the arrangement and size of the particles. Due to the high permeability of coarse-grained soils, settlement occurs very quickly; this is immediate

settlement. In contrast, fine-grained soils (clays), which are less permeable, expel water from the voids very slowly. In this case, compression and volume reduction will develop over time; this is called consolidation settlement.

To design foundations for structures, it is necessary to assess the total amount of settlement and the time at which it will occur. The total settlement St of a loaded soil has three components:

$$S_t = S_i + S_c + S_s \qquad (6.1)$$

Where Si: immediate settlement,

Sc: primary consolidation settlement,

Ss: secondary consolidation settlement.

6.3.1 Immediate settlement

Immediate settlement occurs directly after the application of a load. In partially saturated soils, this settlement is primarily due to gas expulsion, elastic compression, and particle rearrangement. In saturated soils, immediate settlement is considered the result of vertical compression of the soil before any volume change occurs.

The measurement of immediate settlement is generally based on the theory of elasticity. For a perfectly flexible foundation placed on homogeneous soil, the settlement can be expressed as follows:

$$S_i = q \times B' \frac{1-\nu^2}{E_S} \times m \times I_S \times . I_f \qquad (6.2)$$

q: external load in kN/m^2

$B' = {}^{B}/_{2}$ B : Width of the foundation

ν : Poisson'scoefficient

E_S : soil deformation modulus calculated up to $Z = 5B$

$$E_S = \frac{E_1H_1 + E_2H_2 + E_3H_3 + \cdots}{H}$$

m = 4: settlement calculated at the center of the footing.

m = 2: settlement calculated on the side.

m = 1: settlement calculated at the corner.

I_S et I_f: Influencing factors depend on L/B, soil Poisson's ratio, embedment D, and the distance between the base of the foundation wedge and the substratum H;

$$I_S = I_1 + \frac{1-2\nu}{1-\nu} \times I_2 \qquad (6.3)$$

Where:

$$I_1 = \frac{1}{\pi}\left[M \ln \frac{\left(1+\sqrt{M^2+1}\right)\sqrt{M^2+N^2}}{M\left(1+\sqrt{M^2+N^2+1}\right)} + \ln \frac{\left(M+\sqrt{M^2+1}\right)\sqrt{1+N^2}}{M+\sqrt{M^2+N^2+1}}\right]$$

$$I_2 = \frac{N}{2\pi} \tan^{-1}\left(\frac{M}{N\sqrt{M^2+N^2+1}}\right)$$

and $\tan^{-1}$ in radian

$$M = \frac{L}{B} \quad \text{and} \quad N = \frac{H}{B'}$$

The I_f factor can be calculated using Table 2.1.

Table 6.1. Variation of I_f as a function of B/L, D/B and ν

B/L				
ν	**D/B**	**0.2**	**0.5**	**1.0**
0.3	0.2	0.95	0.93	0.9
	0.4	0.9	0.86	0.91
	0.6	0.85	0.8	0.74
	1.0	0.78	0.71	0.65
0.4	0.2	0.97	0.96	0.93
	0.4	0.93	0.89	0.85
	0.6	0.89	0.84	0.78
	1.0	0.82	0.75	0.69

The immediate settlement assessment for a rigid foundation is:

$$S_{i\ rigid} = 0{,}93\ S_{i\ flexible} \qquad (6.4)$$

6.3.2 Primary consolidation settlement

The primary consolidation of a saturated fine-grained soil corresponds to settlement as pore water dissipates. During this process, the effective stresses in the soil increase in anticipation of the total stress increase are fully inverted. Conversely, the pore water pressure decreases and, at the end of consolidation, tends towards hydrostatic pressure.

The assessment of consolidation settlement is generally based on a laboratory test, the oedometer consolidation test. In contrast, field tests are conducted over very short periods, and their results cannot be relied upon.

6.3.2.A. Oedometer test

The oedometer test (XP P 94-090-1) is carried out on intact samples of saturated fine soil. It consists of subjecting a soil specimen trapped in an oedometer cell to a series of loading and unloading stages of constant intensity and measuring, for each of them, the change in height ΔH of this specimen over time (Figure 6.1). The oedometer cell shown in Figure 6.2 comprises a rigid annular ring containing the soil specimen, and two porous stones ensuring drainage of both the upper and lower faces. Comparators used to measure vertical displacements.



Figure 6.1. Oedometer apparatus



Figure 6.2. Oedometer cell

Once the specimen is placed in the oedometer cell, which prevents any lateral deformation, the load is applied at the piston in 24-hour increments, and the settling is measured over time. After 24 hours, the next increment is applied; standards recommend doubling the load, i.e., 12, 25, 50, 100, and 200 kPa. At the end of the test, the specimen is unloaded and weighed before and after oven drying (to determine the void ratio). The oedometer test allows us to plot two types of curves:

- Compressibility curve: variation of e as a function of the log σ'.
- Consolidation curve: variation of settlement as a function of the logarithm of time.

6.3.2.B. Compressibility curve

The oedometer compressibility curve of a granular soil has the general shape shown in Figure 6.3.a. In the AB portion of the curve, we observe that the volume change is quite significant and arises

primarily from the rearrangement of the grains. In the EF portion, it is small results from the elastic deformation of the grains.

For fine-grained soils, permeability is generally low, and water does not flow instantaneously through the soil pores. The deformation of the specimen under each applied load therefore depends on time. Figure 6.3.b shows the compressibility curve of a fine-grained soil; the curve is composed of two roughly straight sections, AB with a shallow slope and CD with a steep slope.

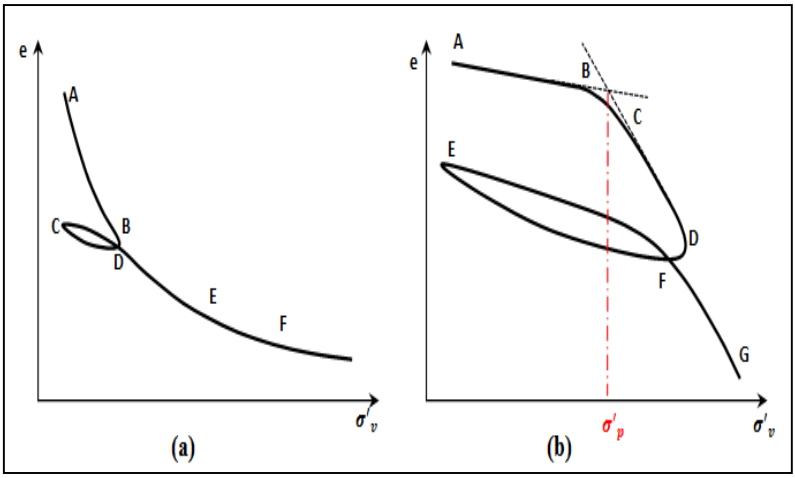


Figure 6.3. Compressibility curve for: (a) granular soils, (b) fine soils

To plot the oedometer curve and interpret the test results, we calculate the initial void ratio of the soil, the initial soil stress in place, and the height H of the specimen. For one-dimensional compression, the cross-section of the specimen remains constant; we can write:

$$\frac{\Delta V}{V} = \frac{S\,\Delta H}{S\,H} = \frac{\Delta H}{H} \qquad (6.5)$$

We have:

$$e = \frac{V_v}{V_s} \qquad (6.6)$$

$$\frac{\Delta H}{H} = \frac{\Delta V_v}{V_v + V_s} = \frac{\Delta e\, V_s}{V_v + V_s} = \frac{\Delta e}{1 + e_0} \qquad (6.7)$$

Therefore, we find that:

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0} \qquad (6.8)$$

6.3.2.C. Compressibility curve parameters

The compressibility curve allows us to determine three characteristics of the soil:

- The pre-consolidation pressure,
- The compression ratio,
- The swelling index.
- ***The pre-consolidation pressure σ'_p :***

The abscissa of the point of intersection P of the two straight parts of the oedometric compressibility curve (Figure 6.3b) is called the pre-consolidation pressure.

In practice, there are different methods for evaluating pre-consolidation pressure (Casagrande (1936), Schmertmann (1953) and Butterfield (1979) methods) which depend on the accurate estimation of certain geometric characteristics of the oedometer curve.

For a soil sample taken intact at a depth Z, one can, on the one hand, calculate the initial natural stress exerted at that depth σ'_{v0}, and on the other hand, determine the preconsolidation pressure σ'_{p} by an oedometer test. By comparing these two values, three soil cases can be distinguished:

1. Normally consolidated soil:

When $\sigma'_{v0} = \sigma'_{p}$, the soil is normally consolidated, it has never been subjected to stress greater than the geostatic stress. The soil has only been consolidated by the weight of the overlying layers currently present.

2. Over-consolidated soil:

When $\sigma'_{v0} < \sigma'_{p}$, the soil is over-consolidated, it is said to have been subjected to a greater pre-consolidation pressure in the past than the weight of the overlying layers now present on the ground. This over-consolidation can be due to the additional load exerted by soils that were subsequently eroded, or to a temporary load such as the weight of a now-vanished glacier.

The over-consolidation ratio (OCR) that characterizes the initial state of the soil is equal to:

$$OCR = \frac{\sigma'_{p}}{\sigma'_{v0}} \qquad (6.9)$$

This ratio of pre-consolidation pressure to natural vertical stress will be equal to 1 in normally consolidated soils, and greater than 1 in the case of an over-consolidated soil.

3. Soil currently being consolidated:

When $\sigma'_{v0} > \sigma'_{p}$, the soil is consolidating under its own weight (recent, poorly or un-compacted fill, recently formed silt or peat), these soils are generally unsuitable for construction; they continue to deform even without additional load.

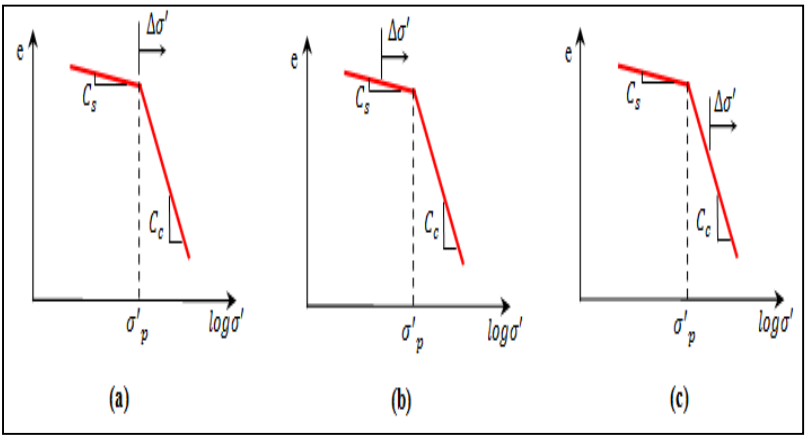


Figure 6.4. Compressibility curves for: (a) Normally consolidated soil, (b) Over-consolidated soil, (c) Under-consolidated soil

- ***Compression ratio Cc :***

The slope of the virgin consolidation curve (Figure 6.4) is called the compression index, Cc; it is noted that once the pressure exceeds the pre-consolidation stress, the settlement increases and the void index decreases considerably.

The coefficient Cc is a dimensionless number. It allows the calculation of the final settlement due to the primary consolidation of a normally consolidated soil layer for a given change in effective stress. The value of Cc can be determined by :

$$C_c = \left|\frac{(e_1 - e_2)}{(\log \sigma_1 - \log \sigma_2)}\right| = \left|\frac{\Delta e}{(\Delta \log \sigma')}\right| \qquad (6.10)$$

As a guide, here are some Cc values depending on the soil type:

- Sands: 0.01 < Cc < 0.10
- Kaolinites: 0.10 < Cc < 0.25
- Illites: 0.25 < Cc < 0.80
- Montmorillonites: 0.80 < Cc < 2.50

We can also consider that a soil is:

- Slightly compressible when Cc < 0.2
- Compressible when 0.2 < Cc < 0.7
- Highly compressible when 0.7 < Cc.

In the absence of the oedometer test, an estimate of the compression index can be obtained from the empirical relationship proposed by Skempton (1944), valid in the case of normally consolidated clays:

$$C_c = 0{,}009(W_L - 10) \qquad (6.11)$$

Where: W_L is the water content at the liquid limit expressed as a percentage.

- ***Swelling index Cs:***

This is the average slope of an unloading/reloading cycle, which also corresponds to the slope of the consolidation curve for a stress value lower than the pre-consolidation stress (Figure 6.4) calculated over an unloading/reloading cycle:

$$C_s = \left|\frac{(e_1 - e_2)}{(\log \sigma_1 - \log \sigma_2)}\right| = \left|\frac{\Delta e}{(\Delta \log \sigma')}\right| \qquad (6.12)$$

6.3.2.D. Compressibility coefficients and oedometer modulus

If we want to draw the compressibility curve in linear coordinates rather than semi-logarithmic coordinates, we use compressibility parameters defined as follows:

- ***Compressibility coefficient av :***

$$a_v = \frac{\Delta e}{\Delta \sigma'} \qquad (6.13)$$

- ***Volume change coefficient mv :***

$$m_v = \frac{a_v}{1 + e_0} \qquad (6.14)$$

- ***The oedometer modulus Eoed:*** is another representation of soil compressibility as measured by the oedometer. It is defined as follows:

$$E_{oed} = \frac{(1 + e_0)\Delta \sigma'}{\Delta e} = \frac{1}{m_v} \qquad (6.15)$$

6.3.2.E. Calculation of primary settlement using the oedometer method

After plotting the oedometer compressibility curve and calculating the various necessary parameters, the settlement of a homogeneous layer of thickness H0 (on the order of a few meters, at most) can be determined using the following formulas:

- *Normally consolidated soil:* $\sigma'_{v0} = \sigma'_p$

$$S_c = \frac{C_c}{1 + e_0} H_0 \log\left(\frac{\sigma'_{v0} + \Delta\sigma}{\sigma'_{v0}}\right) \qquad (6.16)$$

- *Over-consolidated soil:* $\sigma'_{v0} < \sigma'_p$

Case 1: when $\sigma'_{v0} + \Delta\sigma < \sigma'_p$:

$$S_c = \frac{C_s}{1 + e_0} H_0 \log\left(\frac{\sigma'_{v0} + \Delta\sigma}{\sigma'_{v0}}\right) \qquad (6.17)$$

Case 2: when $\sigma'_{v0} + \Delta\sigma > \sigma'_p$:

$$S_c = \frac{C_s}{1 + e_0} H_0 \log\left(\frac{\sigma'_p}{\sigma'_{v0}}\right) + \frac{C_C}{1 + e_0} H_0 \log\left(\frac{\sigma'_{v0} + \Delta\sigma}{\sigma'_p}\right) \qquad (6.18)$$

6.3.3 Correction of consolidation settlement

The assumption that the soil does not undergo lateral deformation, made in the one-dimensional consolidation study, is true for the oedometer sample. However, this condition cannot hold true for the stratified layer. This condition is valid if the clay layer is thin and sandwiched between layers of rigid granular material, or if

the loaded area is large relative to the thickness of the compressible layer. Otherwise, lateral deformation occurs, and the consolidation will no longer be one-dimensional, which could lead to errors in the calculated settlements.

Skempton and Bjerrum (1957) proposed a semi-empirical correction based on parameter A of Skempton's generation of pore water pressures. The correction factor can be obtained from Figure 6.5 for different H/B ratios; H the thickness of the clay layer and B the width of the foundation.

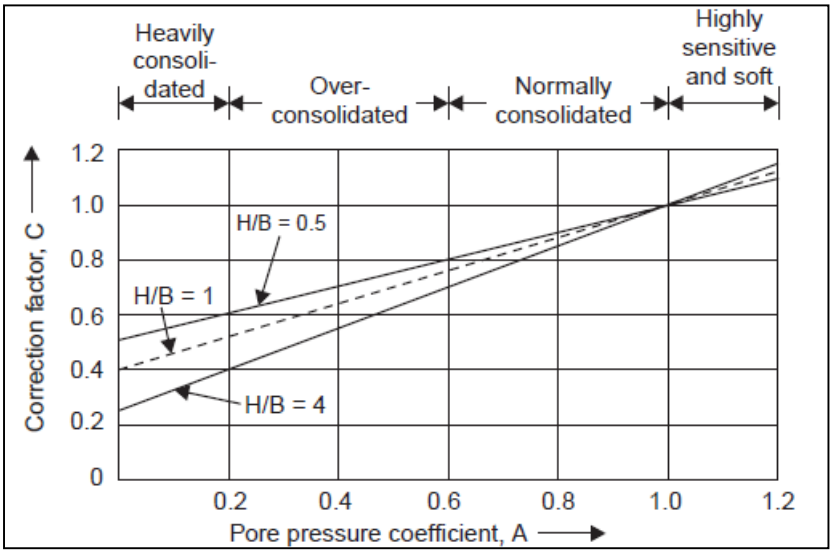


Figure 6.5. Correction factor for the effect of lateral stress on consolidation settlement (after Skempton and Bjerrum, 1957)

The corrected consolidation settlement can be calculated as follows:

$$S_{cc} = C \times S_c \qquad (6.19)$$

Where C is the correction factor and Sc is the calculated consolidation settlement. Accurately evaluating parameter C can be difficult; in this case, it is advisable to not apply the correction due to the uncertainty.

6.4 EVOLUTION OF SETTLEMENT OVER TIME

6.4.1 One-dimensional consolidation theory

The preceding equations, used to calculate the total settlement caused by primary consolidation, provide no information regarding the rate of primary consolidation, that is, the estimated settlement at any given time.

To address this, Terzaghi (1925) proposed the first theory that considers the one-dimensional velocity of consolidation for saturated clay soils. The mathematical method is based on six assumptions.

Consider a compressible clay layer with a thickness of 2H and bounded by two permeable layers. A structure exerting a surcharge $\Delta\sigma$ rests on this soil (Figure 6.6).

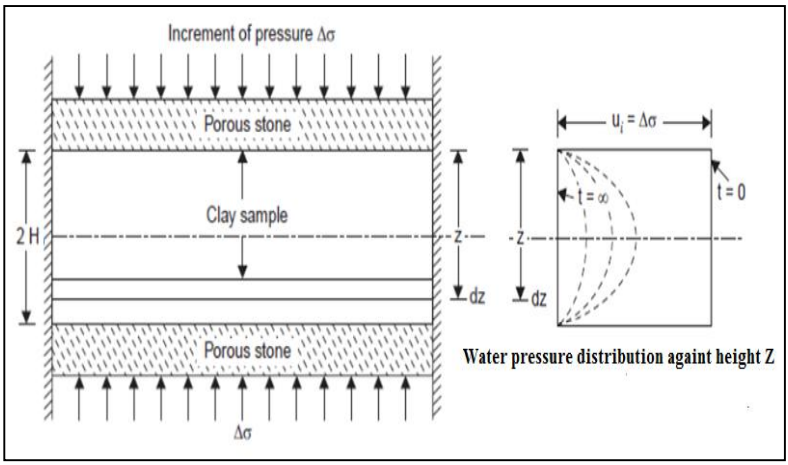


Figure 6.6. Consolidation theory (Consolidation of a clay sample with double drainage)

The flow velocity through the element is given by Darcy's law as:

$$v = k \times i = -k \times \frac{\partial h}{\partial z} \qquad (6.20)$$

In a fixed position z, any change in the total hydraulic head (h) is solely due to a change in the water pressure:

$$v = -\frac{k}{\gamma_w} \times \frac{\partial h}{\partial z} \qquad (6.21)$$

Therefore, the continuity condition can be expressed as:

$$-\frac{k}{\gamma_w} \times \frac{\partial^2 u}{\partial z^2}\, dxdydz = \frac{dV}{dt} \qquad (6.22)$$

The change in volume can be expressed using the coefficient mv:

$$m_v \times \frac{\partial \sigma'}{\partial t}\, dxdydz = \frac{dV}{dt} \qquad (6.23)$$

The increase in effective stress has intensity equal to the decrease in pore water pressure ($\Delta\sigma' = -\Delta u$), and u remains a function of both z and t, the change in volume can be expressed as follows:

$$m_v \times \frac{\partial u}{\partial t}\, dxdydz = \frac{dV}{dt} \qquad (6.24)$$

Combination of Equations 6.23 and 6.24:

$$m_v \times \frac{\partial u}{\partial t} = \frac{k}{\gamma_w} \times \frac{\partial^2 u}{\partial z^2} \qquad (6.25)$$

Where :

$$\frac{\partial u}{\partial t} = c_v \times \frac{\partial^2 u}{\partial z^2} \qquad (6.26)$$

With :

$$c_v = \frac{k}{m_v \times \gamma_w} \qquad (6.27)$$

c_v is the consolidation coefficient, expressed in m^2/year. Since k and m_v are assumed to be constants, c_v is also constant during consolidation.

The differential equation of Terzaghi's consolidation theory (Equation 6.26) can be solved with the following boundary conditions:

$z = 0, u = 0$

$z = 2H, u = 0$

$t = 0, u = u_0$

The solution is:

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_0}{M} \times \sin\left(\frac{M\,z}{H_{dr}}\right)\right] \times e^{-M^2 T_v} \qquad (6.28)$$

With: m: Integer

$M = (\pi/2)(2m+1)$

u_0: initial over-pressure of the water

H_{dr}: Lenght drainage path (function of boundary condition (see Figure 6.9)

and:

$$T_v = \frac{c_v \times t}{H_{dr}^2} \qquad (6.29)$$

Tv is Time factor (dimensionless number).

Where: t is the time after application of surcharge

- ***Degree of consolidation:***

The degree of consolidation at a distance z at a time t is:

$$U_z = \frac{u_0 - u_z}{u_0} = 1 - \frac{u_z}{u_0} \qquad (6.30)$$

u_z: the excess pore water pressure at time t.

Equations 6.28 and 6.30 can be combined to obtain the degree of consolidation at any depth z. (Figure 6.7).

The average degree of consolidation for the entire depth of the clay layer at any time t can be written from Equation 6.30 as follows:

$$U = \frac{S_{c(t)}}{S_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right) \times \int_0^{2H_{dr}} u_z d_z}{u_0} \qquad (6.31)$$

U: average degree of consolidation

Sc (t): settlement at time (t)

Sc: final primary consolidation settlement

The variation of the average degree of consolidation with the time factor Tv represents the case where u_0 is constant for the entire depth of the consolidation layer (Figure 6.8).

The values of the time factor and their corresponding average degrees of consolidation for the case presented in Figure 6.9 can also be approximated by the following simple relationship:

When U = 0 to 60%, $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$

When U > 60%, $T_v = 1{,}781 - 0{,}933\log(100 - U\%)$

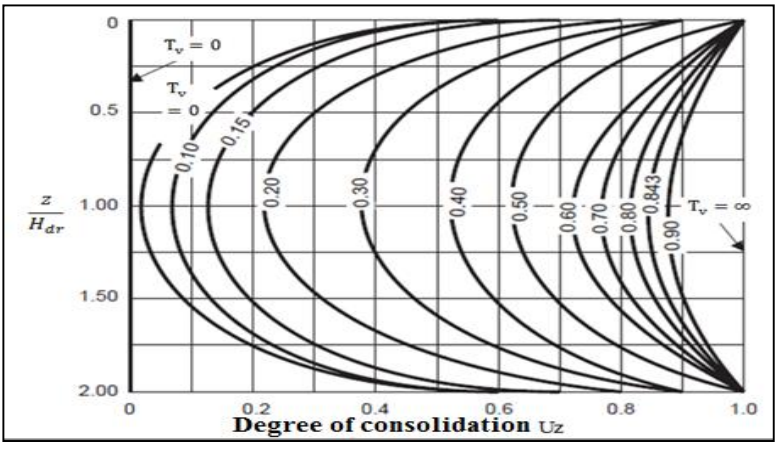


Figure 6.7. Variation of Uz as a function of Tv and z/H_{dr}

Table 6.2 gives the numerical values of U as a function of Tv.

Table 6.2. Some values of U as a function of Tv

U	10%	20%	40%	50%	60%	80%	90%	100%
Tv	0.008	0.031	0.127	0.197	0.287	0.567	0.848	≅2

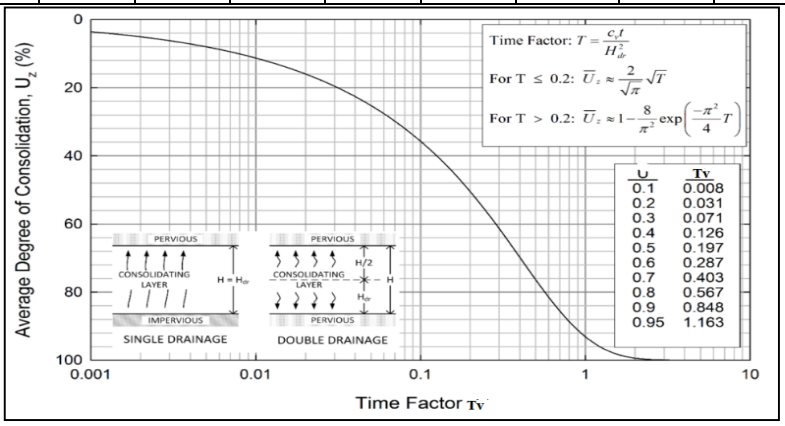


Figure 6.8. Chart representing the function U = f(Tv)

6.4.2 Determination of the consolidation coefficient

The coefficient of consolidation (cv) is determined based on the consolidation curve (settlement over time for a constant load) following an oedometer test. Two graphical methods are commonly used to determine cv: the first is the logarithm of time method developped by Casagrande and Fadum (1940), and the second is the square root of time method given by Taylor (1942). More recently, at least two other methods have been proposed: the hyperbola method (Sridharan and Prakash, 1985) and the early stage log-t method (Robinson and Allam, 1996).

It should be noted that once the value of cv has been determined, the permeability coefficient can be calculated from Equation 6.23, the oedometer test being a useful method for obtaining the permeability of fine-grained soils.

6.4.2.A. Casagrande method

In this method, the settlement is plotted as a function of the logarithm of time (Figure 6.9). We observe that the curve begins with a downward concave segment, followed by a short linear segment and finally a segment with an upward curvature. The objective of this method is to determine the time required to reach 50% of the primary consolidation settlement (t50). This is done by following these steps:

1) Extend the linear portions of the primary and secondary consolidations to intersect at C. The ordinate of C is represented by d100, that is, the deformation at the end of the primary consolidation at 100%.
2) The initial strain d0 corresponding to U = 0% is determined. To do this, two points on the curve (t_1, A) and (t_2, B) are taken; the value of t_2 is four times that of t_1, and the value of B should be between one-quarter and one-half of the total settlement produced during the loading plateau. The initial strain d_0 corresponds to the value of A minus the difference between strains A and B.

3) We calculate the deformation d_{50} and the time t_{50} corresponding to 50% of the primary consolidation settlement.

4) For 50% primary consolidation, the time factor Tv = 0.197; therefore:

$$c_v = \frac{0{,}197 \times H_{dr}^2}{t_{50}} \qquad (6.32)$$

For samples drained from both the top and bottom, Hdr is equal to half the average sample height during consolidation. For samples drained from only one side, Hdr is equal to the average sample height during consolidation (Figure 6.10).

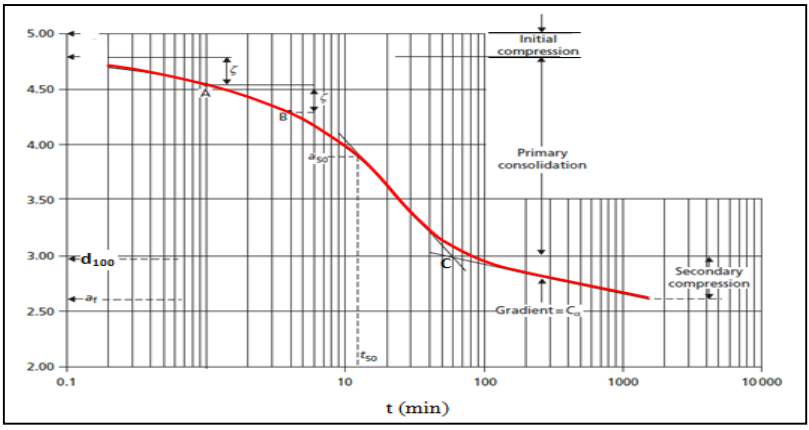


Figure 6.9. Consolidation curve (evolution of settlement over time) (Casagrande method)

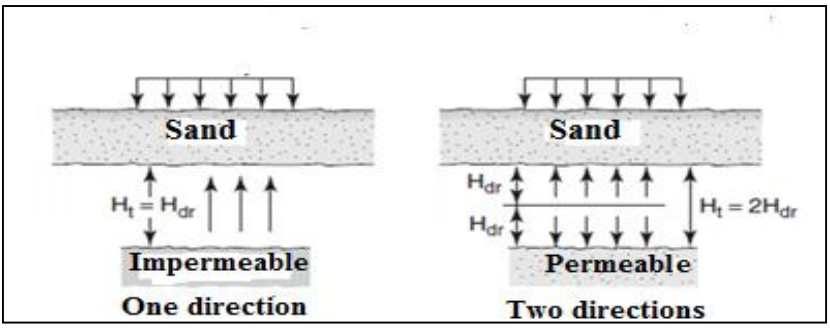


Figure 6.10. Drainage length in soils

2.4.2.B. Taylor's method

The relationship between deformation and the square root also allows us to estimate the consolidation coefficient. The shape of the curve in Figure 6.11 shows that the settlement progresses rapidly at first, then increasingly slowly, eventually reaching zero. Since the curve does not become perfectly horizontal, Taylor (1942) developed a graphical method to evaluate the time required for 90% of the settlement to occur (t90).

The steps to follow this method are:

1) Draw a line AB through the first part of the curve
2) Draw a line AC whose slope deviates by 15° from that of AB. The x-coordinate of point D, which is the intersection of AC and the consolidation curve, gives the square root of the time for 90% consolidation.
3) For 90% consolidation, the Tv = 0.848; therefore:

$$c_v = \frac{0{,}848 \times H_{dr}^2}{t_{90}} \qquad (6.33)$$

Orders of magnitude of Cv depending on soil type:

- Clays: $2 \times 10^{-8} < c_v < 4 \times 10^{-7}\ m^2/s$
- Sandy clays: c_v order of $10^{-7}\ m^2/s$
- Silts: c_v order of $5 \times 10^{-6}\ m^2/s$

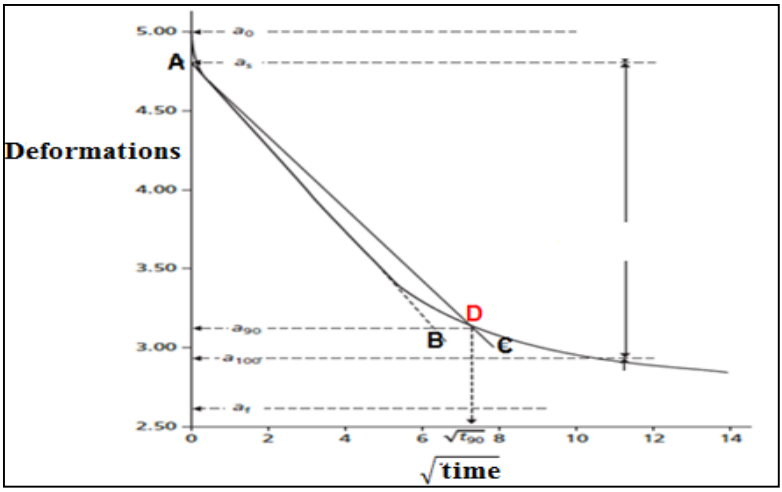


Figure 6.11. Square root method for determining cv (Taylor's method)

6.5 SECONDARY CONSOLIDATION SETTLEMENT

Secondary settlement is the change in volume that occurs immediately after the primary consolidation process. The difference between primary and secondary consolidation lies in the fact that the latter occurs under constant effective stress, after all pore water pressures have dissipated. Secondary settlement can be attributed to compression of the bonds between clay particles or to other phenomena occurring at the microscopic level.

Referring to the consolidation curve shown in Figure 6.10, we see that the secondary settlement segment is linear; from this, we can define a secondary compression index as follows:

$$C_\alpha = \frac{\Delta e}{\log(t_2/t_1)} \qquad (6.34)$$

With Δe: variation of the void index between times t_1 and t_2.

This definition is analogous to that of the primary compression index Cc given by equation 6.10. The modified secondary compression index C'_α will be further defined as follows:

$$C'_\alpha = \frac{C_\alpha}{1 + e_p} \qquad (6.35)$$

Where e_p: the index of gaps at the beginning of the linear segment of the curve of e as a function of t.

The secondary consolidation settlement is therefore calculated using the following formula:

$$S_s = C'_\alpha \times H_0 \times \log\left(\frac{t_2}{t_1}\right) \qquad (6.36)$$

Mesri and Godlewski (1977) gave the $C\alpha$/Cc ratio for a number of natural clays. It is noted that the average value of this ratio is 0.05 and never exceeds 0.1.

- Inorganic silts: 0.035 ≈ 0.06
- Mexico clay: 0.03 ≈ 0.035
- Varved clay (Canada): 0.03 ≈ 0.06
- Organic clayey silt from New Haven: 0.04 ≈ 0.075

6.6 EXERCISES

Exercise 6.1

Consider a soil mass composed of a 2 m thick layer of saturated clay, drained on both sides. Following surface loading, the layer achieved 50% of its final settlement in 85 hours.

1- Determine the time required to reach 70% of the final settlement.

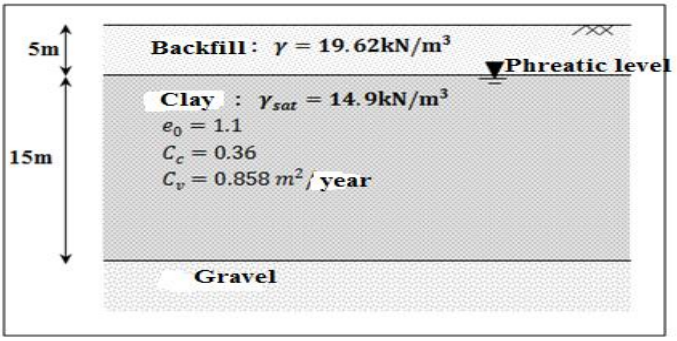
2- Evaluate the degree of UV consolidation after 10 hours.

Exercise 6.2

The soil profile illustrated below. The water table is 5m below the soil surface. In this case, the clay is normally consolidated.

1- Calculate the consolidation settlement at mid-height of the clay layer produced by the backfill by considering: A) a single layer, B) the clay layer is divided into 5 layers.

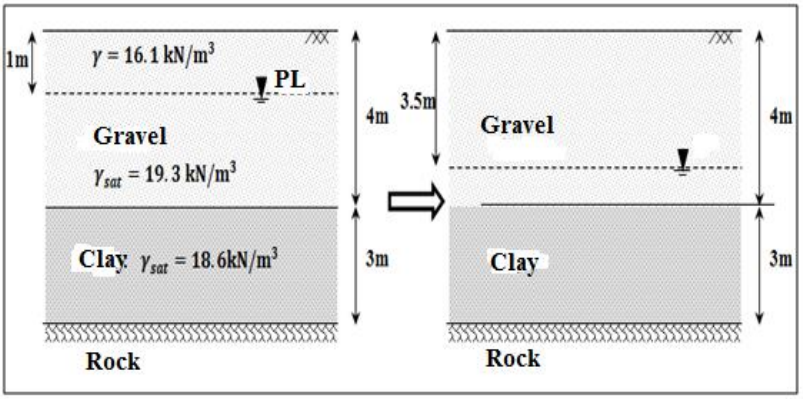
2- Calculate the settlement and the time to reach it during consolidation.



Exercise 6.3

Consider the soil profile illustrated below, consisting of a 4m thick layer of gravel overlying a 3m thick layer of clay resting on bedrock. Initially, the water table is 1m below the soil surface. Due to construction work, its level is lowered by 2.5m over a period of 2 years. At the end of this period, the clay has consolidated by 50%, and the corresponding settlement is 70mm.

Calculate the coefficient C_v $(m^2/year)$ and k (m/s)?



Exercise 6.4

A clay sample is subjected to an oedometer test, the results of which are shown in the table above. The initial height of the sample is 25 mm, and its void ratio $e_0 = 1{,}01$.

1- Plot the compressibility curve (e-logσ'_v).

2- Calculate the compression index at the stress interval of 640 kN/m^2 and 1280 kN/m^2.

Stresses (kN/m^2)	Settlement $\Delta H \quad (mm)$
10	0.02
20	0.03
40	0.05
80	0.1
160	0.19
320	0.43
640	1.09
1280	1.78
160	1.58
40	1.43
10	1.22

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