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To the jury members for their interest in our research by agreeing to examine our work and enrich it.

Dedication

We dedicate this humble work to:

- ✓ *To our parents*
- ✓ *To our brothers*
- ✓ *To our sisters*
- ✓ *To our families*

You have given your all for me without hesitation. In recognition of every sacrifice made by each one of you to help me reach this stage of my life. You have all contributed, directly or indirectly, to my academic and personal growth.

To all our friends and colleagues in the lecture halls, and to everyone belonging to the Faculty of Science and Technology

at Echahid cheikh Larbi Tebessi Universit-, Tebessa.

Sebti Bourahla & Zarkane Nidhal

Abstract:

This thesis explores the application of a Fuzzy Inference System (FIS) for the classification of electrocardiogram (ECG) signals, addressing the challenges posed by their nonlinearity, variability, and susceptibility to noise. The study is structured into four chapters: First, it introduces the fundamentals of nonlinear signals, emphasizing their importance in modeling complex biological systems like the ECG. The second chapter delves into cardiac physiology and the ECG's role as a diagnostic tool, detailing its waveform components and recording methodologies. The third chapter compares traditional signal classification methods (e.g., Discrete Fourier Transform, Broida, and Strejc techniques) with artificial intelligence approaches, highlighting the advantages of fuzzy logic in handling uncertainty. Finally, the fourth chapter presents simulation results using MATLAB, demonstrating the efficacy of the proposed FIS-based method in identifying and classifying ECG signals with minimal error. The research underscores the potential of fuzzy logic for enhancing diagnostic accuracy in clinical settings, paving the way for future integration into AI-assisted healthcare tools.

Keywords:

Electrocardiogram (ECG), Fuzzy Inference System (FIS), Nonlinear Signals, Signal Classification, Biomedical Signal Processing

Résumé :

Cette thèse explore l'application d'un Système d'Inférence Floue (SIF) pour la classification des signaux d'électrocardiogramme (ECG), en abordant les défis posés par leur non-linéarité, leur variabilité et leur sensibilité au bruit. L'étude est structurée en quatre chapitres : Premièrement, elle introduit les fondamentaux des signaux non linéaires, en soulignant leur importance pour la modélisation de systèmes biologiques complexes comme l'ECG. Le deuxième chapitre approfondit la physiologie cardiaque et le rôle de l'ECG comme outil de diagnostic, détaillant les composantes de sa forme d'onde et les méthodologies d'enregistrement. Le troisième chapitre compare les méthodes traditionnelles de classification des signaux (par exemple, la Transformation de Fourier Discrète, et les techniques de Broida et Strejc) avec les approches d'intelligence artificielle, mettant en avant les avantages de la logique floue pour gérer l'incertitude. Enfin, le quatrième chapitre présente les résultats de simulation obtenus avec MATLAB, démontrant l'efficacité de la méthode proposée basée sur le FIS pour identifier et classer les signaux ECG avec une erreur minimale. La recherche souligne le potentiel de la logique floue pour améliorer la précision diagnostique en milieu clinique, ouvrant la voie à son intégration future dans des outils de santé assistés par l'IA.

Mots-clés :

Électrocardiogramme, système d'inférence floue, signaux non linéaires, classification des signaux, traitement du signal biomédical.

الملخص:

تستكشف هذه الأطروحة تطبيق نظام الاستدلال الضبابي لتصنيف إشارات تخطيط القلب الكهربائي ، مع معالجة التحديات التي تفرضها طبيعتها غير الخطية وتغيرها وحساسيتها للضوضاء. تنقسم الدراسة إلى أربعة فصول: أولاً، تقدم أساسيات الإشارات غير الخطية، مؤكدةً على أهميتها في نمذجة الأنظمة البيولوجية المعقدة مثل إشارة تخطيط القلب. ويغطي الفصل الثاني فسيولوجيا القلب ودور تخطيط القلب كأداة تشخيصية، مع تفصيل مكونات الموجة وطرق التسجيل. ويقارن الفصل الثالث بين طرق التصنيف التقليدية للإشارات (مثل تحويل فوربيه المنفصل، وتقنيات برويدا وستريتس) ومناهج الذكاء الاصطناعي، مسلطاً الضوء على مزايا المنطق الضبابي في التعامل مع عدم اليقين. أخيراً، يعرض الفصل الرابع نتائج محاكاة باستخدام برنامج ماتلاب ، موضحاً فعالية الطريقة المقترحة القائمة على نظام الاستدلال الضبابي في تحديد وتصنيف إشارات تخطيط القلب بأقل حد من الخطأ. وتؤكد الأبحاث على إمكانات المنطق الضبابي لتعزيز دقة التشخيص في البيئات السريرية، ممهدةً الطريق لدماجه المستقبلي في أدوات الرعاية الصحية المعززة بالذكاء الاصطناعي.

الكلمات المفتاحية:

تخطيط كهربائية القلب، نظام الاستدلال الضبابي، إشارات غير خطية، تصنيف الإشارات، معالجة الإشارات الحيوية الطبية.

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Abbreviations:

ECG : electrocardiogram

EMG *:* Electromyogram

EEG : Electroencephalography

MUEMG : multiple unit electromyogram

fNIRS: functional near-infrared spectroscopy

MRI: magnetic resonance imaging

PET: positron emission tomography

AVN: atrioventricular node

SN: sinus node

TIA: transient ischemic attack

DFT: Discrete Fourier Transform

FIS: fuzzy inference system

PSO: Particle Swarm Optimization

IFSA: International Fuzzy Systems Association

ANNs: Artificial Neural Networks

MSE: Mean Square Error

Ai: Intelligence Artificielle

MSE: Mean Square Error

AFib: Atrial fibrillation

MI: Myocardial Infarction

AFL: Atrial flutter

LVH: Left Ventricular Hypertrophy

MISO: multi_inputs signle output



General introduction

General introduction:

The heart, this vital organ whose rhythms govern human existence, has captivated scientists since antiquity **[1]**. In the early 20th century, Willem Einthoven revolutionized cardiology by inventing the electrocardiograph (1903) **[2]**, enabling the first translation of cardiac electrical activity into a visual tracing: the electrocardiogram (ECG) **[3].** This biomedical signal a subtle reflection of cardiomyocyte depolarization and repolarization has become the cornerstone diagnostic tool for cardiac pathologies, from arrhythmias to myocardial infarction **[4]**. Nevertheless, its interpretation remains a major challenge: characterized by nonlinear morphology, pronounced interindividual variability, and sensitivity to artifacts (respiration, motion, electrical noise) **[5],** ECG demands analytical methods capable of modeling the inherent uncertainty of biological systems .

The electrocardiogram (ECG) constitutes a fundamental diagnostic tool in cardiology, capturing the heart's electrical activity as complex nonlinear signals. Accurate classification of these signals distinguishing normal from pathological rhythms relies on robust analytical methods able to manage their variability and dynamic nature **[6] .**

This thesis proposes to explore the application of a Fuzzy Inference System (FIS) for ECG signal classification, leveraging its capacity to model uncertainty and expert knowledge through back propagation algorithm. This approach aligns with the rise of artificial intelligence (AI) techniques applied to biomedical systems, offering a flexible alternative to neural networks or optimization methods such as Particle Swarm Optimization (PSO).

The study is structured into four chapters :

- **Nonlinear Signals**: this chapter introduces the fundamental concepts of nonlinear signals and systems, contrasting them with linear systems by highlighting their complex, non-additive behaviors and rich dynamics. It explores the importance of nonlinear signals in modeling real-world phenomena, such as biomedical applications (ECG, EMG, EEG), and discusses their role in capturing intricate interactions that linear systems cannot represent. The chapter concludes by emphasizing the necessity of nonlinear analysis for accurate system representation and advanced signal processing.
- **ECG Signal:** this chapter explores the heart's physiology, detailing its dual-pump structure, cardiac cycle, and electrical conduction system, which coordinates rhythmic

contractions. It explains the electrocardiogram (ECG) as a vital diagnostic tool, describing its waves (P, QRS, T) and their relation to cardiac activity. The chapter also covers ECG recording techniques, electrode placement, and key intervals (PR, QT, RR) used to assess heart function and detect abnormalities.

- **Identification and Classification of Dynamical Systems:** this chapter delves into signal identification and classification, comparing traditional methods (e.g., Discrete Fourier Transform, Broida, and Strejc techniques) with AI-based approaches like Fuzzy Logic, and Particle Swarm Optimization (PSO). It explains how Fuzzy Logic handles uncertainty through graded membership. The chapter also explores the differences between Mamdani and Sugeno inference systems and highlights PSO's role in optimizing classification tasks. These methods are pivotal for applications in biomedical signal processing, telecommunications, and industrial automation. The next chapter will apply Fuzzy Logic to ECG signal classification.
- **Simulation Results:** In this chapter, we'll apply the algorithm used (FIS) in MATLAB, presenting a discussion of the results, and finally, a general conclusion.

The originality of this work lies in adapting the FIS to the specificities of ECG signals notably their morphology and stochasticity to optimize diagnostic accuracy. The simulations presented in Chapter 4 demonstrate this method’s potential for future integration into AI-assisted clinical tools .

The originality of this work lies in adapting the FIS to the specificities of ECG signals notably their morphology and stochasticity to optimize diagnostic accuracy. The simulations presented in Chapter 4 demonstrate this method’s potential for future integration into AI-assisted clinical tools.



chapter1:

Nonlinear signals

1. Introduction:

The fundamental distinction between signals and systems, is their function: a signal conveys information, whereas a system manipulates or processes that information. The signal acts as the input to the system, which then generates an output signal often an altered or transformed version of the original input.

Nonlinear systems and signals play a central role in understanding and modeling complex phenomena, whether in engineering, physics, biology, or other scientific fields. Unlike linear systems, which adhere to the principles of additivity and homogeneity, nonlinear systems are characterized by richer and often unpredictable dynamic behaviors. These behaviors emerge from complex interactions where effects are not simply additive but can include phenomena such as saturation, feedback loops, or even chaos **[7].**

In this chapter, we will explore the fundamental concepts of nonlinear systems and signals, highlighting their differences compared to linear systems. We will also discuss the reasons why nonlinear signals are essential in many fields, as well as the different types of nonlinear signals and their practical applications.

2. Nonlinear systems and signals: basic concepts:

A nonlinear system is defined as any system that does not meet the criteria of linearity, specifically additivity and homogeneity. In this context, a nonlinear signal is one that cannot be generated by a single-input, single-output linear system unless the system is driven by an input that is a linear transformation of the original nonlinear signal. Essentially, the only way to produce a nonlinear signal as the output of a linear system is to provide the system with an input that has a linear relationship to the nonlinear signal. While nonlinear signals must originate from nonlinear systems, it is worth noting that nonlinear systems can, in many cases, produce signals that are indistinguishable from those generated by linear systems **[8].**

3. The difference between nonlinear and linear system:

	Parameter	Linear Systems	Non-Linear Systems
01	Principle	Linear systems follow the principle of superposition, meaning that the response of the system to any input signal is the sum of the responses to each individual component of the input signal.	Nonlinear systems do not follow the principle of superposition and the output of the system can be affected by the interaction of multiple input signals.
02	Proportionality	The response of a linear system is proportional to the input signal.	The response of a nonlinear system is not proportional to the input signal.
03	Input-Output Relationship	The input-output relationship of a linear system is described by a linear function.	The input-output relationship of a nonlinear system is described by a nonlinear function.
04	Coefficients	Linear systems have constant coefficients.	Nonlinear systems may have variable coefficients.
05	Additivity	The input-output relationship of a linear system is additive.	The input-output relationship of a nonlinear system may not be additive.
06	Homogeneity	The input-output relationship of a linear system is homogeneous, meaning that the output signal is a scaled version of the input signal.	The input-output relationship of a nonlinear system may not be homogeneous.
07	Stability	Linear systems are generally stable.	Nonlinear systems can be either stable or unstable.
08	Time-Invariance	Linear systems are time-invariant, meaning that the response of the system to any input signal is independent of the time at which the input is applied.	Nonlinear systems may be time-varying, meaning that the response of the system to any input signal is dependent on the time at which the input is applied.
09	Boundedness	The response of a linear system to a bounded input signal is also bounded.	The response of a nonlinear system to a bounded input signal may not be bounded.
10	Linearity	Linear systems are linear	Nonlinear systems are nonlinear

Table 1: Linear systems and non-linear systems [9].

4. Why we choose nonlinear signals:

The choice of using or analyzing nonlinear signals often arises in various fields such as signal processing, control systems, communications, and physics. Nonlinear signals are essential because they capture behaviors and phenomena that linear systems and signals cannot represent. Below are the key reasons for choosing nonlinear signals:

4.1 Modelling Real-World Phenomena:

Many real-world systems and signals are inherently nonlinear. For example, biological systems, mechanical systems with friction, and electronic circuits with saturation or hysteresis exhibit nonlinear behavior. Linear systems are often approximations of these nonlinear systems, but they fail to capture the full complexity of the real world **[10].**

4.2 Capturing Complex Dynamics:

Nonlinear signals can exhibit rich and complex dynamics such as chaos, bifurcations, and harmonics, which are not possible in linear systems. These dynamics are critical in fields like weather prediction, fluid dynamics, and economic modeling **[11].**

4.3 Improved Signal Processing and Control:

Nonlinear signal processing techniques can provide better performance in tasks like noise reduction, signal detection, and system identification compared to linear methods. For example, nonlinear filters can handle non-Gaussian noise and non-stationary signals more effectively **[12].**

4.4 Nonlinear System Identification and Control:

Many control systems, such as robotics, aerospace systems, and autonomous vehicles, require nonlinear models to achieve precise control. Linear approximations often fail to provide the necessary accuracy for these applications **[7].**

4.5 Energy Efficiency and Performance Optimization:

Nonlinear systems and signals can often achieve higher energy efficiency and performance compared to linear systems. For example, nonlinear oscillators and resonators are used in energy harvesting and MEMS (Micro-Electro-Mechanical Systems) **[13].**

5 Types of nonlinear signals :

Nonlinear signals can be classified into different types based on their characteristics,

generation mechanisms and applications, here are some types of nonlinear signals:

5.1 Deterministic Signal:

Deterministic signals are predominantly featured in theoretical and textbook contexts but are less prevalent in practical, real-world applications. A deterministic signal is characterized by the property that its future values can be precisely predicted with absolute certainty, provided sufficient historical data is available. In many cases, only a minimal amount of past information is required. For instance, knowledge of a single cycle of a sinusoidal waveform enables the exact prediction of its values at any future point in time. Commonly referenced signals in academic literature, such as impulse functions, step functions, and exponential functions, are inherently deterministic. More broadly, any signal that can be explicitly represented in a closed-form mathematical expression as a function of time is classified as deterministic **[14].**

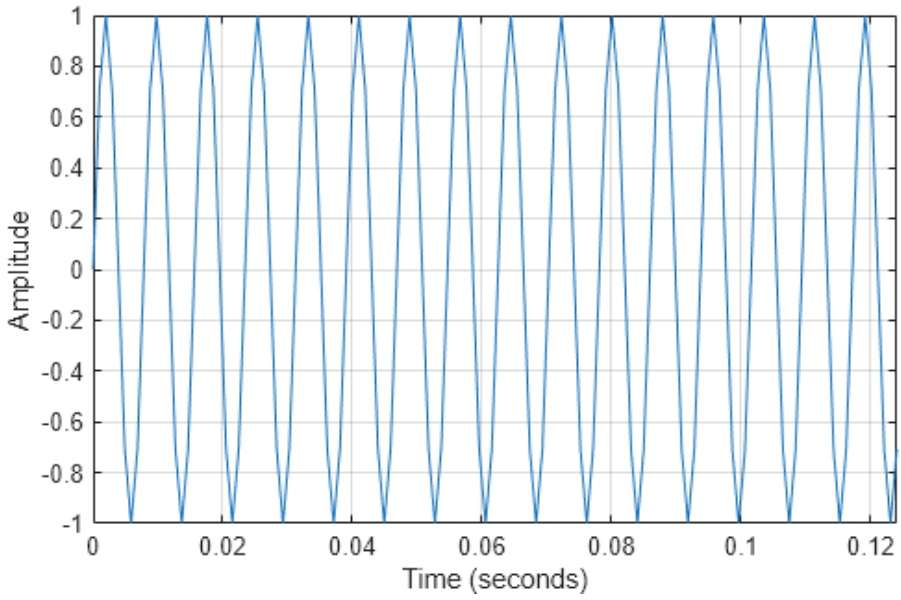


Figure 1.1: Deterministic signal [15].

5.2 Chaotic signals:

Chaotic signals are a class of deterministic signals whose future behavior cannot be precisely predicted, despite their deterministic nature. This apparent contradiction arises from the property of sensitive dependence on initial conditions, which is a hallmark of chaotic systems. For certain deterministic signals, their future trajectories are so highly sensitive to their initial conditions that even infinitesimal uncertainties in the specification of past values render

long-term prediction impractical. Consequently, while these signals are theoretically deterministic, their predictability diminishes rapidly over time, as even minor errors in initial conditions lead to exponentially growing deviations in the predicted trajectory. As a result, chaotic signals exhibit behavior that visually resembles randomness, although they are fundamentally distinct from random signals (but random signals are not chaotic and chaotic signals are not random) **[16].**

Applications: Used in secure communications, encryption, and modeling complex systems like weather patterns **[8].**

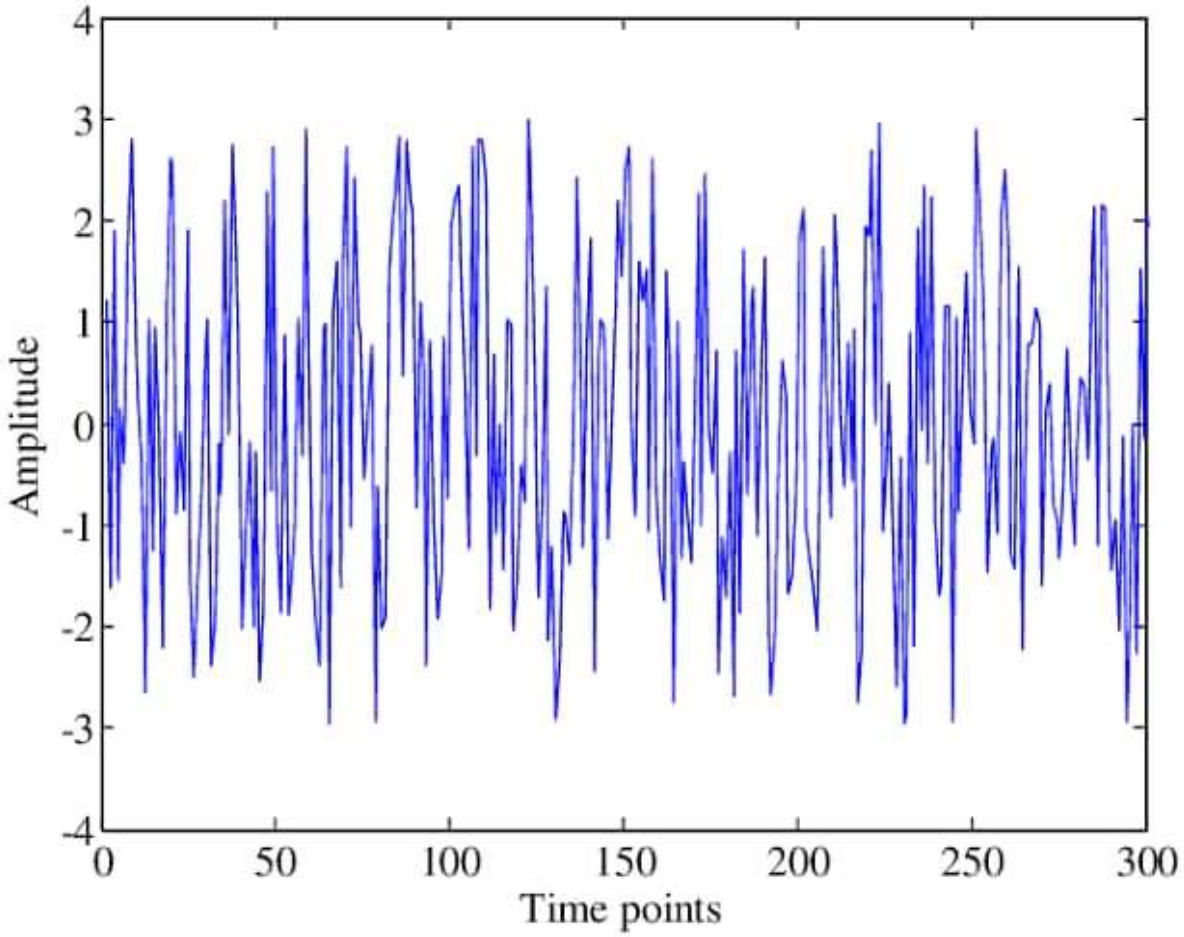


Figure 1.2: Chaotic signal [17].

5.3 Random signals and noise:

In the field of electrical engineering, it is common to encounter signals that lack a precise mathematical description, as they evolve as random functions of time. This randomness may arise from a single random variable or, more frequently, from the combined influence of multiple random variables. In some cases, the underlying causes of randomness are unclear,

making it impossible to provide a definitive description, and the signal can only be characterized through empirical measurements **[18].**

A random time function can represent either a desired signal, such as an audio or video signal, or an unwanted signal that is inadvertently superimposed onto a desired (information-carrying) signal, thereby distorting it. The desired signal is referred to as a random signal, while the unwanted signal is termed noise. However, noise often does not conform to the classical definition; instead, it behaves more like interference. In such instances, it also carries information but is considered undesirable. Generally, it is not possible to completely separate a desired signal from noise (or interference). However, through well-designed signal processing techniques in a receiver, the desired signal can be maximally enhanced while the disturbance is minimized to the greatest extent possible **[19].**

In all scenarios, a clear description of the signals is essential to analyze their impact on the performance of the system in question. This challenge is particularly prevalent in communication theory. The inherent randomness of these signals as a function of time complicates their description, as well as their processing or filtering. Despite these challenges, it is crucial to characterize such signals using a limited set of deterministic parameters, enabling system users to evaluate system performance effectively. The key tool for addressing both random signals and noise is the concept of the stochastic process **[20].**

5.3.1 The Concept of the Stochastic Process:

In probability theory, a random variable is a rule that assigns a number to every outcome of an experiment, such as rolling a die. This random variable $\boldsymbol{X}$ is associated with a sample space $\boldsymbol{S}$, such that for each event $\boldsymbol{s}$ in $\boldsymbol{S}$, a numerical value is assigned according to a well- defined procedure. **(Fig (1.3))** illustrates a few sample functions of a stochastic process, which represents a collection of random variables indexed by time or another parameter: **[21]**

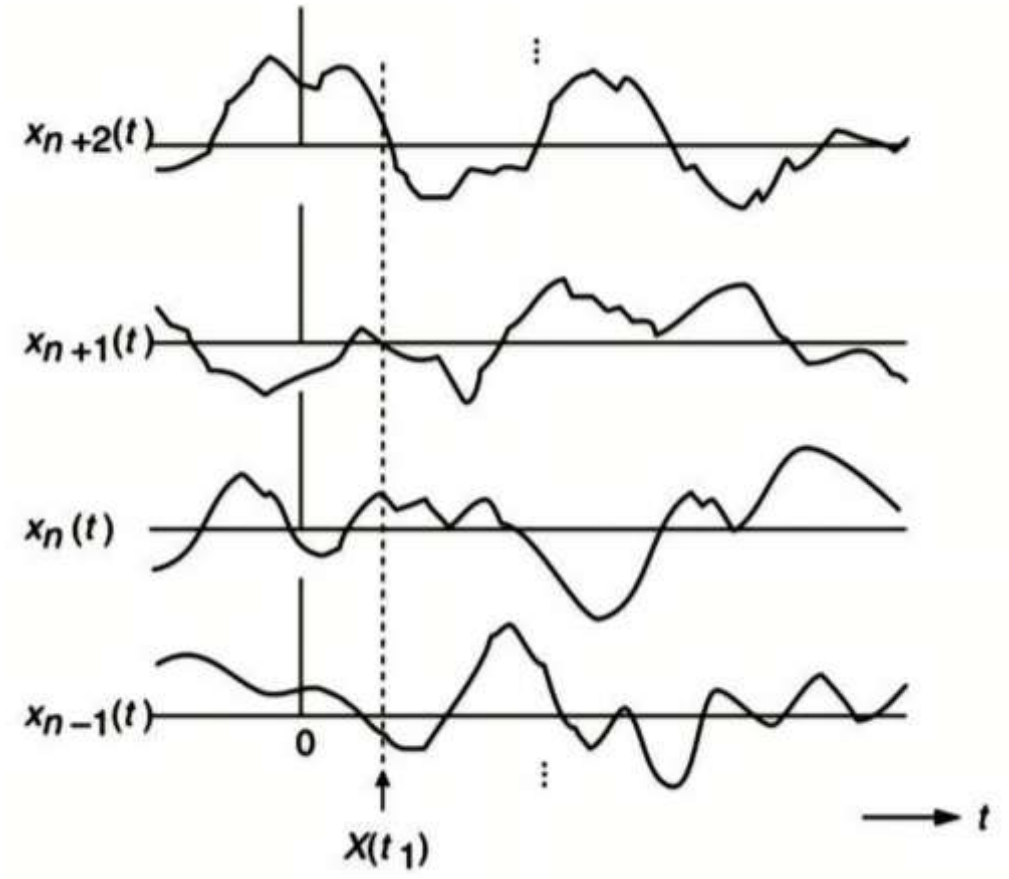


Figure 1.3: A few simple functions of a stochastic process [22].

In probability theory, a random variable **X** assigns a number $\mathbf{X}(\boldsymbol{s})$ to each outcome s in a sample space $\boldsymbol{S}$. For stochastic processes, however, a time-dependent function $\mathbf{X}(\boldsymbol{t}, \boldsymbol{s})$ is assigned to each outcome in the sample space. Within the context of an experiment, the collection (or ensemble) of all possible functions that can be realized is referred to as the stochastic process, denoted by $\mathbf{X}(\boldsymbol{t}, \boldsymbol{s})$. A specific waveform from this collection is called a sample function or a realization of the stochastic process, denoted as $\boldsymbol{X_{in}}(\boldsymbol{t})$. When referring to a realization in general, the subscript $\boldsymbol{n}$ is omitted. **(Fig (1.3))** illustrates a few sample functions that form an ensemble. While the figure shows a finite number of realizations, the ensemble may consist of an infinite or even uncountable number of realizations **[23].**

A realization itself is sometimes also referred to as a stochastic process. Additionally, a stochastic process generates a random variable when time t is fixed and s is allowed to vary. For example, the random variable $\mathbf{X}(\boldsymbol{t_1}, \boldsymbol{s}) = \mathbf{X}(\boldsymbol{t_1})$ is obtained by examining the ensemble of realizations at a fixed time $\boldsymbol{t_1}$ (see **Fig (1.3)**). This random variable $\mathbf{X}(\boldsymbol{t_1})$, also denoted as $\boldsymbol{x_1}$, describes the statistical properties of the process at time $\boldsymbol{t_1}$. The expectation of $\boldsymbol{x_1}$is called the ensemble mean, expected value, or simply the mean of the stochastic process at

time t_1. Since t_1 can be chosen arbitrarily, the mean of the process is generally not constant and may vary with time **[24].**

Finally, a stochastic process can also represent a single number when both t and s are fixed.

Thus, the term "stochastic process" can have four distinct interpretations:

- An ensemble of time functions: Both t and s are variables.
- A single time function (sample function or realization): t is a variable, and s is fixed.
- A random variable: t is fixed, and s is variable.
- A single number: Both t and s are fixed **[25].**

5.3.2 Classification of Stochastic Processes:

The classification of stochastic processes depends on the nature of their realization values $\mathbf{X}(t)$ and the time parameter t, both of which can be either continuous or discrete. Based on these characteristics, stochastic processes can be categorized into the following classes:

1. Continuous Stochastic Process: Both the values of $\mathbf{X}(t)$ and the time parameter t are continuous.
2. Discrete-Time Process (Continuous Random Sequence): The values of $\mathbf{X}(t)$ are continuous, but the time parameter t is discrete
3. Discrete Stochastic Process: The values of $\mathbf{X}(t)$ are discrete, while the time parameter t is continuous.
4. Discrete Random Sequence: Both the values of $\mathbf{X}(t)$ and the time parameter t are discrete **[26].**

$\mathbf{X}(t)$	Time	
	Continuous	Discrete
Continuous Discrete	Continuous stochastic process Discrete stochastic process	Discrete-time process Discrete random sequence

Table 2: Summary of names of different processes [22].

5.4 Biomedical Signals:

The term BIOMEDICAL SIGNAL is commonly used to refer to time-based

measurements obtained from biological mechanisms, essentially representing a form of energy produced by these mechanisms. Examples of such signals include the electroencephalogram (EEG), which corresponds to the electrical activity of the brain recorded by electrodes placed on the scalp; the electrocardiogram (ECG), which represents the electrical activity of the heart recorded from the chest; and the electromyogram (EMG), which is recorded from the skin and corresponds to the electrical activity generated by skeletal muscles **[27].**

Nowadays, biomedical signals such as EEG and ECG are extensively analyzed to diagnose conditions like cardiac arrhythmias in the case of ECG, or epilepsy, memory impairments, and sleep disorders in the case of EEG **[28].**

5.4.1 Some Typical Sources of Biomedical Signals:

The sources of biomedical signals are infinite in variety and we shall identify only a few as examples of the different types of these signals. A large class of biomedical signals comprises those which are electrical in nature **[29].**

5.4.1.1 ECG Signal:

The electrocardiogram (ECG), which can be recorded by placing electrodes on the chest, is an example of this type of signal. It records the electrical potential at the electrode (or the potential difference between two electrodes) induced by the presence of time-varying electrical activity within the cardiac muscle, associated with the generation and propagation of action potentials. Each heartbeat produces a sequence of electrical waves (the P, Q, R, S, and T waves), and by examining the shape of the ECG waveforms **(Fig (1.4)),** a physician can gain significant insights into whether the heart's contractions are occurring normally. In practice, the physician typically compares electrocardiogram signals from multiple sites on the body. Although visual inspection of ECG signals is clinically useful, the analysis of electrocardiograms can be highly sophisticated and involve extensive signal processing. For instance, there is considerable interest in detecting small changes in the shape of the ECG waveform that occur after the main peak (the R-wave) during the period when the ventricles are recovering from their contraction [**30].**

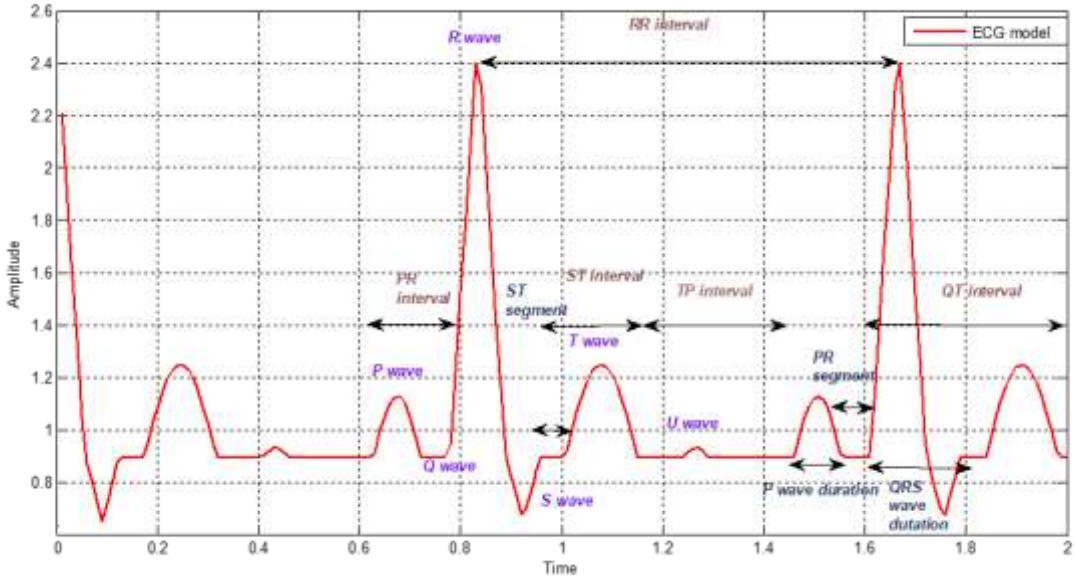


Figure 1.4: ECG signal [31].

5.4.1.2 EMG Signal:

Electromyogram (EMG) signals are recorded by placing electrodes in, on, or near a muscle and amplifying the electrical potential (or the potential difference between two electrodes) generated by the creation and propagation of action potentials along muscle fibers. A multiple-unit EMG (MUEMG) captures the electrical activity from numerous muscle fibers simultaneously **(Fig (1.5)).** The presence of MUEMG activity indicates that a muscle is actively contracting. Many researchers have worked on developing techniques to filter MUEMG signals to produce a signal proportional to the force generated by the muscle. This objective is significant because it would allow the estimation of muscle force without the need to attach a force transducer directly to the muscle a task that is often impractical, especially in human subjects. (For instance, how could one attach a force transducer to the extraocular muscles?)

MUEMG signals are also used in rehabilitation engineering to control prosthetic devices. The subject is trained to voluntarily contract a muscle they can still control, and the amplitude of the MUEMG signal is used as an indicator of the desired level of prosthesis activation. For example, it can determine the gripping force of an artificial hand. However, since MUEMG signals are inherently noisy, they must be filtered to extract a smoothed signal that varies with the amplitude of the MUEMG **[32].**

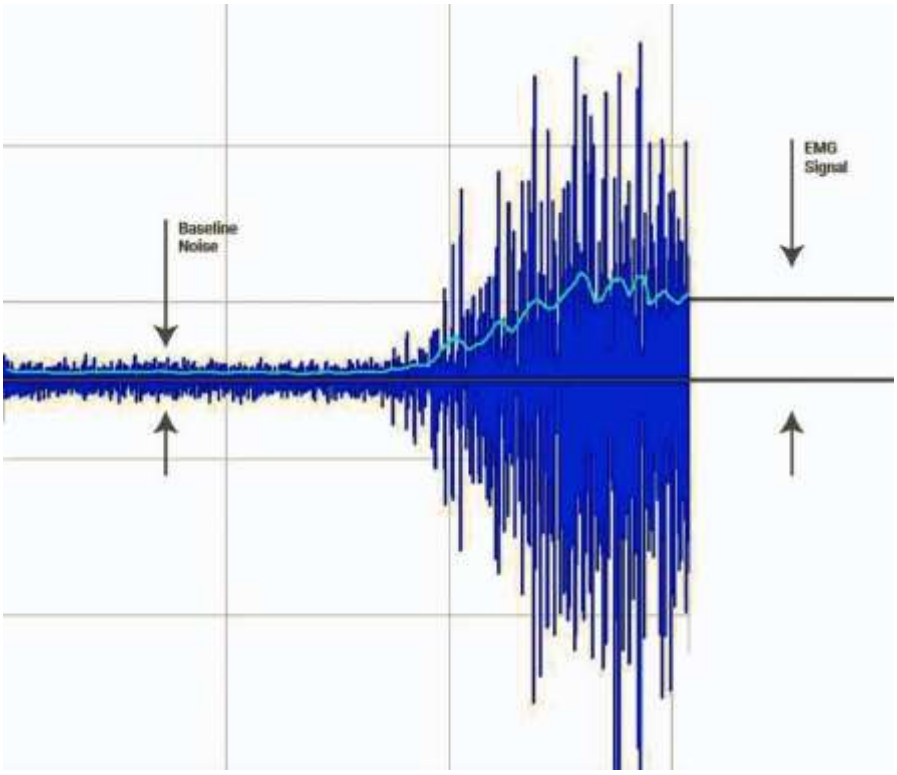


Figure 1.5: EMG SIGNAL [33].

5.4.1.3 EEG Signal:

EEG (Electroencephalography) is a noninvasive neuroimaging technique that involves placing electrodes on the scalp to record the brain's electrical activity. This method allows researchers to measure and analyze the electrical signals produced by the brain, providing valuable insights into its functioning. EEG signals are instrumental in identifying various neurological disorders and studying cognitive processes such as perception, attention, and memory. Due to its noninvasive nature and safety, EEG has become a widely used tool for investigating the electrical activity of the human brain. Furthermore, EEG signals can be combined with other imaging modalities, such as magnetic resonance imaging (MRI), functional near-infrared spectroscopy (fNIRS), and positron emission tomography (PET), to enhance the understanding of brain function and structure **[34].**

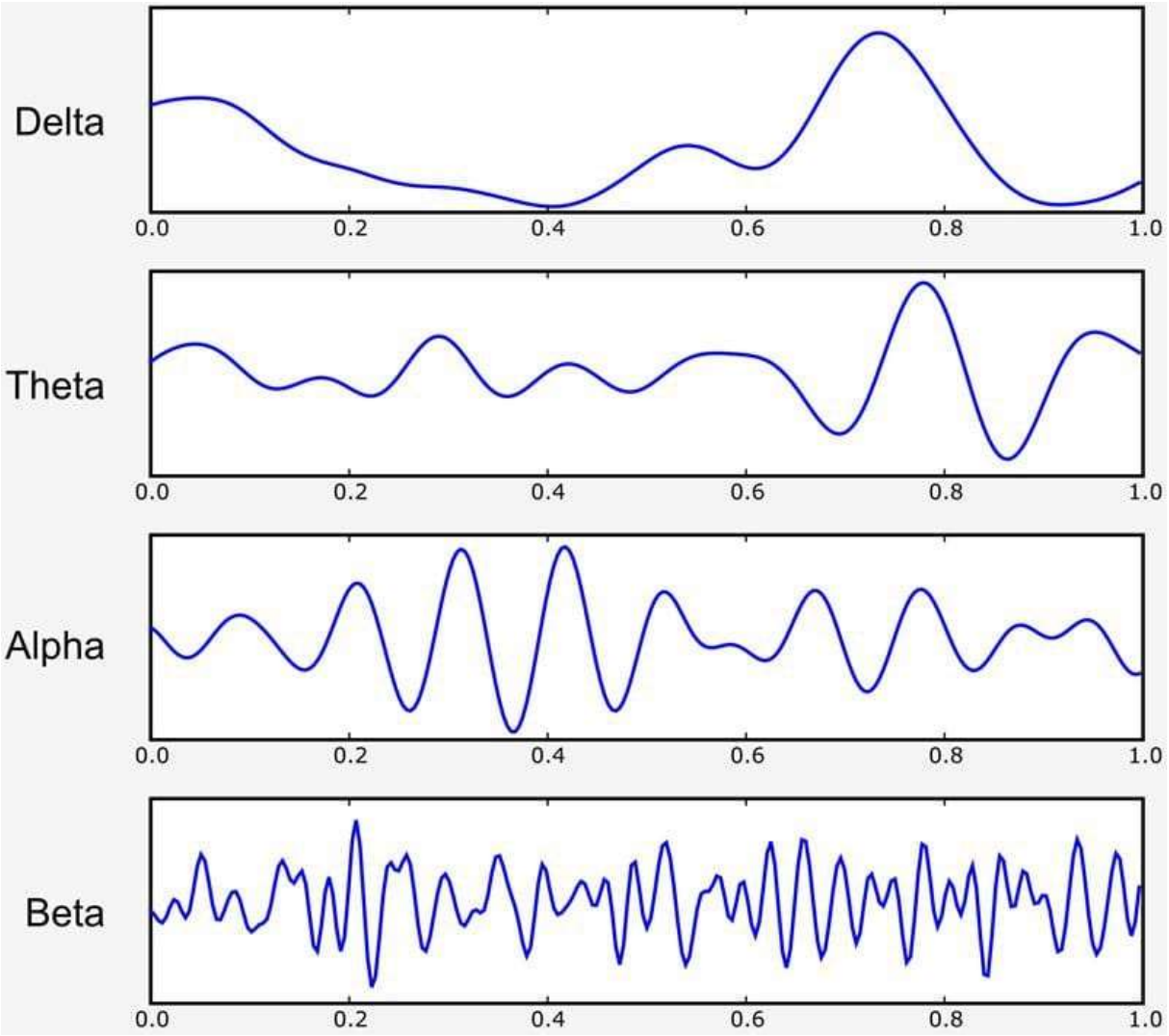


Figure 1.6: Different types of EEG Signals [35].

6. Conclusion:

In this chapter, we have explored the fundamental concepts of nonlinear signals and their distinctions from linear signals, which follow the principle of superposition, where the response to a combination of inputs is equal to the sum of the individual responses. In contrast, nonlinear signals do not follow this principle, making them more complex but also more suitable for modeling real-world phenomena where interactions are non-additive.

There are numerous reasons to opt for nonlinear signals. First, many natural and artificial systems exhibit nonlinear behaviors, such as biological systems, advanced electronic circuits, or chaotic phenomena. Second, nonlinear signals enable a more accurate representation of complex dynamics and rich interactions that cannot be captured by linear models. Finally, the analysis of nonlinear signals provides powerful tools for extracting subtle and often hidden information from data, which is particularly valuable in fields such as biomedical signal processing.

Among the types of nonlinear signals, biomedical signals hold significant importance. Within this category, we have highlighted the electrocardiogram (ECG), a quintessential example of a nonlinear signal. The ECG records the electrical activity of the heart and reflects the complex interactions among various components of the cardiac system.

In the next chapter, we will delve into the details of ECG signals, exploring their characteristics and properties in depth.



chapter2:

ECG signal

1. Introduction:

Even though the heart is considered the fundamental organ of the circulatory system, it appears to be the organ essential to life, as it distributes blood throughout the body. The study of the physiological function of the heart, its cardiac cycle, and the functioning of the electrical mechanisms that govern it must therefore precede the cardiological component. This chapter will therefore focus on cardiac physiology, the cardiac cycle, the organ's electrical architecture, and the electrocardiographic signal, a key diagnostic tool **[36].**

2.The Heart as a Pump:

The heart, illustrated in (Fig. 2.1), functions as two distinct pumps: the right heart, which circulates blood through the lungs, and the left heart, which pumps blood through the systemic circulation to supply the rest of the body's organs and tissues. Each of these pumps consists of two chambers an atrium and a ventricle and operates in a pulsatile manner.

The atria act as weak primer pumps, assisting the flow of blood into the ventricles. The ventricles, in turn, generate the primary force that drives blood either (1) through the pulmonary circulation via the right ventricle or (2) through the systemic circulation via the left ventricle. Encasing the heart is the pericardium, a protective double-layered sac that stabilizes its position. Specialized cardiac mechanisms sustain a continuous cycle of contractions known as cardiac rhythmicity by propagating electrical impulses through the heart muscle, ensuring its regular, rhythmic beating **[37].**

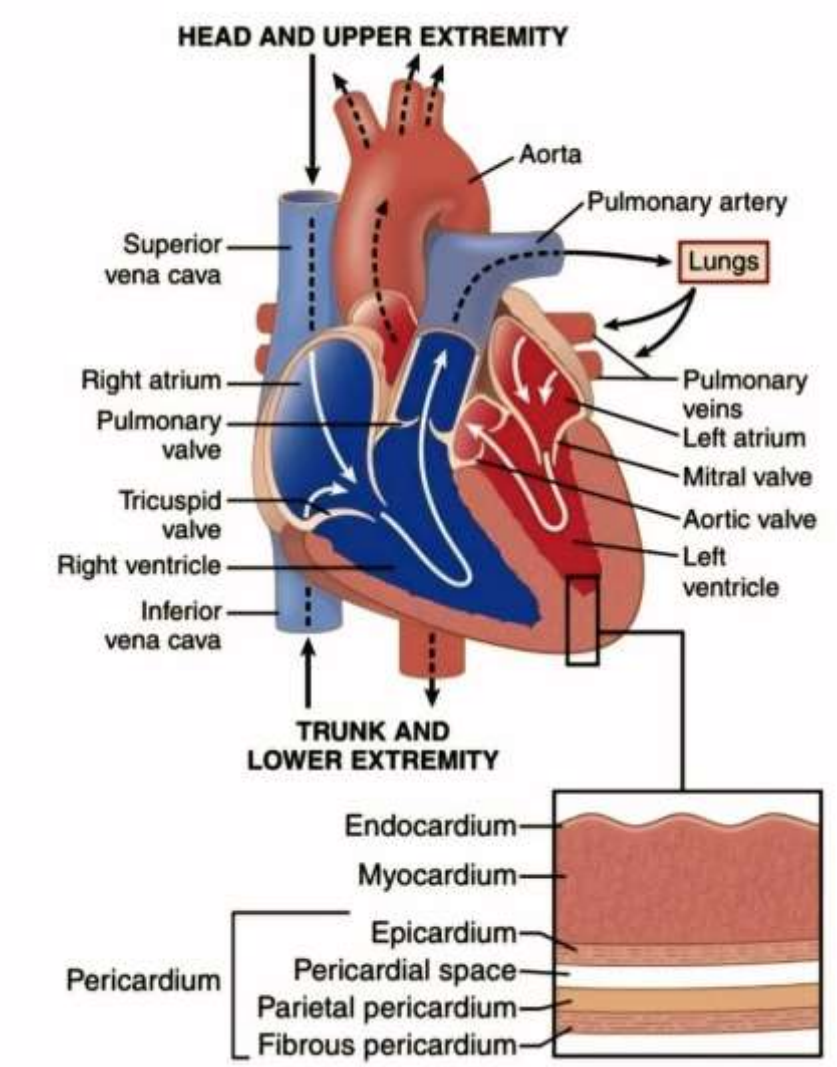


Figure 2.1: Structure of the heart and course of blood flow through the heart chambers and heart valves [37].

2.1 Physiology of cardiac muscle:

The heart consists of three main types of cardiac muscle: atrial muscle, ventricular muscle, and specialized excitatory and conductive fibers. While atrial and ventricular muscles contract similarly to skeletal muscle, their contractions last longer. In contrast, the specialized excitatory and conductive fibers contract only weakly due to their sparse contractile fibrils. Instead, these fibers generate and transmit rhythmic electrical impulses (action potentials), creating an intrinsic pacing system that regulates the heart's rhythmic contractions. **[38].**

2.2 Cardiac Cycle:

The sequence of events from the start of one heartbeat to the next is known as the cardiac cycle. Each cycle begins when an action potential spontaneously arises in the sinus node a small region located in the upper lateral wall of the right atrium, near the superior vena cava's opening. This electrical impulse spreads rapidly through both atria before passing into the ventricles via the A-V bundle. Due to the specialized structure of the cardiac conduction system, the signal experiences a brief delay (over 0.1 seconds) as it moves from the atria to the ventricles, ensuring coordinated heart function.

This brief delay ensures that the atria contract before the ventricles, allowing them to fill the ventricles with blood prior to the more forceful ventricular contraction. In this way, the atria serve as priming pumps, while the ventricles generate the primary force that propels blood throughout the circulatory system **[39].**

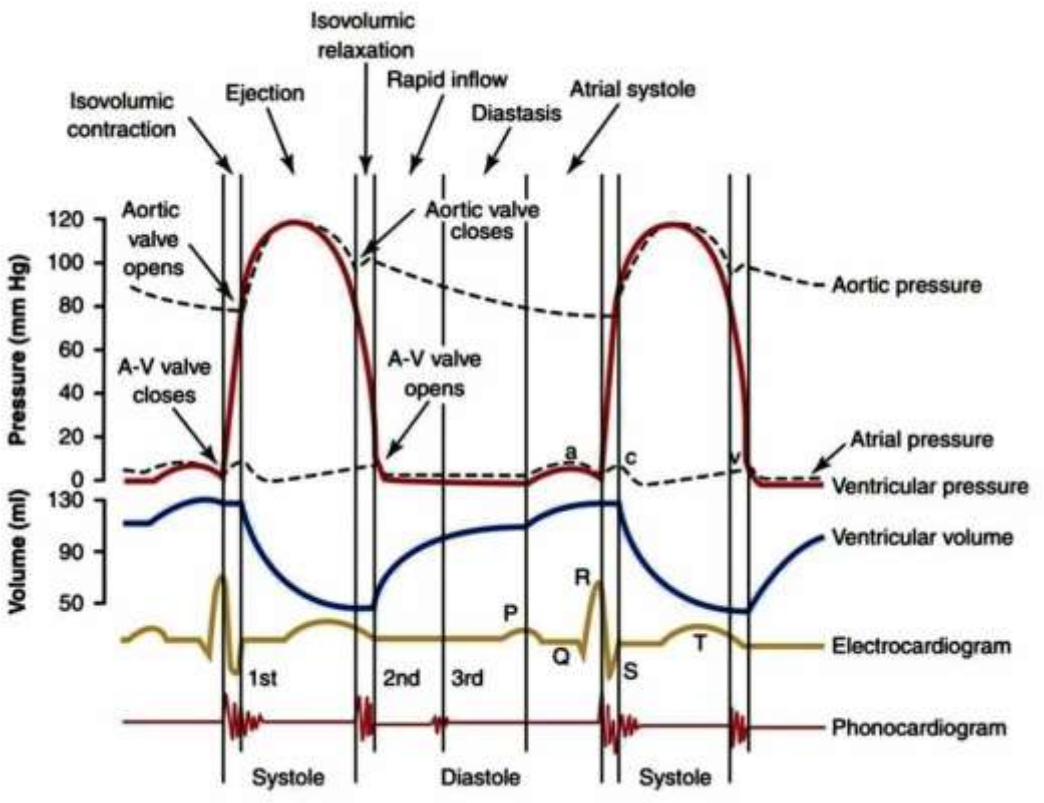


Figure 2.2.: Events of the cardiac cycle for left ventricular function, showing changes in left atrial pressure, left ventricular pressure, aortic pressure, ventricular volume, the electrocardiogram, and the phono- cardiogram. A- V, Atrioventricular [40].

3. Electrical conduction of the heart:

Cardiac muscle contraction is triggered by an electrical impulse that spreads through the heart in a precise sequence, optimizing contraction efficiency. This coordinated activity is governed by a specialized conduction system, which includes: the sinus node (the heart's primary pacemaker), internodal pathways, the atrioventricular node (AVN), the bundle of His, right and left bundle branches, and Purkinje fibers **(Fig.2.3)**. Together, these components ensure synchronized and effective pumping of the heart.

Normal electrical activity of the heart follows the following activation sequence: **[41]**

3.1 The Sinus Node (S N):

The sinus node (SN), situated in the superior portion of the right atrial wall near the superior vena cava junction, serves as the heart's natural pacemaker by spontaneously generating electrical impulses. These impulses propagate through both atria, with transmission enhanced by specialized internodal pathways that bridge the sinus node to the atrioventricular node **[42].**

3.2 The Atrioventricular Node (AVN):

Located in the inferior portion of the right atrium, the AVN consists of specialized cells characterized by slow electrical conduction. This nodal tissue introduces a critical physiological delay (approximately 100 ms) to impulse transmission between the atria and ventricles. This delay serves two vital functions: (1) it prevents ventricular overstimulation by limiting rapid atrial impulses, and (2) permits complete atrial emptying, thereby optimizing ventricular filling and subsequent contraction **[43].**

3.3 The Bundle of His:

Situated in the superior aspect of the interventricular septum, the Bundle of His penetrates through the fibrous cardiac skeleton - the electrically insulating connective tissue that creates anatomical and electrical separation between atrial and ventricular chambers. Under normal physiological conditions, this structure, together with the AV node, forms the sole conduction pathway bridging atrial and ventricular electrical activity, collectively termed the atrioventricular junction. The Bundle of His features a proximal trunk that bifurcates into right

and left bundle branches, which respectively distribute the electrical impulse to the right and left ventricular myocardium **[44].**

3.4 Purkinje Fibers:

The right and left bundle of his branches ultimately arborize into an extensive network of specialized Purkinje fibers that permeate the ventricular myocardium. These fibers terminate by forming intricate anastomotic connections with ventricular muscle cells, ensuring rapid and synchronized electrical activation of the entire ventricular mass **[45].**

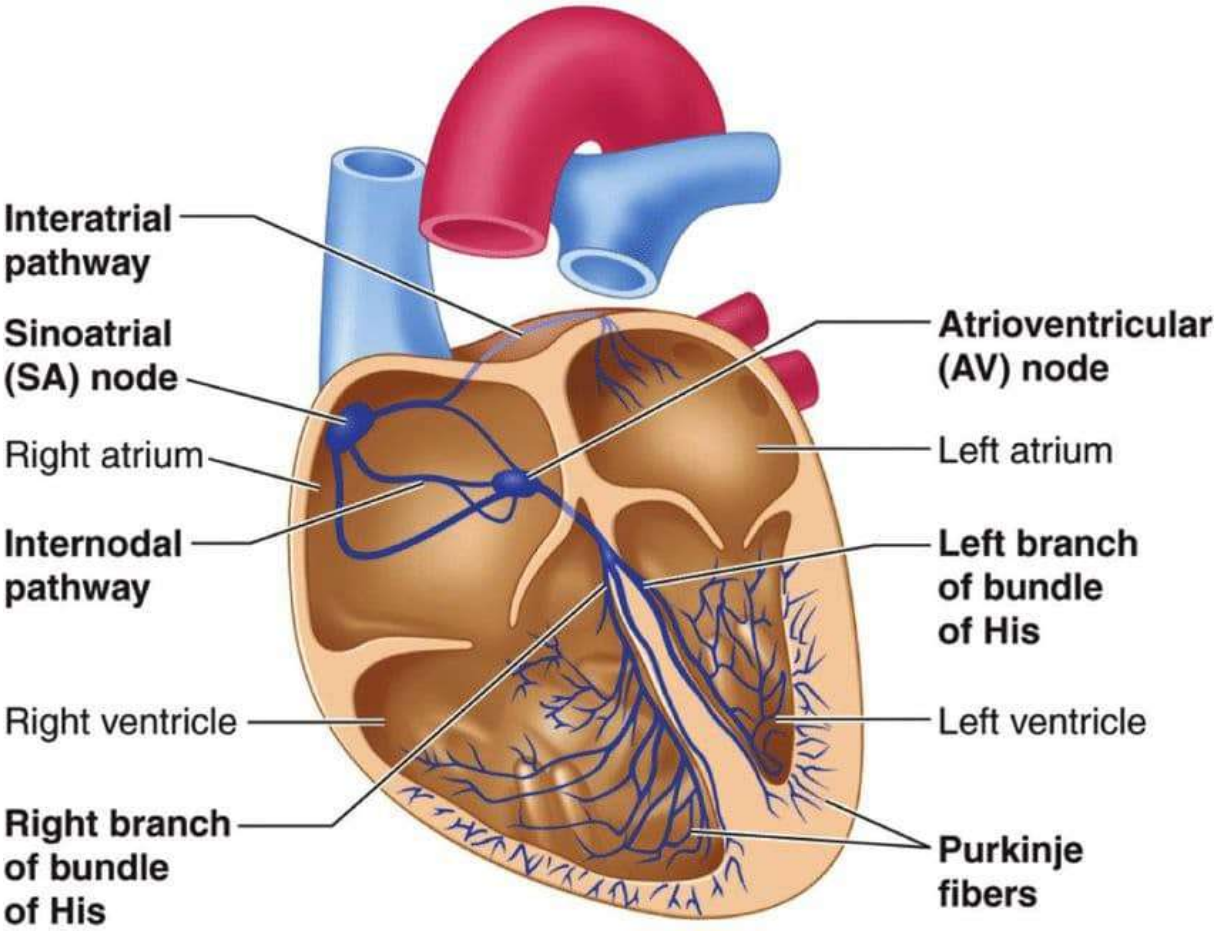


Figure 2.3: The specialized excitation system [46].

4. The electrocardiogram (ECG)

4.1 Definition of ECG

As illustrated in **(Fig2.4)** the electrocardiogram (ECG) represents the electrical activity of the heart, recorded through electrodes placed on the skin surface. First established by Einthoven in 1901, this diagnostic technique captures characteristic waveforms including the P wave, QRS complex, and T wave, each corresponding to specific phases of cardiac electrical activation. These morphological features enable clinicians to assess cardiac rhythm and identify pathological conditions. Notably, ECG morphology varies according to electrode placement configurations, as demonstrated in Figure 2.5, with different lead systems providing distinct perspectives of the heart's electrical activity **[47].**

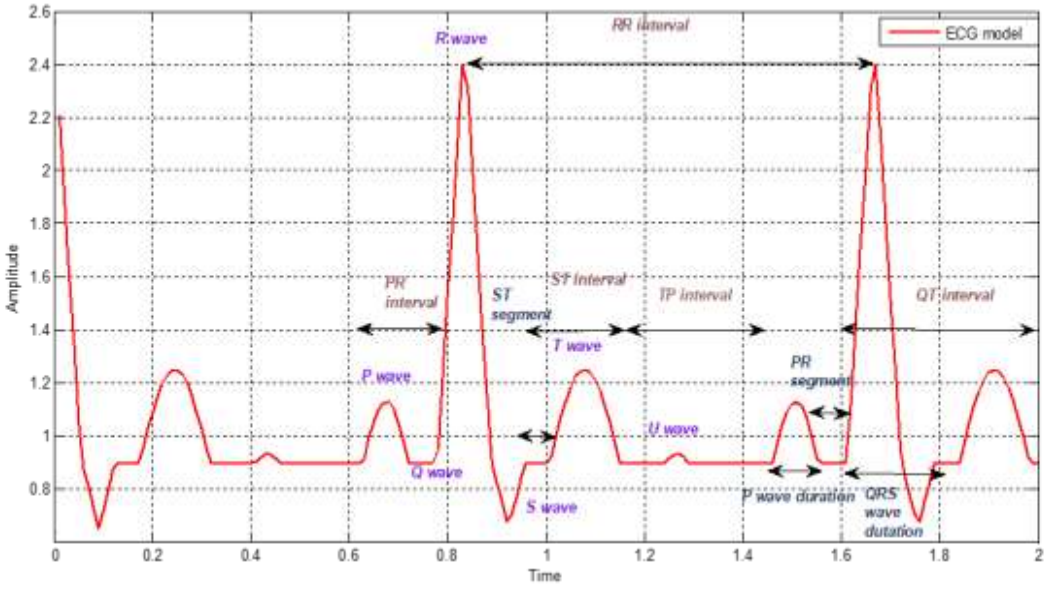


Figure 2.4: Example of different morphologies of the ECG signal [48].

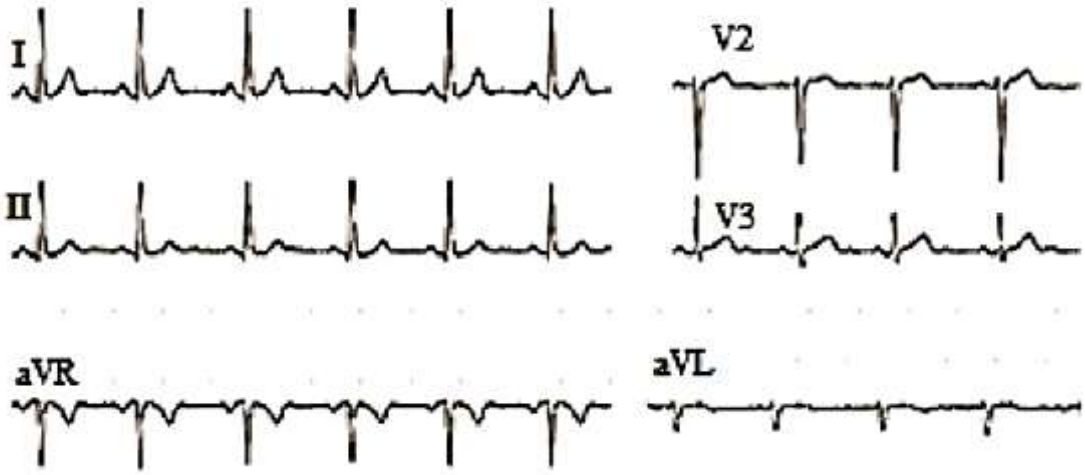


Figure 2.5: Example of different morphologies of the ECG signal [49].

4.2 Historical Development of Electrocardiography:

The foundations of cardiac electrophysiology were established in the late 18th century, though practical measurement techniques only emerged in the late 1800s. While the first electrocardiogram was recorded in 1895, widespread clinical adoption began following Willem Einthoven's pioneering work in the early 20th century. The 1940s marked significant advancements through the contributions of researchers like Thomas Lewis and Norman Wilson, culminating in the development of the modern 12-lead ECG system. Post-World War II technological progress led to device miniaturization and automation, while contemporary systems now incorporate computerized interpretation and remote monitoring capabilities. Initially focused on rhythm and conduction disorders, electrocardiography evolved dramatically after 1950 with new insights into ventricular repolarization. Today, the field continues to transform with emerging approaches to arrhythmia analysis and novel diagnostic applications **[50].**

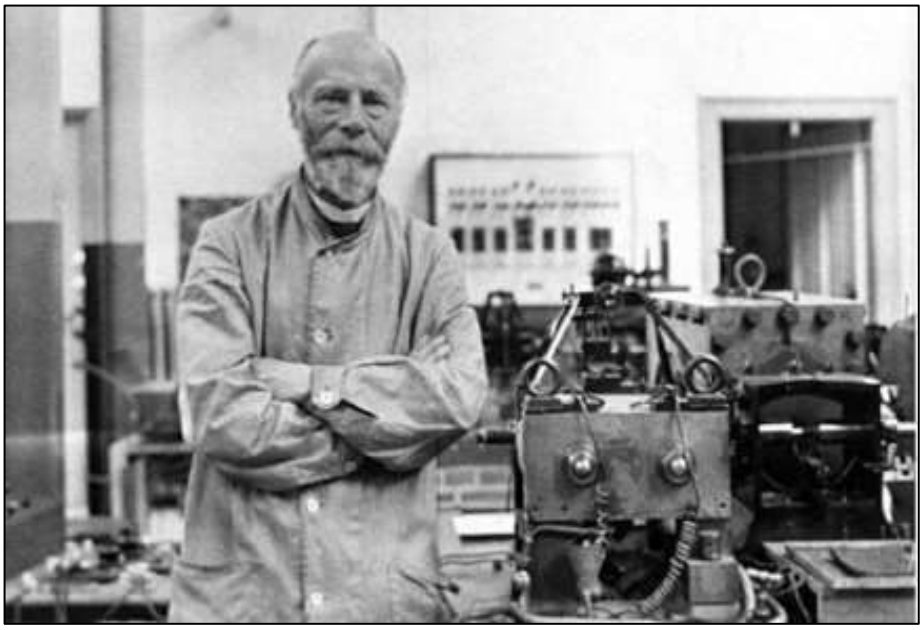


Figure 2.6: Willem Einthoven [51].

4.3. When do we need an ECG?

An ECG should be recorded whenever a patient has chest pain, palpitations, breathlessness or dizziness, or if the patient has had an episode of syncope (blackout) or an unexplained fall. In addition, a patient with a stroke or a transient ischemic attack (TIA) must have an ECG as these may be due to an irregular heart rhythm. Remember that the patient's symptoms and physical signs will guide interpretation of the ECG **[52].**

4.4. Relation of the Monophasic Action Potential of Ventricular Muscle to the QRS and T Waves in the Standard Electrocardiogram

The monophasic action potential in ventricular cardiomyocytes typically maintains a duration of 250-350 milliseconds. As demonstrated in the upper panel of **(Fig2.5)** intracellular microelectrode recordings from individual ventricular muscle fibers reveal distinct electrophysiological phases:

1. The rapid upstroke phase (depolarization) mediated by sodium ion influx

2. The prolonged repolarization phase reflecting potassium efflux

These transmembrane potential changes represent the fundamental electrical activity underlying ventricular contraction and recovery **[53].**

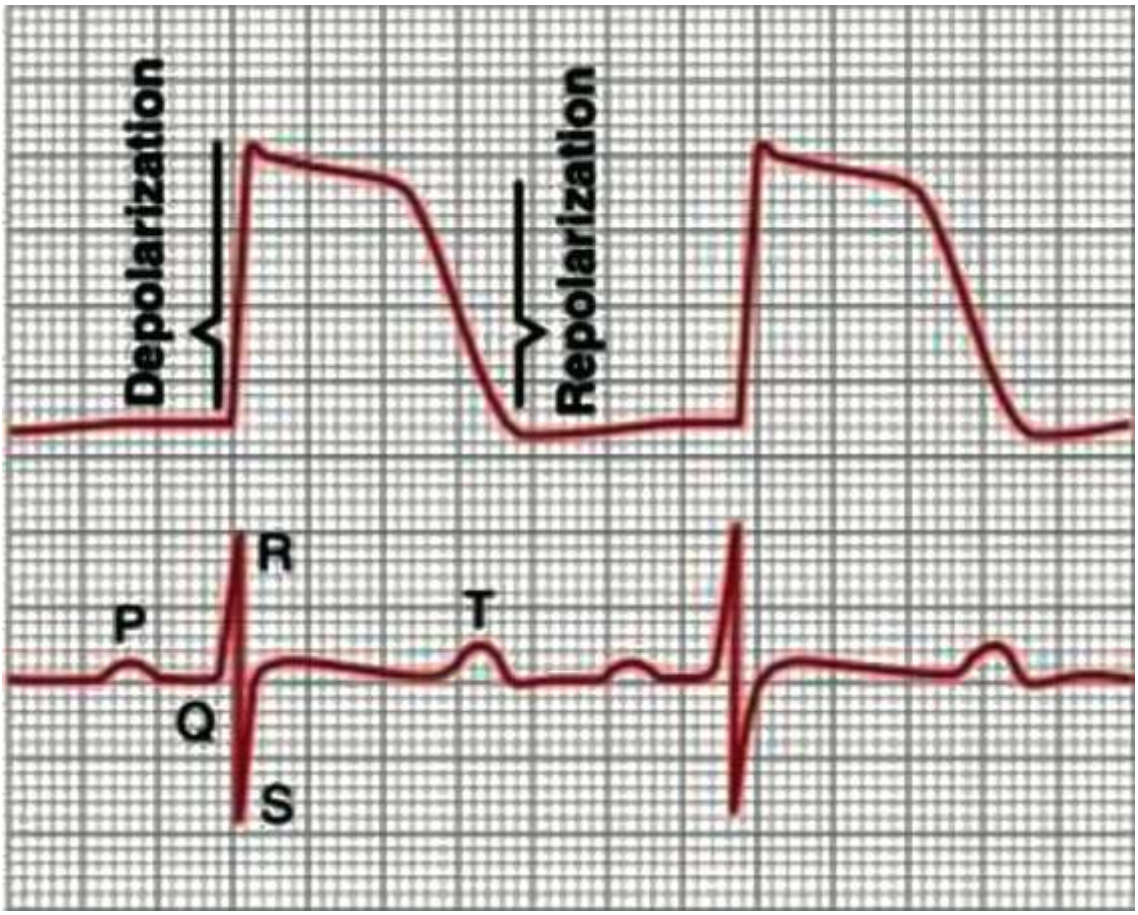


Figure 2.7: Top, Monophasic action potential from a ventricular muscle fiber during normal cardiac function showing rapid polarization and then repolarization occurring slowly during the plateau stage but rapidly toward the end. Bottom, Electrocardiogram recorded simultaneously [54].

The lower panel of **(Fig2.7)** demonstrates synchronous recordings of ventricular action potentials and corresponding ECG tracings. Key observations include:

1. Precise temporal alignment where:

   - QRS complex onset coincides with initial depolarization

   - T-wave manifestation corresponds with repolarization completion

2. Electrically silent periods occur during:

   - Full resting membrane potential (complete polarization)

   - Plateau phase of action potential (complete depolarization)

3. ECG deflection generation requires:

   - Regional voltage gradients during partial polarization/depolarization

   - Resultant current flow through ventricular tissue volume conductors

   - Detectable body surface potential differences **[53].**

4.5 Relationship of atrial and ventricular contraction to the waves of the Electrocardiogram:

Before contraction of muscle can occur, depolarization must spread through the muscle to initiate the chemical processes of contraction. Refer again to **(fig2.4),** the P wave occurs at the beginning of contraction of the atria, and the QRS complex of waves occurs at the beginning of contraction of the ventricles. The ventricles remain contracted until after repolarization has occurred that is, until after the end of the T wave.

The atria repolarize about 0.15 to 0.20 second after termination of the P wave, which is also approximately when the QRS complex is being recorded in the ECG. Therefore, the atrial repolarization wave, known as the atrial T wave, is usually obscured by the much larger QRS complex. For this reason, an atrial T wave is seldom observed on the ECG.

The ventricular repolarization wave is the T wave of the normal ECG. Ordinarily,

ventricular muscle begins to repolarize in some fibers about 0.20 second after the beginning of the depolarization wave (the QRS complex), but in many other fibers, it takes as long as 0.35 second. Thus, the process of ventricular repolarization extends over a long period, about 0.15 second. For this reason, the T wave in the normal ECG is a prolonged wave, but the voltage of the T wave is considerably less than the voltage of the QRS complex, partly because of its prolonged length **[55]**.

4.6 Electrocardiogram Recording Methodology:

Electrodes are placed on the chest and limbs of the patient to record different views of the heart's electrical activity. Each view of the heart is described as a 'lead'. The word 'lead' does not refer to the electrodes. The rhythm of the heart can be determined from only one view, i.e.one lead (this requires two electrodes). For a full picture of the heart's electrical activity, a 12-lead view is conventional. One electrode is attached to each limb.

These four electrodes provide six 'limb leads' or six different views of the heart in a vertical plane. These are called leads I, II, III, VL, VF and VR. VL, VF and VR used to be called AVL, AVF and AVR, respectively, but the A is essentially meaningless and is redundant.

Six electrodes are attached to the chest, recording leads V1 to V6. Accurate placement of these electrodes is essential for comparing later ECGs. These leads 'look at' the heart from the front in a horizontal plane **(Fig 2.8) [56].**

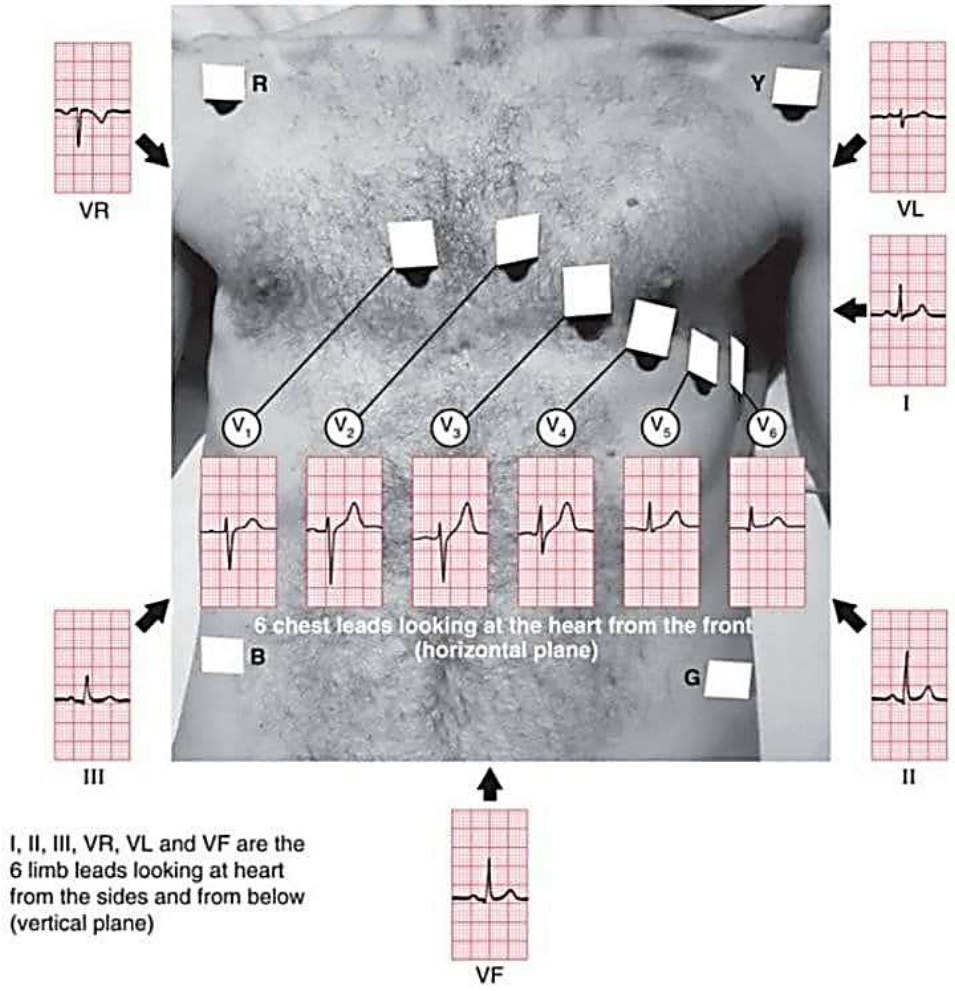


Figure 2.8: Lead positions for a 12-lead ECG with 12 views of the heart [57].

4.6.1. Top tips for recording an ECG:

- To record a '3-lead ECG' using only limb electrodes, remember the mnemonic: "Ride Your Green Bike". R for red and right arm. Y for yellow, G for green and B for black. Apply the first red electrode to the right arm and work clockwise to left arm, left leg and finally right leg (the black electrode is the earth electrode).
- Placement of the limb electrodes is easy: there is no specific position to remember Try to find the least hairy area: anywhere from the shoulder or outer clavicle down to the wrist is fine for the upper limb; anywhere from the lower abdomen to the foot on right and left side is fine for the lower limb electrodes.
- Placement of the chest electrodes MUST be accurate and standardized for every recording.
- Make sure the patient is warm and relaxed.

- Check machine settings: standard is paper speed of 25mm/second; the voltage calibration should be set so that 1 mV causes 1 cm of upwards deflection.
- Write the patient's symptoms and BP on the ECG **[58].**

4.7. Waves, Segments, and Intervals:

There are certain elements in the ECG waveform **(fig (2.9)):**

- The isoelectric line: a horizontal line when there is no electrical activity on ECG.
- Segments: the duration of the isoelectric line between waves.
- Intervals: the time between the same segments of adjacent waves **[59].**

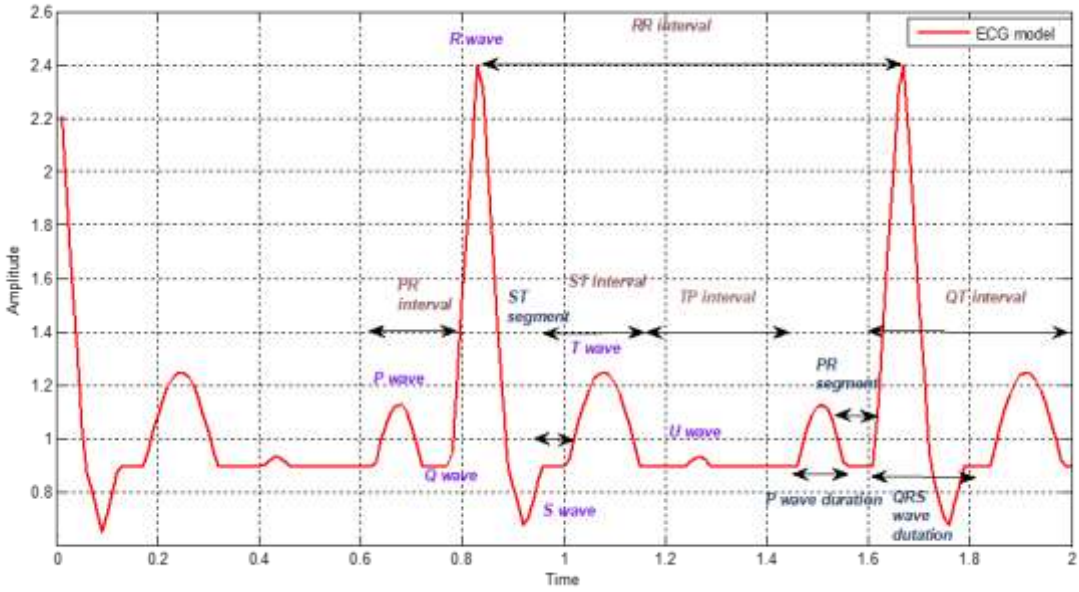


Figure 2.9: The ECG waves, segments, and intervals [60].

a. The P-wave:

The P-wave represents atrial depolarization and typically appears as a small, smooth positive deflection in most leads, though it is normally inverted in lead AVR due to the heart's electrical axis. In leads III, V_1, and AVL, minor variations such as biphasic or negative deflections may still fall within normal limits. A normal P-wave does not exceed 0.12 seconds in duration or 0.25 mV in amplitude in limb leads (0.15 mV in precordial leads). Abnormalities in P-wave morphology—such as notching, excessive amplitude (>0.25 mV), prolonged duration (>0.12 s), or atypical polarity—often indicate underlying pathology. These deviations may arise from atrial enlargement (e.g., left atrial hypertrophy causing a "notched" P mitrale

pattern), ectopic atrial rhythms, or conduction disturbances. Consistent evaluation of P-wave characteristics is critical for detecting atrial abnormalities and guiding further diagnostic assessment **[61].**

b. The PR Interval:

The PR interval reflects the total conduction time from atrial depolarization onset (beginning of the P wave) to ventricular activation initiation (start of the QRS complex). This interval incorporates both atrial depolarization and the essential physiological delay at the atrioventricular (AV) node, which coordinates proper cardiac timing. In healthy adults, the PR interval typically ranges between 120-200 milliseconds (3-5 small boxes at standard 25 mm/s ECG paper speed). Clinically, this interval serves as a key indicator of AV conduction integrity, with deviations suggesting potential conduction system abnormalities **[62].**

c. The PR Segment:

The PR segment corresponds to the electrical pause between the completion of atrial depolarization (end of P wave) and the initiation of ventricular activation (start of QRS complex). This segment typically appears as an isoelectric line that maintains continuity with the initial baseline of the P wave, reflecting the conduction delay through the AV node and His-Purkinje system. In standard clinical practice, the PR segment serves as an important reference baseline for assessing subsequent ST segment deviations **[62].**

d. The QRS complex:

The QRS complex represents the electrical depolarization of the ventricles, which precedes their mechanical contraction. As the most prominent waveform on the ECG, it typically consists of three deflections:

- ✓ Q wave: Initial negative deflection (when present)
- ✓ R wave: Prominent positive deflection
- ✓ S wave: Subsequent negative deflection following the R wave

In normal conduction, the QRS duration ranges from 85 to 95 milliseconds (ms). This brief interval reflects the rapid spread of the electrical impulse through the ventricular myocardium via the His-Purkinje system. The configuration and duration of the QRS complex provide

critical diagnostic information about ventricular conduction pathways and potential abnormalities **[63].**

e. The QRS Interval:

The QRS interval measures the total duration of ventricular depolarization, corresponding to the width of the QRS complex on the ECG. In healthy adults, this interval typically ranges between 60 to 100 milliseconds (0.06–0.10 seconds). Values exceeding this range may indicate conduction abnormalities such as bundle branch blocks or ventricular hypertrophy. Precise measurement of the QRS interval is essential for evaluating electrical synchrony across the ventricles **[63].**

f. The ST Segment:

The ST segment corresponds to the period between ventricular depolarization completion (end of QRS complex) and repolarization onset (beginning of T wave). Under normal conditions, this segment appears isoelectric or slightly ascending across all leads, reflecting the myocardial plateau phase of action potentials. Clinically, ST segment morphology serves as a critical indicator of myocardial status, with deviations suggesting ischemia, injury, or electrolyte imbalances **[64].**

g. The T Wave:

The T wave reflects ventricular repolarization, an energy-intensive process contrasting with passive depolarization. Its morphology varies significantly due to cardiac (e.g., ischemia) and systemic (e.g., hormonal) influences. Normally, repolarization propagates inversely to depolarization, creating positive T waves in leads with prominent R waves (typically 1/3 to 2/3 the R wave amplitude) as both waves generate positive deflections when approaching electrodes **[62].**

h. The QT interval:

The QT interval measures the time between consecutive R-wave peaks on an ECG, serving as a key metric for assessing cardiac electrical activity. Clinically, it provides critical information for two main purposes: (1) arrhythmia detection, particularly for identifying prolonged QT syndromes that may predispose to dangerous ventricular rhythms, and (2) heart

rate variability analysis, which offers insights into autonomic nervous system influence on cardiac function **[64].**

i. The RR interval:

This interval measures the time between the peaks of two successive R waves on an ECG. It is crucial for determining the instantaneous heart rate and plays a key role in diagnosing arrhythmias and analyzing heart rate variability **[64].**

5. Conclusion:

The heart, being such a critical and sophisticated organ, is the centerpiece of blood circulation and maintenance of life. Here in this chapter, we delved into the basics of its physiology, detailing the mechanisms underlying its function. The cardiac cycle, through its systole and diastole, demonstrates the accuracy with which the heart contracts to pump blood. In addition, the heart's electrical anatomy, as directed by the conduction system, demonstrates the significance of electrical impulses in synchronizing cardiac contractions.

The electrocardiogram (ECG), through the recording of such electrical impulses, offers a useful diagnostic tool for quantifying cardiac activity and detecting potential abnormalities. Characteristics of the ECG signal, i.e., the P, QRS, and T waves, reflect the different phases of the cardiac cycle and allow precise evaluation of cardiovascular health.

In the next chapter we will talk about signal classification and we will mention and discuss in detail both traditional and AI methods of signal classification.



chapter3:

Identification and classification of dynamicals systems

1. Introduction:

Signal classification is a crucial task in various engineering and scientific applications, including telecommunications, biomedical signal processing, industrial automation, and machine learning. Traditional methods rely on mathematical models and deterministic approaches, while modern artificial intelligence (AI) techniques leverage adaptive learning and optimization to improve classification accuracy and robustness.

This chapter explores in detail both classical and AI-based signal classification methods **[65].**

2. What is a signal:

A signal refers to a physical quantity that changes over time, space, or another independent variable. Mathematically, it is represented as a function dependent on one or more such variables. For instance, the equations:

$$S1(t) = 5t \quad \dots (3.1)$$

$$S2(t) = 20t \quad (1) \dots (3.2)$$

describe signals varying linearly and quadratically with time(t), respectively. Similarly, the function:

$$S(x,y) = 3x + 2xy + 10y^2 \dots (3.3)$$

represents a signal dependent on two spatial coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$.

Signals like those in (3.1) and (3.2) fall into a category where their behavior is explicitly defined through functional relationships. However, in practical scenarios, such relationships may either remain unknown or become too intricate to model effectively.

For example, a speech signal **(Fig 3.1)** cannot be succinctly modeled using simple functional expressions like those in (3.1). Instead, a segment of speech is often approximated with high precision as a combination of multiple sinusoids, each with distinct amplitudes and frequencies. Mathematically, this representation can be expressed as **[66]:**

$$\sum_{i=1}^{N} Ai(t) \sin[2\pi Fi(t)t + \theta i(t)] \quad \dots (3.4)$$

Here, $\{Ai(t)\}$, $\{Fi(t)\}$, and $\{\theta i(t)\}$ represent the (potentially time-varying) amplitudes,

frequencies, and phases, respectively, of the constituent sinusoids. Importantly, the information or message embedded within a short-time segment of the speech signal can be interpreted by quantifying these parameters. Specifically, measuring the instantaneous amplitudes, frequencies, and phases within the segment allows us to extract the signal's informational content **[67].**

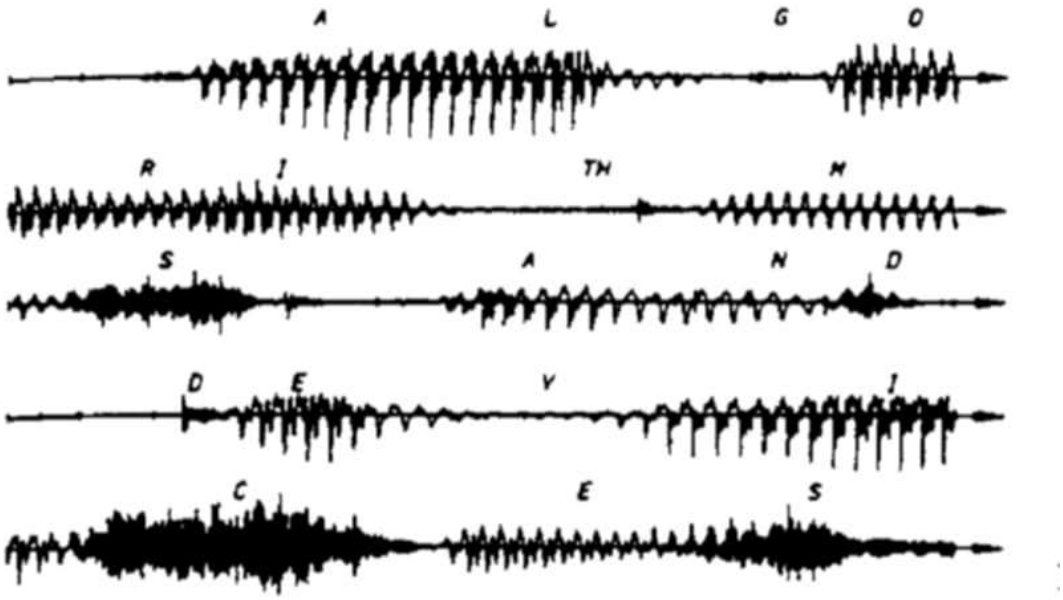


Figure 3.1: Example of a speech signal [67].

A common example of a naturally occurring signal is the electrocardiogram (ECG), which enables healthcare professionals to assess the health and rhythm of a patient's heart. Likewise, the electroencephalogram (EEG) captures electrical patterns in the brain, offering critical insights into neural activity and brain function. Both signals serve as vital diagnostic tools in medical practice **[68].**

Signals like speech, electrocardiogram (ECG), and electroencephalogram (EEG) are one-dimensional, meaning they vary along a single independent variable time and carry critical information about physiological or physical processes. In contrast, image signals depend on two independent variables, such as spatial coordinates (e.g., length and width), to represent visual or structural data. These examples represent a small fraction of the diverse range of natural signals used to analyze and interpret real-world phenomena **[69].**

Natural signals are generated through specific mechanisms tied to their origins. For example, speech signals are produced when air passes through the vocal cords, while images are created by exposing photographic film to light from a scene or object. This process highlights

that signal generation typically involves a system (e.g., the vocal cords and vocal tract for speech) reacting to a stimulus (e.g., airflow, light). Together, the system and its triggering stimulus form what is termed a signal source. Examples include speech sources, image sources, and many others, each representing unique combinations of systems and stimuli that produce distinct types of signals**[70] .**

3. Why signals are" processed:"

Signal processing refers to modifying or analyzing a signal to achieve goals such as extracting information (e.g., patterns, trends), identifying relationships between signals, or transforming the signal into a more useful format. This manipulation is often guided by mathematical models, though qualitative or fuzzy logic-based methods may also be applied. Common motivations for processing signals include :**[71]**

1 .Noise removal: Eliminating unwanted interference to isolate the meaningful component of a signal .

2 .Information enhancement: Revealing hidden details or structuring data in a way that supports analysis (e.g., visualizations, simplified representations) .

3 .Prediction: Forecasting future signal behavior to anticipate system outcomes (e.g., stock trends, weather patterns) .

A core challenge lies in defining what constitutes the "desired" signal versus "noise," as this distinction depends on context and user objectives. For instance, filtering techniques inherently rely on this judgment to separate relevant information from distortions**.[72]**

Signal processing plays a central role in biomedical applications, where its primary goal is to distinguish abnormal signals from normal ones to aid in disease diagnosis. Similar to how a physician identifies irregularities in heart or lung sounds, biomedical engineers often decompose signals into fundamental components to detect anomalies that may indicate

pathology. A common strategy involves analyzing a signal's Fourier transform to uncover unusual frequency patterns. For instance, in conditions like congestive heart failure, blood pressure exhibits slow oscillations (periods of seconds to tens of seconds), which can be identified by examining the power spectrum (derived from the squared magnitude of the Fourier transform) of the blood pressure signal .**[72]**

However, many biomedical signals deviate practically from the assumptions underlying Fourier analysis specifically, the idea that signals can be represented as sums of simple sinusoids. This limitation has spurred significant research into alternative decomposition methods that better align with the complex, non-stationary nature of biomedical signals. Developing these techniques remains a critical focus, as they could unlock more accurate tools for interpreting physiological data and diagnosing disorders**[73] .**

Predicting future signal values is critical in biomedical contexts, primarily for two scenarios **:[74]**

1 .Control Systems: When managing physiological processes like regulating blood glucose via insulin injections delays between intervention and effect necessitate anticipating future states. By predicting outcomes (e.g., glucose levels), adjustments can be timed to align with when their impact will manifest, improving treatment precision **[75]** .

2 .Early Disease Detection: Identifying subtle deviations from normal physiological patterns (e.g., heart rate variability) can signal disease onset. This requires defining boundaries of expected behavior to flag anomalies. For instance, analyzing fractal properties in heart rate data aims to correlate irregular patterns with conditions like heart disease. Despite its potential, reliably detecting such early markers remains a major challenge, driving ongoing research into advanced predictive models tailored to biomedical signals**[76]**.

This field thrives at the intersection of theory and application, as innovations in signal prediction are often motivated by urgent clinical needs—from personalized treatment to proactive diagnostics**[77] .**

4. Classification of signals:

The approach to processing signals or analyzing a system's response to them is largely determined by the unique characteristics of the signal itself. Since many techniques are tailored to specific signal categories (e.g., periodic vs. non-periodic, continuous vs. discrete), effective signal analysis begins with classifying the signals relevant to the application. This step ensures the chosen methods align with the signal's properties, optimizing accuracy and efficiency in tasks like noise removal, feature extraction, or predictive modeling.

Signal classification involves categorizing signals based on their characteristics, such as time-domain, frequency-domain, or statistical properties. Here are some common signal classification methods **[78]:**

4.1 Traditional Signal classification Methods:

4.1.1 Discrete Fourier Transform:

The Discrete Fourier Transform (DFT) converts a finite sequence of (N) discrete samples, $(x(0), x(1), \ldots\ldots, x(N-1))$, into a new sequence $(X(0), X(1), \ldots\ldots X(N-1))$, where each term (X(k)) is defined as:

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{nk}{N}} \quad \ldots(3.5)$$

Here, (j) represents the imaginary unit.

Practical Context:

- The sequence $(x(n))$ often originates from sampling an analog signal at intervals (Te) (sampling period), such that $(x(n) = x(nTe))$.

- The resulting $(X(k))$ approximates the Fourier transform of the original analog signal (scaled by (Te)) at (N) discrete frequencies:

$fk = ke/N$, with $k = 0, 1, \ldots\ldots, N-1$

where $fe = 1/Te$ is the sampling frequency? These frequencies range from 0 to fe , spaced evenly by fe/N **[79].**

4.1.2 Broida Method:

The Broida method is an open-loop graphical identification or classification technique used to model a system's step response. It derives a first-order transfer function with time delay, expressed as:

$$H(p) = \frac{Ke^{-Tp}}{1+\tau p} \quad \dots (3.6)$$

where the parameters to determine are:

K: Static process gain:

Calculated as the ratio of the output change ($\Delta\mathbf{s}$) to the input change ($\Delta\mathbf{e}$):

$$K = \frac{\Delta s}{\Delta e} \quad \dots (3.7)$$

T: Dead time (delay)

Determined using experimental time instances t_1 and t_2:

$$T = 2.8\ t_1 - 1.8\ t_2 \quad \dots (3.8)$$

τ: Time constant

Calculated as:

$$\tau = 5.5\ (t_2 - t_1) \quad \dots (3.9)$$

Comparison with Strejc's Method:

Unlike Strejc's method, which requires plotting a tangent at the step response's inflection point, Broida's method identifies specific time points (t_1) and (t_2), corresponding to 28% and 40% of the output's final steady-state value. These points are measured directly from the experimental response curve to compute ($\boldsymbol{T}$) and ($\boldsymbol{\tau}$) **[80].**

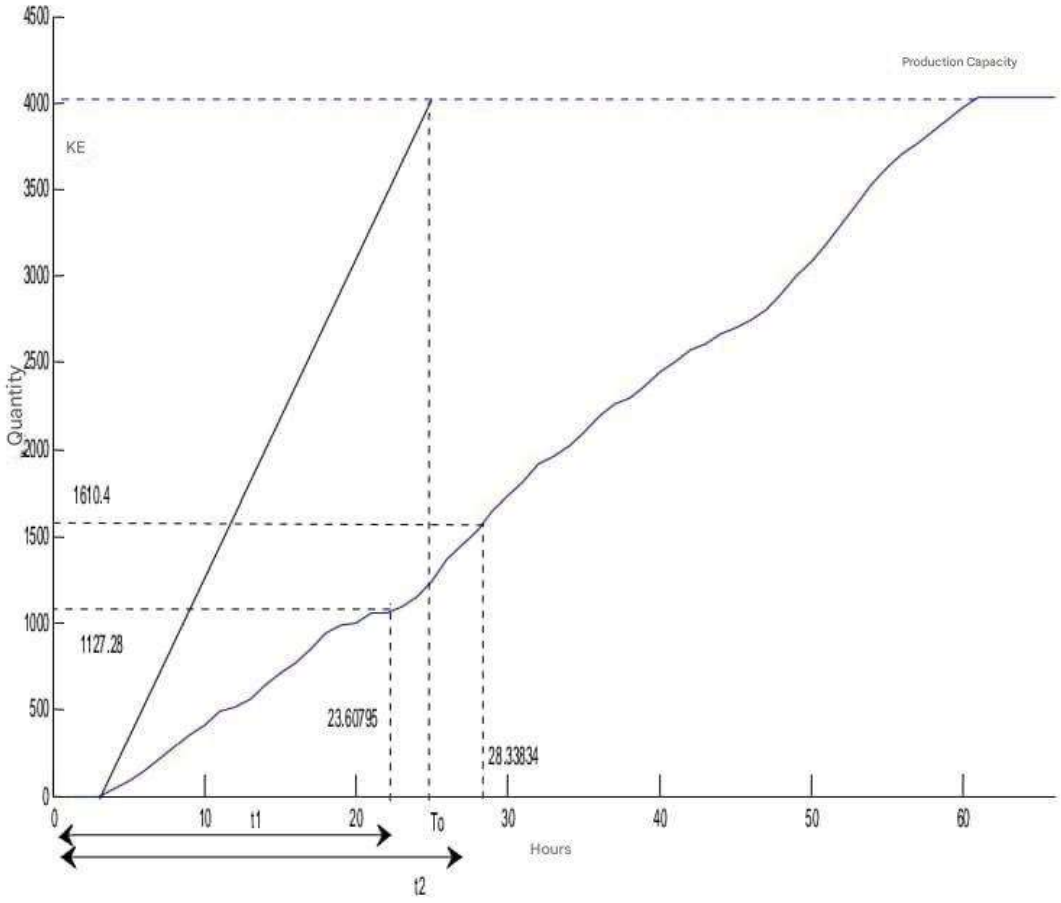


Figure 3.2: Application of the Broida method [81].

We therefore obtain:

	MODEL CHARACTERISTICS	
t1	Time to 28 %	23.60795
t2	Time to 40 %	33.725654
τ	$\tau = 5.5\,(t_2 - t_1)$	26.017152
T	$T = 2.8\,t_1 - 1.8\,t_2$	5.3961047
K	$K = \frac{\Delta s}{\Delta e}$	61.938461

table 3: Model Characteristics [81].

Therefore, the transfer function is as follows:

$$H(p) = \frac{61.938461e^{-5.3961047\ p}}{1+26.017152\ p} \quad \dots (3.10)$$

As for system optimization, the Broida method allows us, through the calculation of the $\frac{T}{\tau}$, to determine which regulator we should use in order to optimize our system **[80].**

4.1.3 Strejc Method:

The Strejc method addresses the challenge of modeling systems with multiple time constants, where directly identifying each individual constant from a step response is impractical. Instead, this approach approximates the system using a simplified model that assumes identical time constants and incorporates a delay term. By optimizing the delay, the model's simulated step response can closely align with the actual experimental response **[82].**

First method:

This approach is suitable for systems with step responses that do not overshoot. The Strejc model characterizes such systems using a transfer function of the form:

$$H(p) = \frac{k.e^{-\tau dp}}{(1+\tau.p)^n} \quad \dots (3.11)$$

where (k) is the gain, (τd) is the time delay, $(\tau)$is the time constant, and (n) denotes the system's order? By normalizing this transfer function (removing the gain and delay terms), it simplifies to:

$$Hn(p) = \frac{1}{(1+\tau.p)^n} \quad \dots (3.12)$$

The corresponding step response, illustrated in **(Fig 3.3),** features an inflection point $Pinf$. At this point, specific geometric properties (e.g., slopes, curvature, or time intervals) are analyzed to estimate the system's order (n) and time constant (τ). These parameters define the dynamic behavior of the system within the Strejc framework.

The analytical determination of the inflection point $Pinf$ involves calculating the second derivative $\ddot{Y}n(t)$of the normalized step response, which is mathematically described as follows **[83]:**

$$\mathrm{Yn(t)} = 1 - \mathrm{e}^{-\mathrm{t}/\tau} \sum_{\mathrm{j}=0}^{\mathrm{n}-1} \frac{1}{\mathrm{J!}} . (\mathrm{t}/\tau)^{\mathrm{j}} \quad \ldots (3.13)$$

$$\dot{Y}n(t) = L^{-1}[p.Hn(p)] = \frac{1}{\tau^n} . e^{-t/\tau} . \frac{t^{n-1}}{(n-1)!} \quad \ldots (3.14)$$

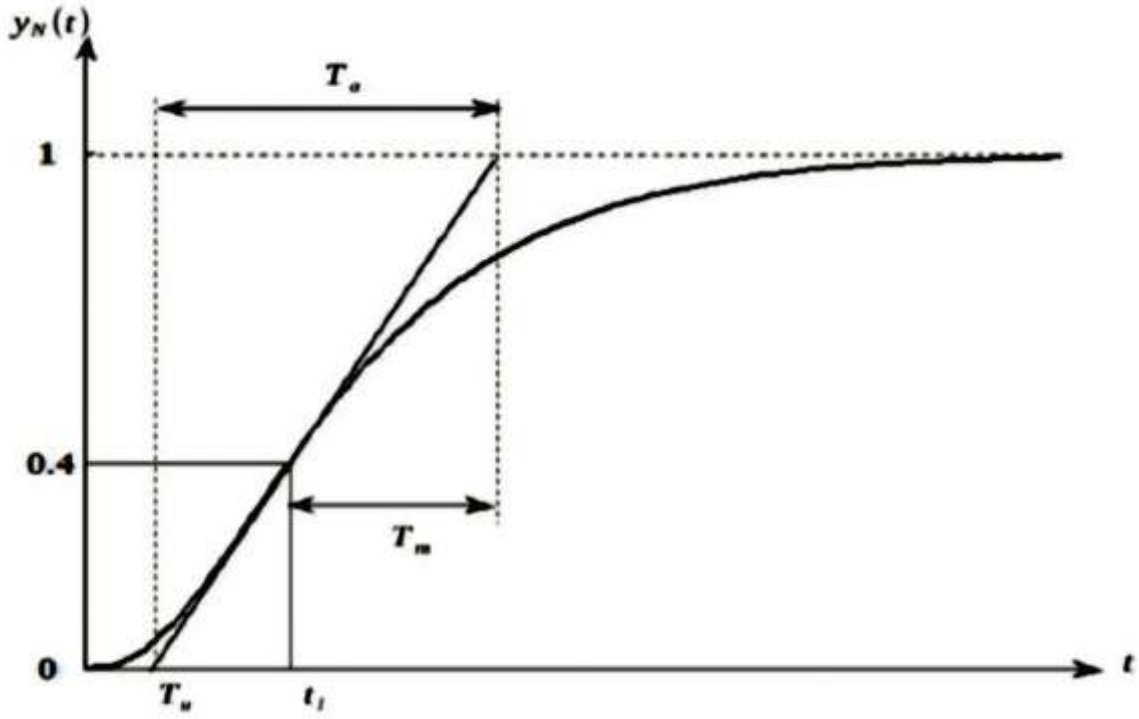


Figure 3.3: Strejc's first method [84].

Hence:

$$\ddot{Y}n(t) = \frac{t^{n-2}}{\tau^n\,(n-1)!} . [n - 1 - \frac{t}{\tau}] e^{-t/\tau} \quad \ldots (3.15)$$

The inflection points $Pinf$ of the normalized response is calculated from $\ddot{y}n(t) = 0$ where it is located

at time ti which corresponds to a tangent value **[83].**

Algorithm:

To identify or classify a system using the Strejc method, follow these steps:

a. Plot the tangent at the inflection point: On the system's response curve, identify the inflection point $((t_i, y_i))$ and draw the tangent at this point. Determine the parameters $(y_i), (Ta)$ (asymmetry time), (Tu) (rise time), and (Tm) (maximum time) from this tangent.

b. Estimate the order (n): Use (yi) or the ratio $\frac{Tu}{Ta}$ to estimate the system's order (n) using

(Table 4)

c. Calculate four estimates of $(\boldsymbol{\tau})$ Based on $(\boldsymbol{n})$, compute four potential values for the time constant $(\boldsymbol{\tau})$ using the relationships derived from the ratios $\frac{\boldsymbol{Ta}}{\boldsymbol{\tau}}$; $\frac{\boldsymbol{Tu}}{\boldsymbol{\tau}}$; $\frac{\boldsymbol{Ti}}{\boldsymbol{\tau}}$; and $\frac{\boldsymbol{Tm}}{\boldsymbol{\tau}}$.

d. Validate consistency of $(\boldsymbol{\tau})$: If the four $(\boldsymbol{\tau})$ values are close, use their average.

e. Adjust for inconsistency: If the values differ significantly, revise the order $(\boldsymbol{n})$ (typically increasing it) or reduce the delay time to improve alignment **[85].**

n	$\boldsymbol{Ta/\tau}$	$\boldsymbol{Tu/\tau}$	$\boldsymbol{Tu/Ta}$	$\boldsymbol{tl/\tau}$	$\boldsymbol{yl}$	$\boldsymbol{Tm/\tau}$	$\boldsymbol{Tm/Ta}$
1	0	0	0	0	0	1	1
2	2.718	0.282	0.104	1	0.204	2	0.736
3	3.695	2.805	0.218	2	0.323	2.500	0.677
4	4.463	1.425	0.319	3	0.353	2.888	0.674
5	5.199	2.100	0.410	4	0.371	3.219	0.629
6	5.699	2.811	0.493	5	0.384	3.510	0.616
7	6.226	3.549	0.570	6	0.394	3.775	0.606
8	6.711	4.307	0.642	7	0.401	4.018	0.509
9	7.146	5.081	0.709	8	0.407	4.245	0.593
10	7.590	5.869	0.773	9	0.413	4.458	0.587

Table 4: Parameters of the normalized Strejc answer [86].

Second method:

This alternative identification technique involves determining the time instances (t_2) and (t_2) at which the normalized system response $(y_n(t))$ attains 5% and 95% of its maximum value, respectively. The method employs a time-axis transformation, where the response is replotted on a rescaled timeline. Similar to the first method, the system's order (n) and time constant (τ) are identified by aligning the measured values with a predefined reference table. A key advantage of this approach is its ability to estimate the pure delay constant directly from the original, non-normalized response data **[87].**

4.2 Artificial Intelligence Methods for Signal Classification:

4.2.1 Artificial Neural Network:

Artificial Neural Networks (ANNs), a foundational pillar of machine learning, are extensively employed across diverse domains and remain central to the evolution of artificial intelligence. A distinctive feature of ANNs lies in their conceptual origin: their architecture is directly modeled after the structural and functional principles of biological neural networks in the human brain **[88].**

4.2.1.1 Definition and Origin of artificial neural Networks (ANNs):

ANNs are designed to model complex systems that are difficult to analyze using traditional statistical methods, they consist of interconnected nodes (artificial neurons) that process and transmit signals in parallel, mimicking the hierarchical and adaptive nature of biological neural networks **[89].**

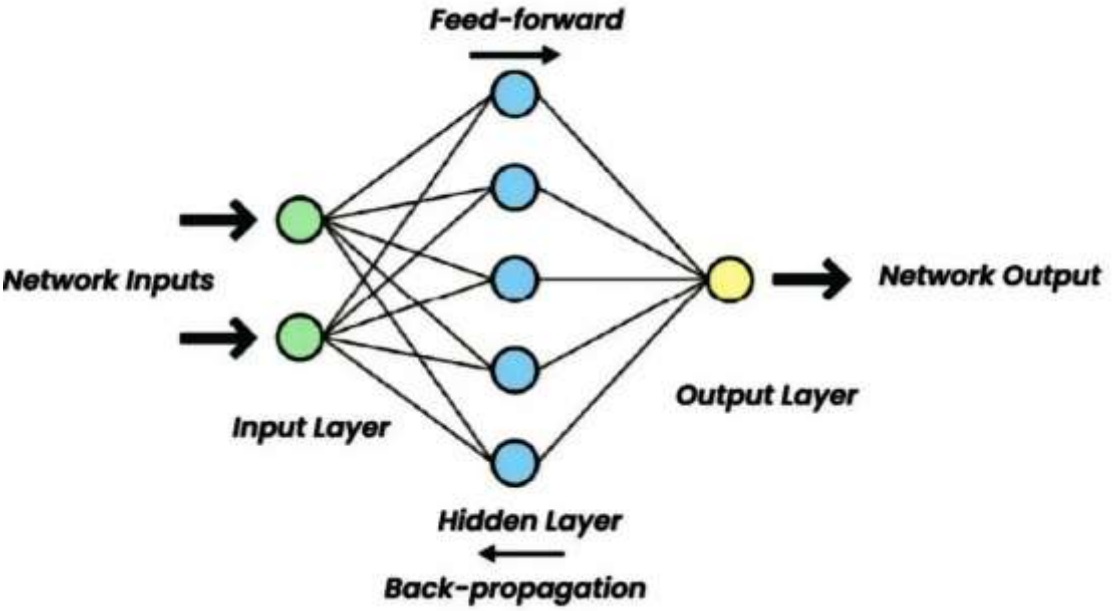


Figure 3.4: articial neural networks [89].

4.2.1.2 Biological inspiration:

- The human brain contains approximately 100 billion neurons, each composed of dendrites (receiving inputs), a cell body (integrating signals), and an axon (transmitting outputs) **[84].**

- ANNs replicate this structure, where artificial neurons receive weighted inputs, apply a threshold, and produce outputs based on activation functions **[91].**

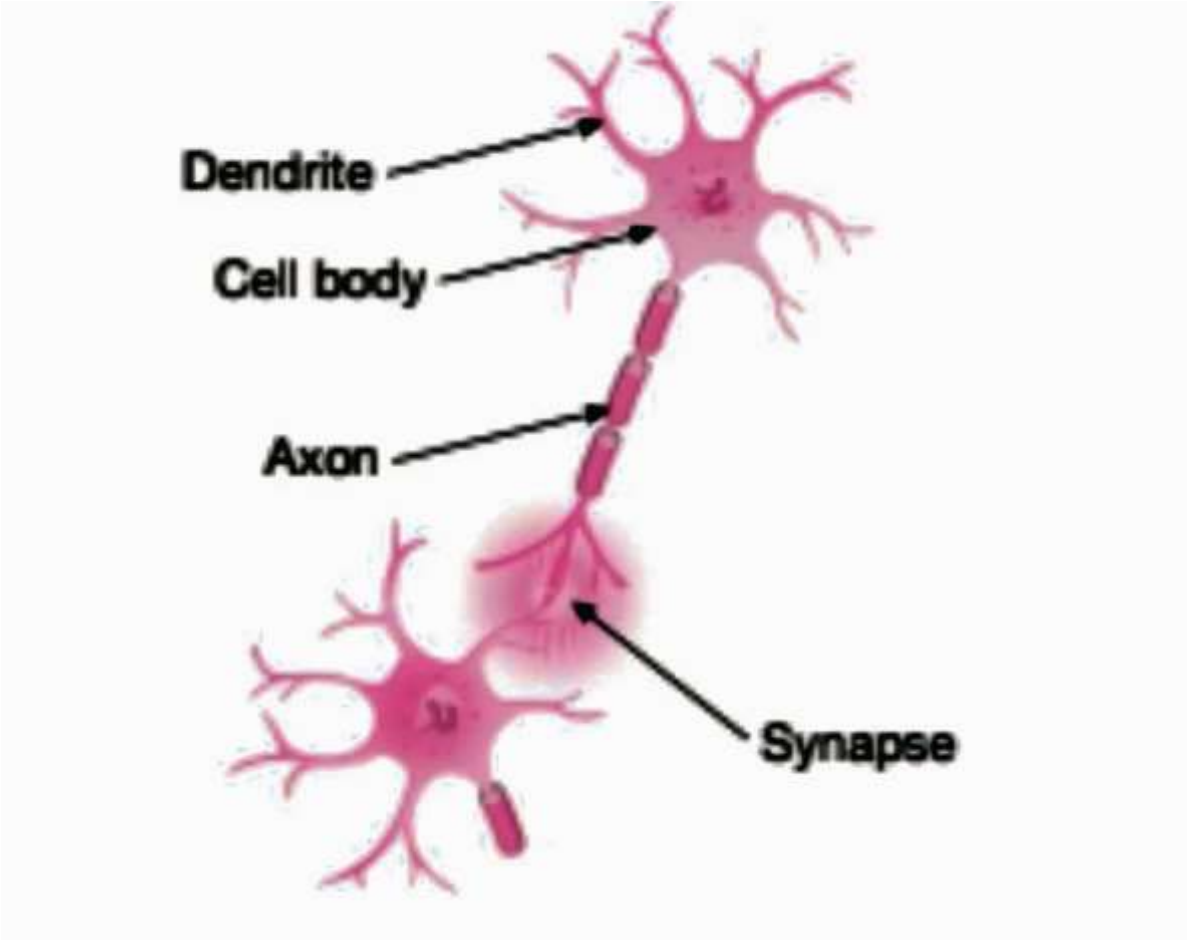


Figure 3.5: Neuron [92].

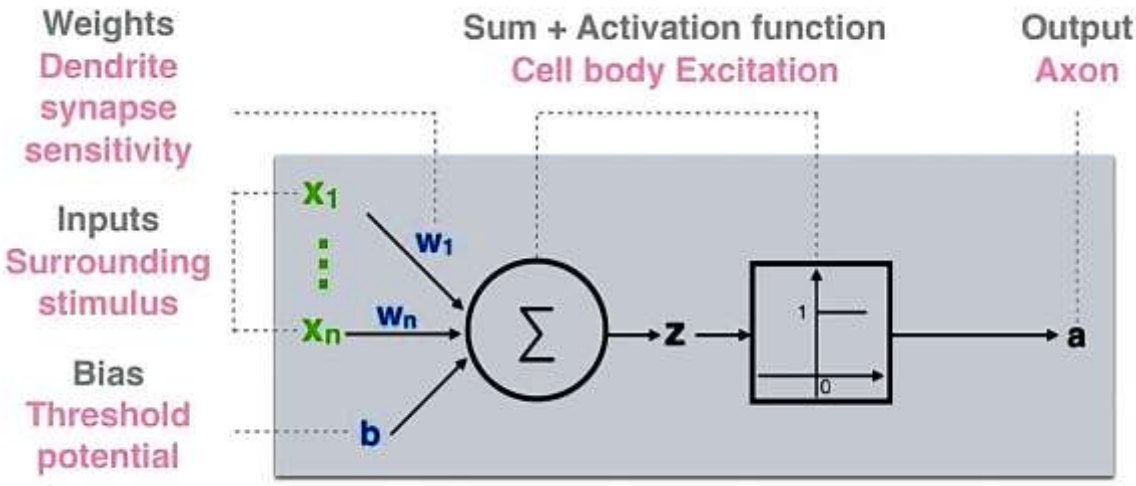


Figure 3.6: perceptron structure [93].

4.2.1.3 Mathematical Model:

- The perceptron is the simplest ANN model, emulating a single neuron.

- It takes binary inputs $(x_1, x_2 \ldots, x_n)$, each associated with a weight $(w_1, w_2 \ldots w_n)$, and computes a weighted sum.

- The output is determined by a step function:

$$a = \begin{cases} 0 \; if \;\; \sum_{i=1}^{n} w_i x_i + b \leq 0 \\ 1 \; if \;\; \sum_{i=0}^{n} w_i x_i + b > 0 \end{cases} \quad \ldots (3.16)$$

where (b) is the bias (threshold)**[94].**

4.2.1.4 Learning Mechanism:

- ANNs learn by adjusting weights and biases through backpropagation, minimizing errors between predicted and actual outputs **[95].**

- The hierarchical structure allows ANNs to extract features and patterns from data automatically, making them powerful tools for tasks like classification and regression **[96].**

4.2.2 Fuzzy logic:

Fuzzy logic, though still an underexplored domain, offers versatile applications across diverse disciplines. Unlike traditional binary logic (True/False), fuzzy logic addresses ambiguity by accommodating partial truth in uncertain scenarios. For example, when a system encounters vague or incomplete data, fuzzy algorithms evaluate all possible states using graded membership values within the range [0.0, 1.0], where **[97]:**

- 0.0 represents absolute falsehood,

- 1.0 denotes absolute truth.

- **Core Mechanism:**

Fuzzy systems assign intermediate values (e.g., 0.3, 0.7) to quantify uncertainty, enabling nuanced reasoning. This approach mirrors human decision-making, where outcomes often exist on a spectrum rather than as rigid binaries **[98].**

- **Applications & Advantages [99]:**

1. Decision-Making: Effective in scenarios requiring adaptive thresholds, such as automated control systems (e.g., temperature regulation).

2. Noise Handling: Robustly processes erratic or noisy data (e.g., sensor inputs, real-world measurements) by tolerating imprecision.

3. Complex Systems: Models intricate relationships in fields like AI, robotics, and medical diagnostics, where rigid rules fail.

By embracing ambiguity, fuzzy logic bridges the gap between computational precision and real-world complexity, proving indispensable for tasks demanding flexibility and resilience **[100].**

4.2.2.1 History and Evolution of Fuzzy Logic:

Fuzzy logic was introduced in 1965 by Lotfi A. Zadeh in his groundbreaking paper "Fuzzy Sets," which proposed a mathematical framework for handling uncertainty by allowing partial truth values between 0 (completely false) and 1 (completely true). This departure from classical binary logic enabled more human-like reasoning in machines. In the 1970s, Ebrahim Mamdani applied fuzzy logic to control systems, demonstrating its effectiveness in real-world scenarios like steam engine regulation. By the 1980s, Japan led industrial adoption, integrating fuzzy logic into consumer electronics (e.g., washing machines, air conditioners) and automated subway systems (e.g., Sendai Subway, 1987). The 1990s saw theoretical advancements, including hybrid neuro-fuzzy systems and the establishment of organizations like the International Fuzzy Systems Association (IFSA). Today, fuzzy logic plays a key role in AI (explainable decision-making), medicine (diagnostic systems), and robotics (adaptive control). Despite early skepticism, its ability to model uncertainty has solidified its place in engineering and AI, with ongoing research in optimizing rule-based systems and enhancing computational

efficiency. Key figures like Zadeh, Mamdani, Bart Kosko, and Didier Dubois have shaped its development, making fuzzy logic a vital tool for complex, real-world problem-solving **[101].**

4.2.2.2 Fuzzy Logic Architecture:

A fuzzy logic system operates through four key components:

- **Rule Base:** Contains expert-defined if-then rules that govern decision-making. Recent advancements in fuzzy theory have optimized controller design by reducing the number of required rules while maintaining system efficiency.

Stores IF-THEN rules and MF parameters

Rule Structure:

$$IF\ x_1\ is\ A_1 and\ \ x_2\ is\ A_2 THEN\ y\ is\ B$$

A_1 , A_2 : input fuzzy sets.

B: Output fuzzy sets (Mamdani) or crisp function (Sugeno).

- **Fuzzification:** Converts precise (crisp) numerical inputs into fuzzy sets. Unlike classical (crisp) sets, where membership is binary (true/false), fuzzy sets allow partial membership, enabling graded interpretations of data.

Key Equations:

- Triangular MF:

$$\mu A(x) = \max\left(0, \min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right)\right) \quad \dots (3.17)$$

a, b, c : Left foot, peak, right foot

- Gaussian MF:

$$\mu A(x) = e^{-\frac{1}{2}\left(\frac{X-C}{\sigma}\right)^2} \quad \dots (3.18)$$

c: Center, σ : width

- Trapezoidal MF:

$$\mu A(x) = \max\left(0, \min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right)\right) \ldots (3.19)$$

a, d : feet, b, c : shoulders

- **Inference Engine:** Evaluates the compatibility between input values and fuzzy rules to determine appropriate control actions. This component, combined with the knowledge base, forms the core fuzzy controller.

- **Defuzzification:** Translates fuzzy output sets back into crisp, actionable values. The choice of defuzzification method (e.g., centroid, max membership) depends on the application to ensure optimal results **[102].**

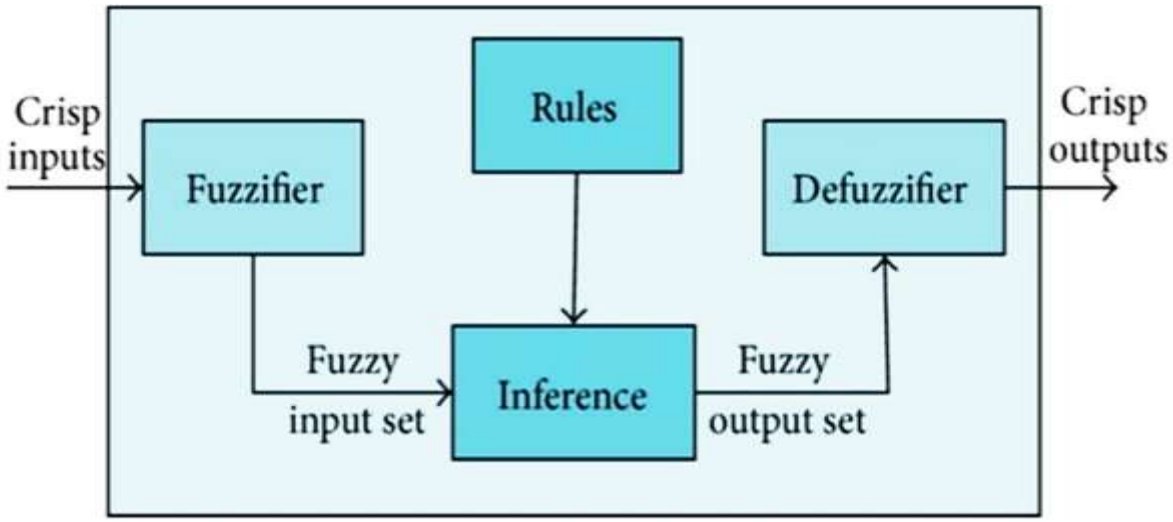


Figure 3.7: Fuzzy Logic architecture [103].

4.2.2.3 Fuzzy inference system:

A fuzzy inference system (FIS) is a framework that uses fuzzy logic to map inputs to outputs, enabling decision-making or pattern recognition. This process relies on three key components: membership functions (which define how input values are classified into fuzzy sets), logical operations (which combine fuzzy values), and if-then rules (which establish the relationships between inputs and outputs). Together, these elements allow the system to handle uncertainty and approximate reasoning effectively **[104].**

4.2.2.4 Mamdani and Sugeno Fuzzy Inference Systems:

4.2.2.4.1 Sugeno Fuzzy Inference Systems:

Sugeno Fuzzy Inference (also known as Takagi-Sugeno-Kang fuzzy inference) employs singleton output membership functions, which can be either constant or a linear combination of the input variables. Unlike Mamdani systems, which require complex centroid-based defuzzification, Sugeno systems simplify the process by using a weighted average or weighted sum of a few data points. This makes Sugeno's defuzzification more computationally efficient **[105].**

In a Sugeno system, each rule follows a structured approach. For example, in a two-input system with variables $\boldsymbol{x}$ and $\boldsymbol{y}$, the output of each rule is determined by either a constant value or a linear equation based on the inputs **[106].**

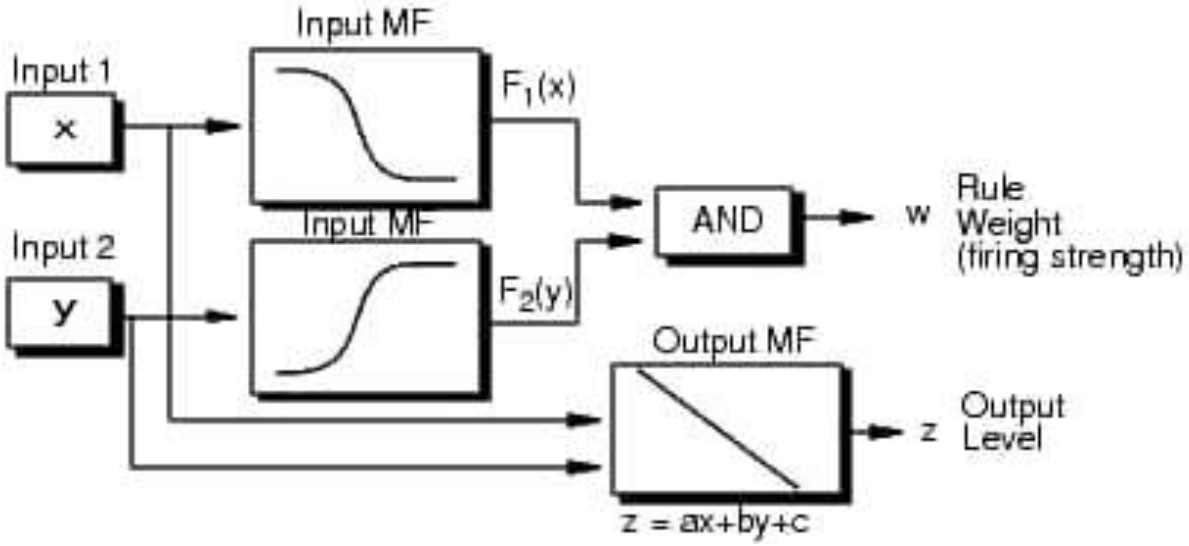


Figure 3.8: Fuzzy rule operation [107].

Each rule generates two values:

z_l Rule output level, which is either a constant value or a linear function of the input values

$$z_l = a_l x + b_l y + c_l \quad \ldots (3.20)$$

Here, x and y are the values of input 1 and input 2, respectively, and a_l, b_l, and c_l are constant coefficients.

For a zero-order Sugeno system, z_l is a constant ($\mathbf{a} = \mathbf{b} = \mathbf{0}$).

$\boldsymbol{\omega_l}$ Rule firing strength derived from the rule antecedent

$\boldsymbol{\omega_l}$=AndMethod $(F_1(\boldsymbol{x}), F_2(\boldsymbol{y}))$

Here, $F_1(\dots)$ and $F_2(\dots)$ are the membership functions for inputs 1 and 2, respectively.

The output of each rule is the weighted output level, which is the product of $\boldsymbol{\omega_l}$and $\boldsymbol{z_l}$.

The easiest way to visualize first-order Sugeno systems (a and b are nonzero) is to think of each rule as defining the location of a moving singleton. That is, the singleton output spikes can move around in a linear fashion within the output space, depending on the input values. The rule firing strength then defines the size of the singleton spike.

The final output of the system is the weighted average over all rule outputs:

Final Output = $\hat{\boldsymbol{f}}(\underline{\boldsymbol{x}}) = \frac{\sum_{l=1}^{M} \boldsymbol{\omega_l} b_l}{\sum_{l=1}^{M} \boldsymbol{\omega_l}}$ … (3.21)

So

Final Output = $\hat{\boldsymbol{f}}(\underline{\boldsymbol{x}}) = \frac{\sum_{l=1}^{M} b_l \prod_{i=1}^{n} \boldsymbol{exp}\left(-\left(\frac{xi - c_i^l}{\sigma_i^l}\right)^2\right)}{\sum_{l=1}^{M} \prod_{i=1}^{n} \boldsymbol{exp}\left(-\left(\frac{xi - c_i^l}{\sigma_i^l}\right)^2\right)}$ … (3.22)

Where:

$$\boldsymbol{\omega_l} = \mu A_1^i(x1) \wedge \dots \wedge \mu A_1^i(xp) \dots (3.23)$$

is the degree of verity of the l^{th} rule and μA_j^i is the membership degree of value xj to fuzzy set A_j^i . The error between the original output and the fuzzy model is given by

$$e = \frac{1}{2}\left(Or - \hat{f}(\underline{x})\right)^2 \dots (3.24)$$

The modified parameters: part of consequent b^l , mean part of antecedent c_i^l , and sigma part of antecedent σ_i^l are presented as:

$$b^l(K+1) = b^l(k) + \alpha_b \frac{e}{B}\omega_l \quad \dots (3.25)$$

$$c_i^l(k+1) = c_i^l(k) + \alpha_c \frac{(d-\hat{f})}{B}\left(b^l - \hat{f}\right)^2 \frac{\left(x_i - c_i^l\right)}{\sigma_i^{l^2}}\omega_l \quad \dots (3.26)$$

$$\sigma_i^l(k+1) = \sigma_i^l(k) + \alpha_\sigma \frac{e}{B}\left(b^l - \hat{f}\right)^2 \frac{\left(x_i - c_i^l\right)}{\sigma_i^{l^2}}\omega_l \quad \dots (3.27)$$

Where:

$$B = \sum_{l=1}^{M} \omega_l \dots (3.28)$$

The following figure shows the fuzzy inference process for a Sugeno system **[108].**

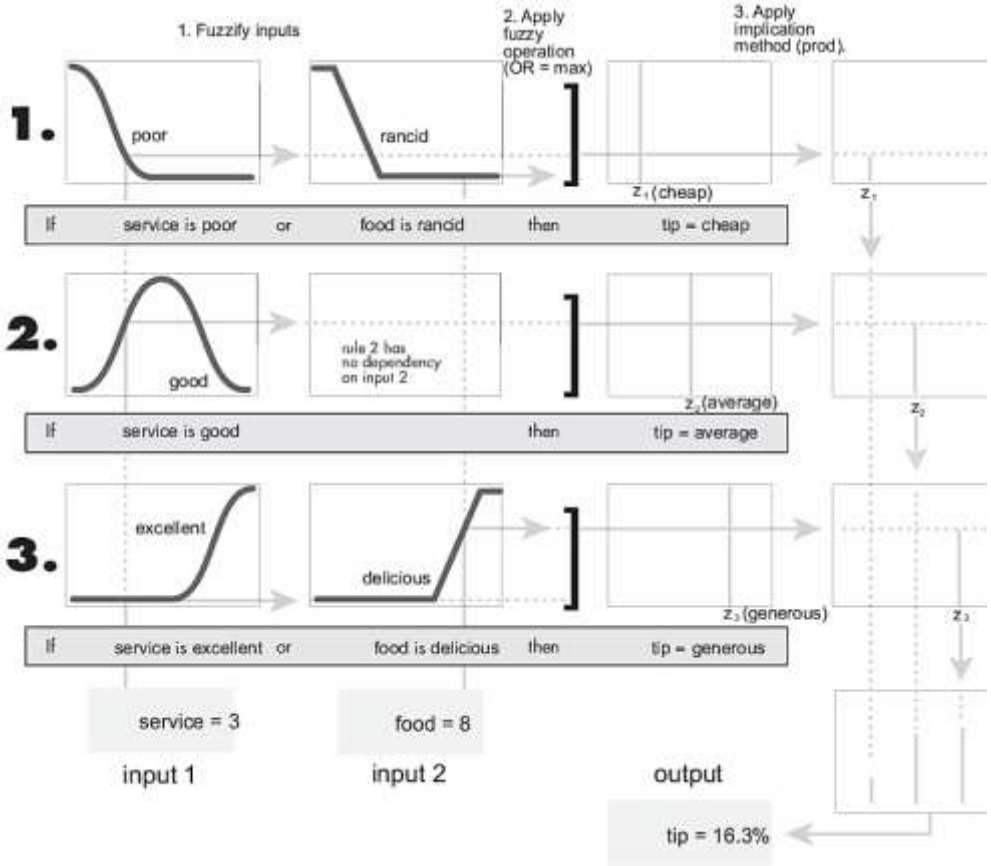


Figure 3.9: fuzzy inference process for a Sugeno system [107].

Note: Sugeno systems always use product implication and sum aggregation **[109]**

The Sugeno fuzzy inference method is particularly effective as an interpolating supervisor for multiple linear controllers in nonlinear dynamic systems. Since each rule depends linearly on the inputs, the Sugeno system can smoothly transition between different control strategies based on varying operating conditions **[110].**

For instance, an aircraft's performance varies significantly with altitude and Mach number. While linear controllers work well for specific flight conditions, they must adapt continuously to the vehicle's changing state. A Sugeno fuzzy system excels at this task by interpolating linear gains across the input space, acting as an efficient gain scheduler **[111].**

Moreover, Sugeno systems are well-suited for modelling nonlinear systems, as they can seamlessly blend multiple linear models to approximate complex behaviors **[112].**

4.2.2.4.2 Mamdani Fuzzy Inference Systems:

Mamdani fuzzy inference was originally developed to design control systems by translating linguistic rules from human operators into automated control strategies. Unlike Sugeno systems, each rule in a Mamdani system produces a fuzzy set as its output **[113].**

Thanks to their intuitive and interpretable rule structure, Mamdani systems are particularly useful in expert systems that rely on human knowledge such as medical diagnostics, where rules are derived from expert judgment.

The diagram below summarizes the Mamdani inference process **[114].**

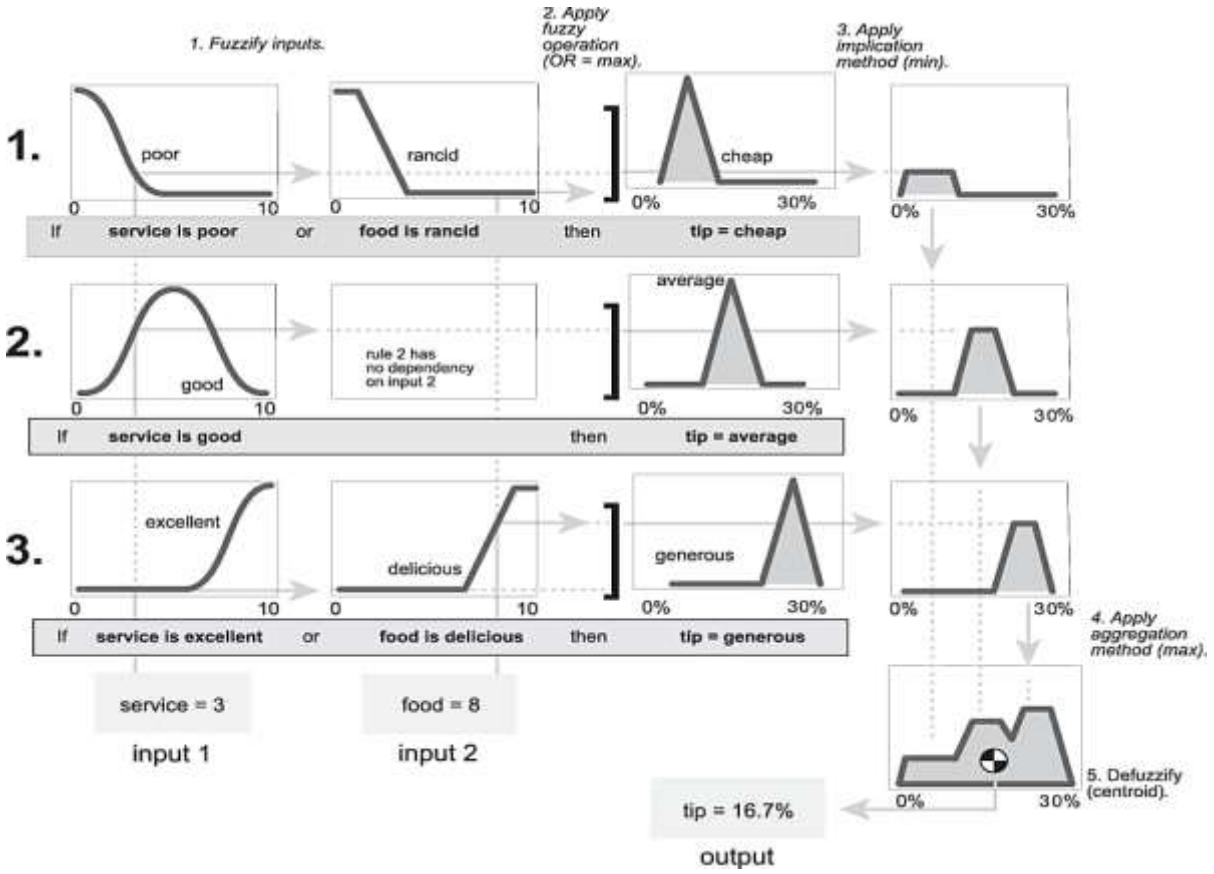


Figure 3.10: Fuzzy inference process for mamdani system [108].

In a Mamdani fuzzy inference system, each rule generates a fuzzy set based on the system's output membership functions and implication method. These individual fuzzy sets are then merged into a single aggregated fuzzy set using the system's chosen aggregation method **[113].**

4.2.2.4.3 Difference Between Mamdani and Sugeno Fuzzy Inference System:

Mamdani FIS	Sugeno FIS
Output membership function is present	No output membership function is present
The output of surface is discontinuous	The output of surface is continuous
Distribution of output	Non distribution of output, only Mathematical combination of the output and the rules strength
Through defuzzification of rules consequent of crisp result is obtained	No defuzzification here. Using weighted average of the rules of consequent crisp result is obtained
Expressive power and interpretable rule consequent	Here is loss of interpretability
Mamdani FIS possess less flexibility in the system design	Sugeno FIS possess more flexibility in the system design
It has more accuracy in security evaluation block cipher algorithm	It has less accuracy in security evaluation block cipher algorithm
It is using in MISO (Multiple Input and Single Output) and MIMO (Multiple Input and Multiple Output) systems	It is using only in MISO (Multiple Input and Single Output) systems
Mamdani inference system is well suited to human input	Sugeno inference system is well suited to mathematically analysis
Application: Medical Diagnosis System	Application: To keep track of the change in aircraft performance with altitude

Table 5: Difference Between Mamdani and Sugeno Fuzzy Inference System [107]

4.2.3 Particle Swarm Optimization (PSO):

The Particle Swarm Optimization (PSO) method for classification is a metaheuristic approach inspired by the social behaviour of bird flocking or fish schooling. PSO is commonly used for optimization problems, but it can also be applied to classification tasks by optimizing classifier parameters or feature selection **[115].**

4.2.3.1 How PSO Works for Classification:

PSO is a population-based optimization algorithm where each candidate solution is called a particle. These particles move through the search space to find the optimal solution (e.g., the best set of features or classifier parameters) **[116].**

4.2.3.2 Algorithm Steps:

1. Initialize particles with random positions and velocities.
2. Evaluate fitness (classification performance) for each particle.
3. Update pbest (individual best) and gbest (global best).
4. Adjust velocity and position using:

$$\boldsymbol{v_i^{t+1} = \omega . v_i^t + c_1 . r_1 . (\mathbf{pbest}_i - x_i^t) + c_2 . r_2 . x_i^{t+1} = x_i^t + v_i^{t+1}} \ldots (3.29)$$

where:

- ✓ $(\boldsymbol{\omega})$ = inertia weight,
- ✓ $(\boldsymbol{c_1, c_2})$ = cognitive & social coefficients,
- ✓ $(\boldsymbol{r_1, r_2})$ = random numbers in [0,1].

5. Repeat until convergence (max iterations or desired accuracy) **[117].**

4.2.3.3 Core PSO Equations:

Component	Equation
Velocity Update	$v_i(t+1) = \omega v_i(t) + c_1 r_1 \left(pbest_i - x_i\ (t)\right) + c_2 r_2 (gbest - x_i(t))$
Position Update	$x_i(t+1) = x_i(t) + v_i(t+1)$
Inertia Weight	$\omega(t) = \omega_{max} - (\frac{\omega_{max} - \omega_{min}}{t_{max}})t$
Constriction PSO	$v_i(t+1) = x\ [v_i(t) + c_l r_l (pbest_i - x_i(t))$
Fully Informed PSO	$v_i(t+1) = \omega v_i(t) + \sum_{k=1}^{K} \frac{cr_k (pbest_k - \ x_i(t))}{K}$

Table 6: Core PSO Equations [118].

4.2.3.4 Parameter tuning tips:

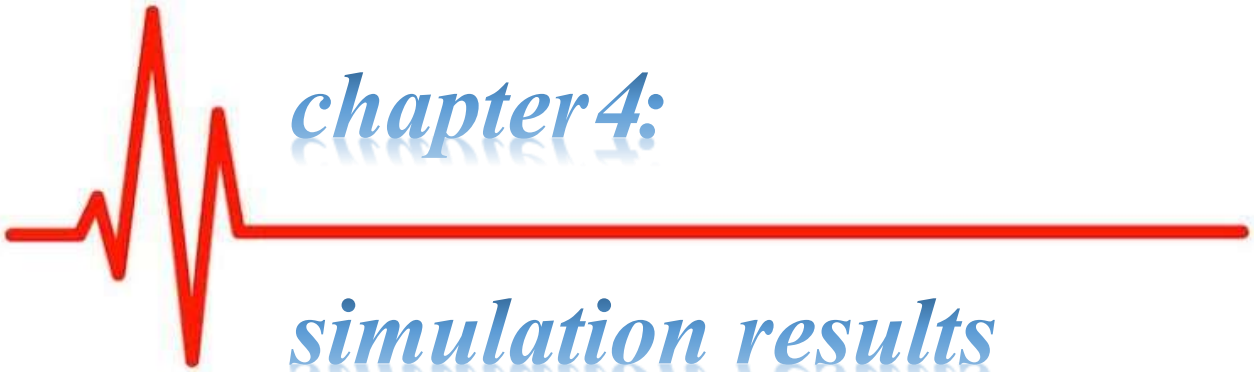
Parameter	Role	Typical Value
$\boldsymbol{\omega}$	Balances exploration (highw) vs.ex ploitation (lown)	0.4- 0.9(or adaptive)
$\boldsymbol{c_1}$	Controlsattraction tobest (individual learning)	1.5 – 2.0
$\boldsymbol{c_2}$	Controls attraction togbest (social learning)	1.5 – 2.0
$\boldsymbol{N}$	Number of particles (higher = better exploration but slower)	20 – 50

Table 7: Parameter tuning tips [119].

5. conclusion:

Signal classification is a fundamental aspect of modern signal processing, with applications spanning telecommunications, biomedical engineering, industrial automation, and artificial intelligence. This chapter explored both classical and AI-based methods, highlighting their strengths, limitations, and practical use cases.

In the next chapter, we will use one of these methods fuzzy inference system, to classify the ECG signal.



chapter 4:

simulation results

1.Introduction:

Several identification methods have been developed for nonlinear signals. The advent of fuzzy logic has made it possible to propose particularly effective approaches for dealing with the nonlinearity of these signals**[120]**.

In this chapter, we apply this identification technique to three nonlinear signals of varying complexity **[121].** Among these signals, we focus particularly on the electrocardiogram (ECG), with the following objectives:

1. Identifying it with minimal error
2. Accurately classifying it To achieve this, we use: The fuzzy backpropagation algorithm and the Sugeno fuzzy system inference model

This chapter also presents simulation results that demonstrate the effectiveness of our approach.

2.Presentation of the signals to be identified:

Signal 1:

It described by the second-order recurrent equation:

$$\boldsymbol{y(k+1) = f[y(k), y(k-1)] + u(k)} \quad \textbf{...(4.1)}$$

Where function f has the form:

$$\boldsymbol{f[y(k), y(k-1)] = \frac{y(k)y(k-1)[y(k)+2.5]}{(1+y(k)^2+y(k-1)^2}} \textbf{...(4.2)}$$

where, u(k) is abounded reference input:

$$u(k) = sin\,(2\Pi k/25) \textbf{...(4.3)}$$

For learning of fuzzy model we took the following parameters:

FIS	Number of rules	20
	Algorithm	Back-propagation
	Random input u(k)	[-2,2]
	Number of inputs	2
	Part of consequent α_c	0.4
	Part of premise α_b	0.9
	Sigma part of antecedent δ	0.9
	Training time (s)	59.56
	Number of iterations	6001
	MSE	$3.832e^{-0.05}$

Table (8): parameters used to identify the output of signal 1

- **Fuzzy identification:**

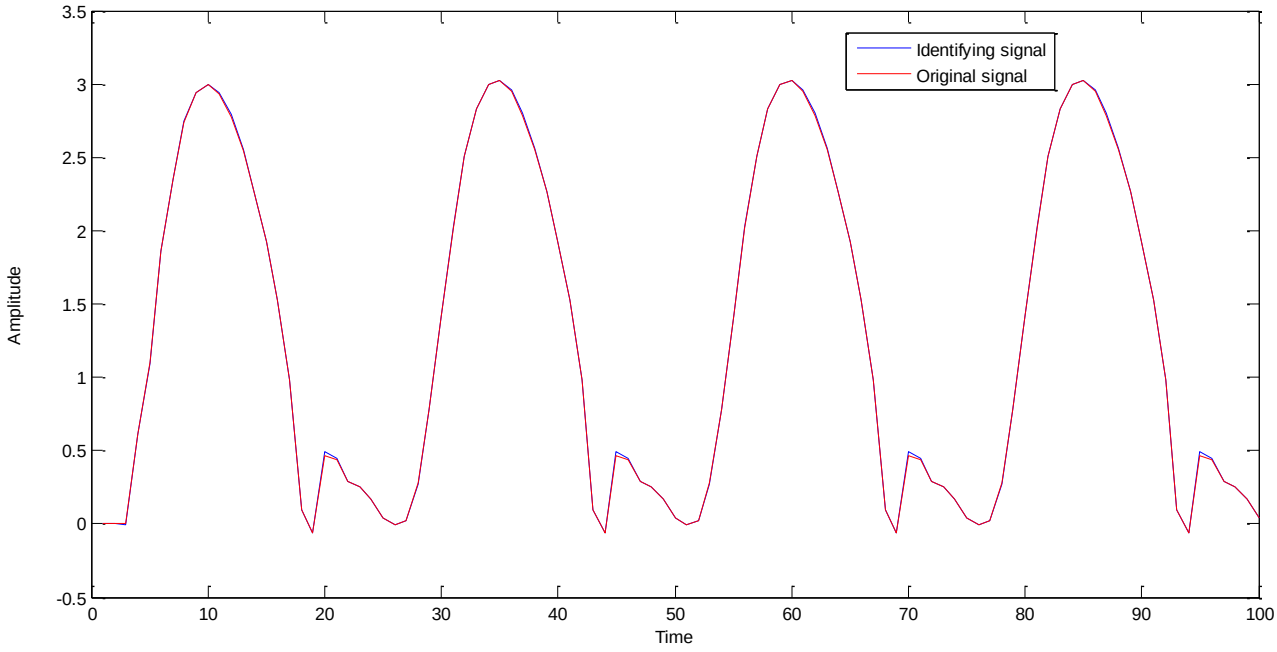


Figure 4.1: identifying signal and original signal

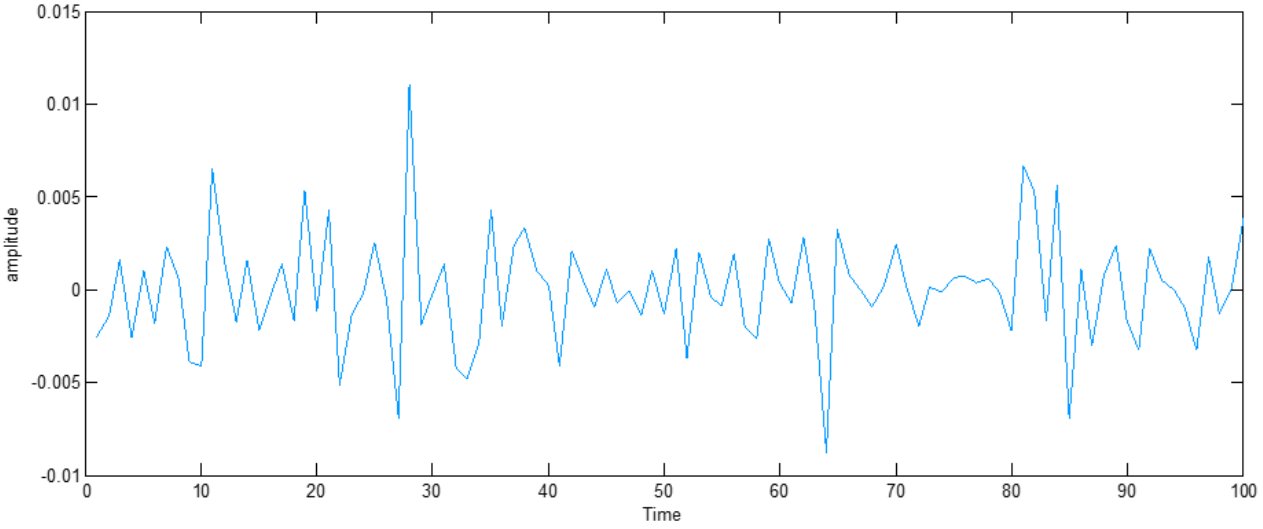


Figure 4.2: MSE between identifying signal and original signal

Signal 2 :

MISO signal described by the recurrent equation:

$$y\,(k\ +1)\ =\ f[y(k), y\,(k\ -\ 1), y\,(k\ -\ 2), u(k), u\,(k\ -\ 1)]\ldots(4.4)$$

Where function f has the form:

$$f\,[y(k), y(k\ -1), y(k\ -2), u(k), u(k\ -\ 1)] = \frac{[y(k)y(k-1)y(k-2)u(k-1)][y(k-2)-1]+u(k)}{1+y(k-2)^2+y(k-1)^2}\ldots\ (4.5)$$

Where u(k) is abounded reference input:

$u(k)\ =\ sin(2\Pi k/250)$ For $1 \leq k \leq 500$

$u(k) = 0.8sin(2\Pi k250)\ + 0.2sin(2\Pi k/25)$ For$500 \leq k \leq 800$

For learning of fuzzy model we took the following parameters:

FIS	Number of rules	20
	Algorithm	Back-propagation
	Random input u(k)	[-1,1]
	Number of inputs	5
	Part of consequent α_c	0.8
	Part of premise α_b	0.8
	Sigma part of antecedent δ	1.2
	Training time (s)	159.86
	Number of iterations	3001
	MSE	$2.7417e^{-0.04}$

Table (9): parameters used to identify the output of signal 2

➢ **Fuzzy identification:**

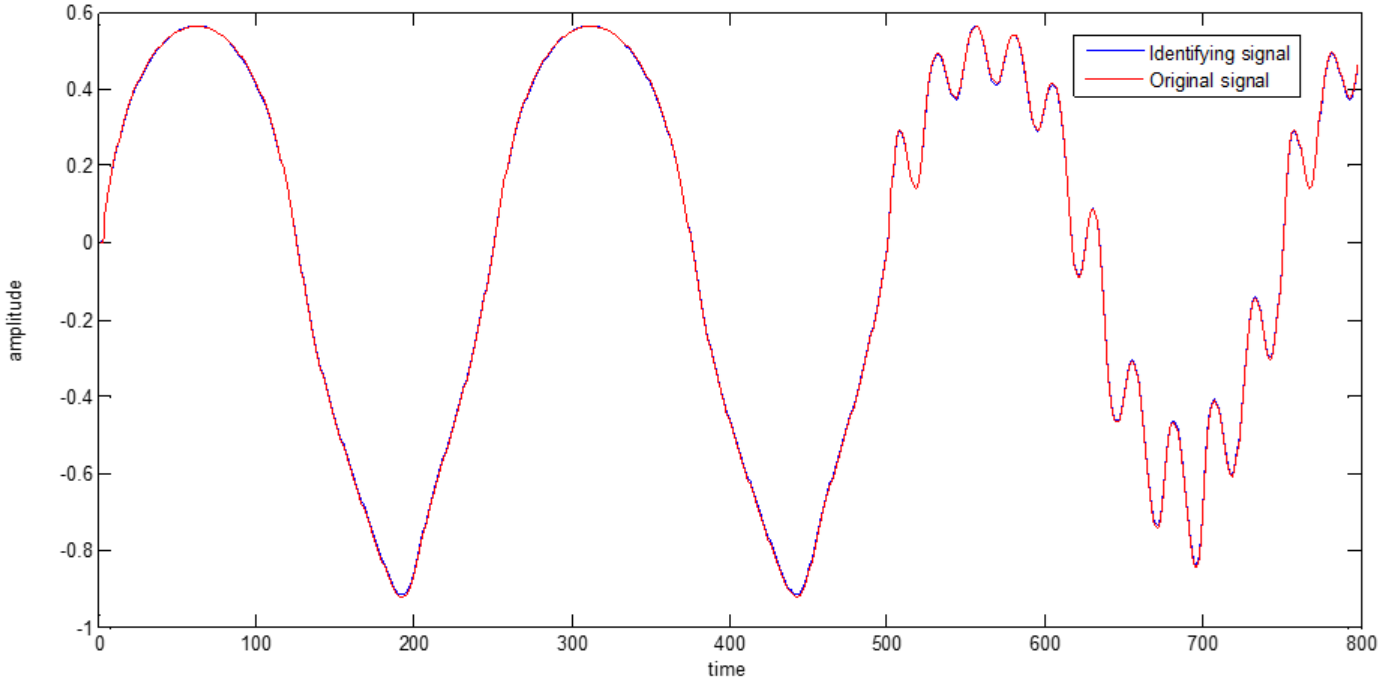


Figure 4.3: identifying signal and MISO signal

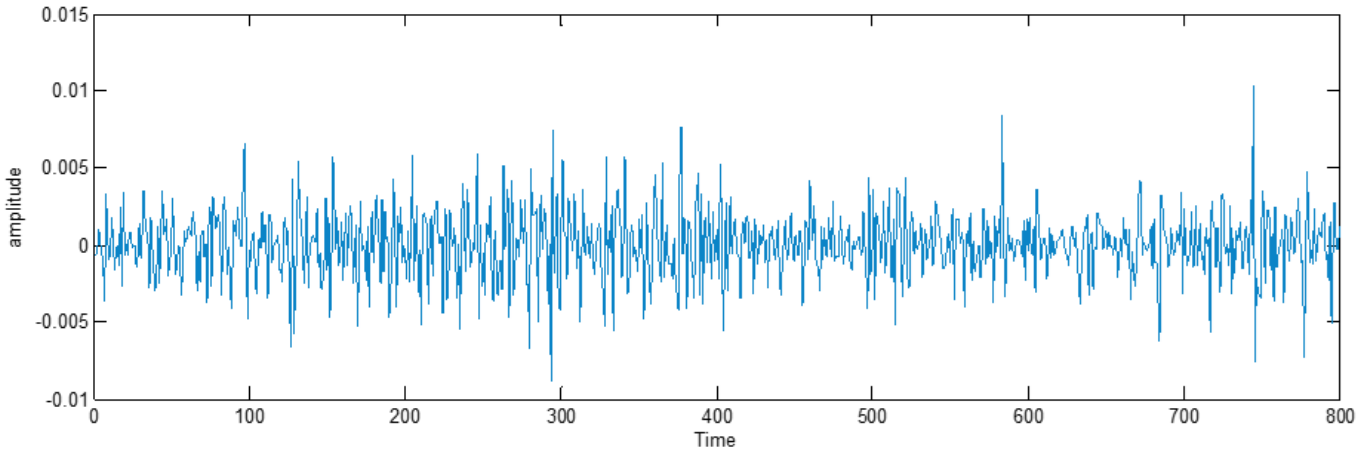


Figure 4.4: MSE between identifying signal and MISO signal

3 .Electrocardiogram (ECG) signal:

to obtain the ECG signal we use the development of Fourier serie.

P wave: using the Fourier serie the p wave can be represented as follows:

$$f(x)=\begin{cases} 0 & -l < x < -l/b \\ \cos\left(\frac{\pi b}{2l}\right) & -l/b < x < -l/b \\ 0 & l/b < x < l \end{cases} \ldots(4.6)$$

where:

b: is an adjustment coefficient related to the duration of the wave.

So, we can evaluate the coefficients of Fourier series as:

$$a_0 = \frac{1}{l}\int cos\left(\frac{\pi bx}{2l}\right) dx \ldots (4.7)$$

So, $a_0 = -4/\pi b$

$$a_n = \frac{1}{l}\int cos\left(\frac{\pi bx}{2l}\right) cos\left(\frac{\pi bx}{2l}\right) dx \ldots (4.8)$$

we find:

$$a_n = \frac{2}{\pi(b-2n)}(sin(\pi b - 2n\pi)/2b) + \frac{2}{\pi(b-2n)}(sin(\pi b - 2n\pi)/2b) \ldots (4.9)$$

$$b_n = \frac{1}{l}\int cos\left(\frac{n\pi x}{2l}\right) sin(\frac{n\pi x}{l})\, dx \ldots (4.10)$$

as the function is pair so:

$$b_n = 0$$

QRS complex:

Using the Fourier serie , the complex QRS can be reprented as the follows:

$$f(x) = \begin{cases} \left(-\frac{bax}{l}\right) + a & 0 < x < l/b \\ \left(-\frac{bax}{l}\right) + a & -l/b < x < 0 \end{cases} \ldots (4.11)$$

so, we can evaluate the coefficients of fourier series as:

$$a_n = 1/l \int f(x)dx \ldots (4.12)$$

so $a_0 = a/b$

$$a_n = 1/l \int f(x)\cos(\frac{\pi bx}{l})dx \ldots (4.13)$$

where

$$a_n = \frac{2ba}{n^2\pi^2}(1 - cos(\frac{n\pi}{b}) \ldots (4.14)$$

and

$$b_n = 1/l \int f(x) sin\left(\frac{n\pi x}{l}\right) dx \ldots (4.15)$$

The generation of the others waves (u,t) follows the same approach of the precceding waves.

Now that all the waves are generated, we can add them together to produce an ECG signal.

The different characteristics of normal Ecg signal are:

Characteristic / waves	Amplitude (mv)	Duration (s)	Time (s)
P	0.25	0.09	0.16
q	0.025	0.066	0.166
Qrs	1.6	0.11	
S	0.25	0.066	0.09
T	0.35	0.142	0.2
u	0.035	0.0476	0.433

Table (10): characteristics of normal Ecg signal

We choose The number of heartbeats= 72 Beats/min

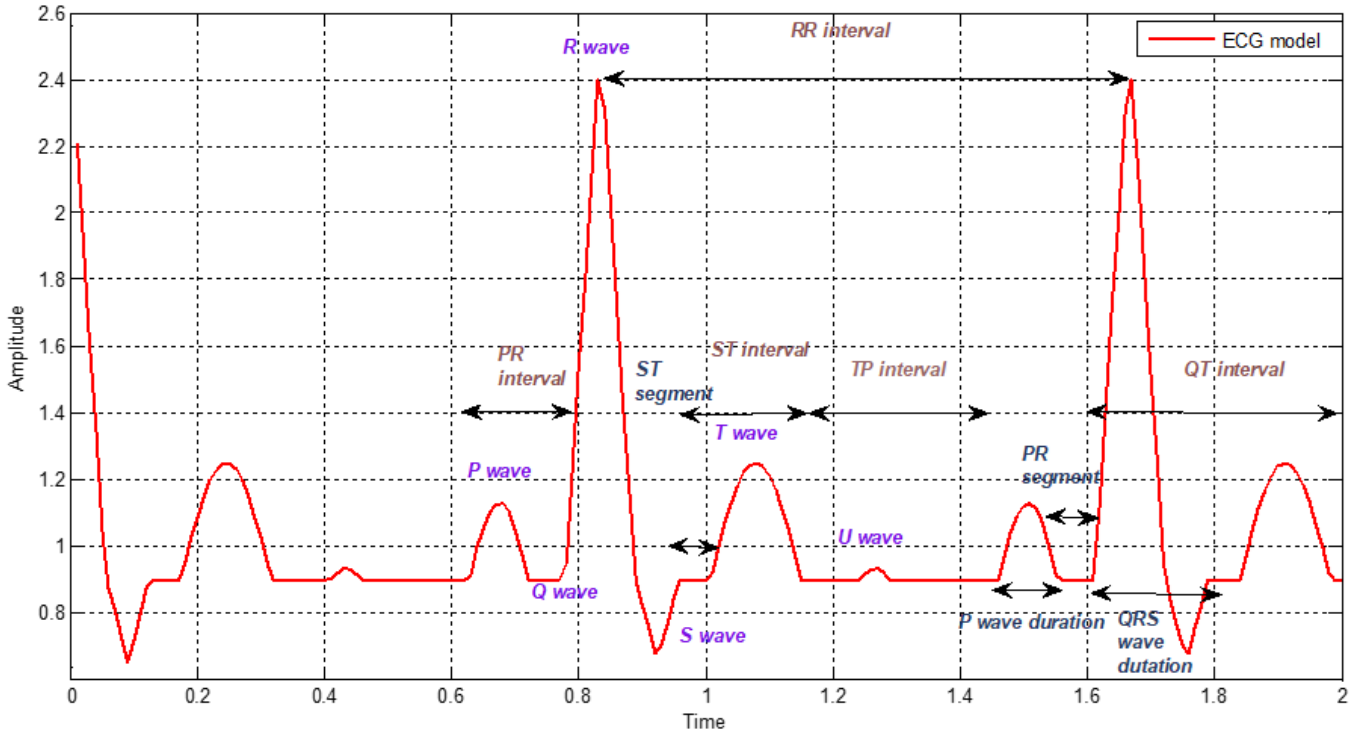


Figure 4.5: trace ecg

3.1 Fuzzy identification of Ecg signal:

For learning of fuzzy model we took the following parametrs:

FIS	Number of rules	30
	Algorithm	Back-propagation
	Part of consequent α_c	0.5
	Part of premise α_b	0.1
	Sigma part of antecedent δ	0.9
	Training time (s)	9422
	Number of iterations	18000
	MSE	0.8818

Table (11): parameters used to identify the output of Ecg signal

As for the random inputs, they are shown in the following table:

Characteristic / waves	Amplitude (mv)	Time (s)
P	[0.1,0.299]	[0.1,0.299]
q		[0.1,0.299]
S		[0.01,0.209]
T	[0.1,0.299]	[0.1,0.299]
U	[0.01,0.209]	

Table(12) : Random inputs

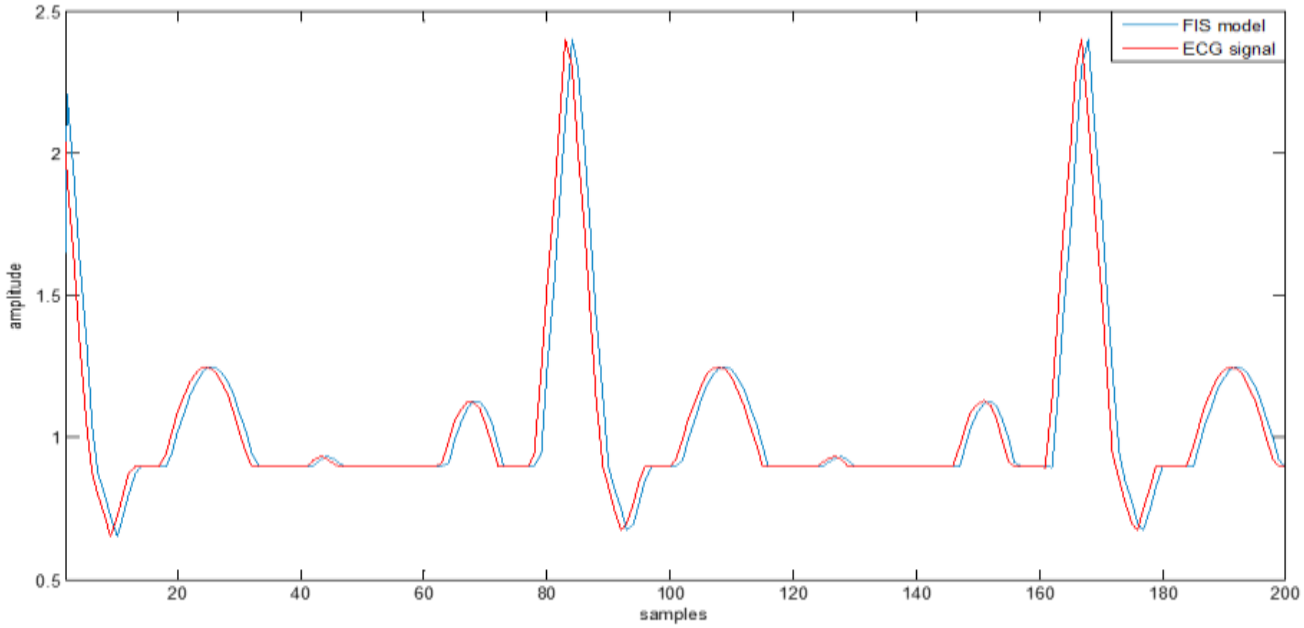


Figure 4.6: Fuzzy model and original ECG

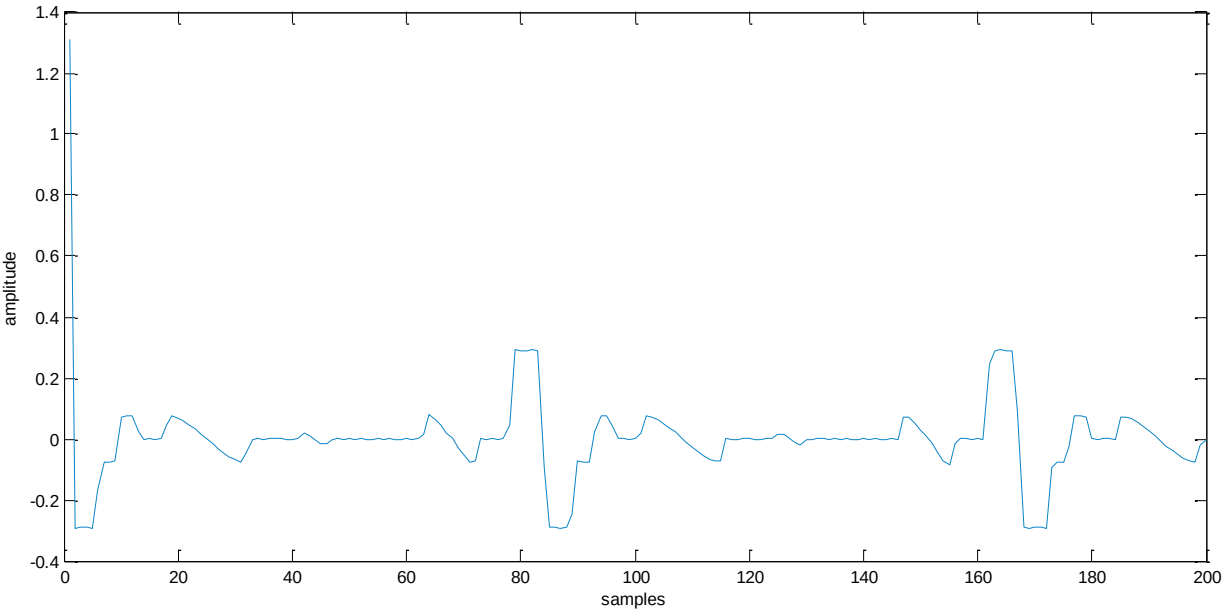


Figure 4.7: MSE between Fuzzy model and original ECG

3.2 Classification of ECG signal:

We took three ECGs from three patients and compared them to the ECG model (identifying ECG) as shown in the following figures:

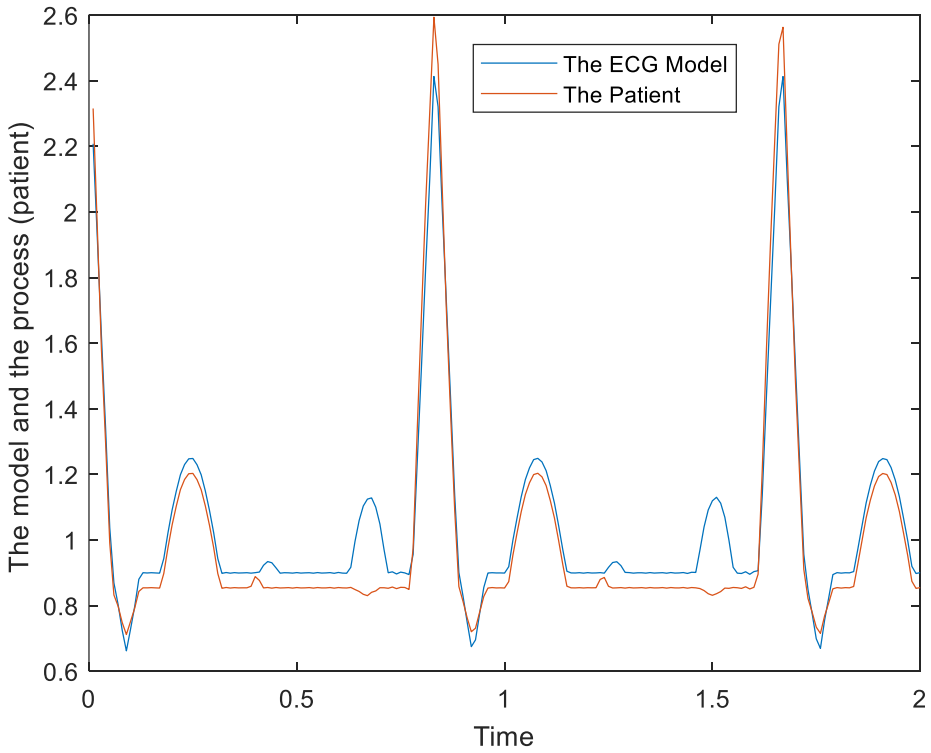


Figure 4.8: the model and the process (patient 1)

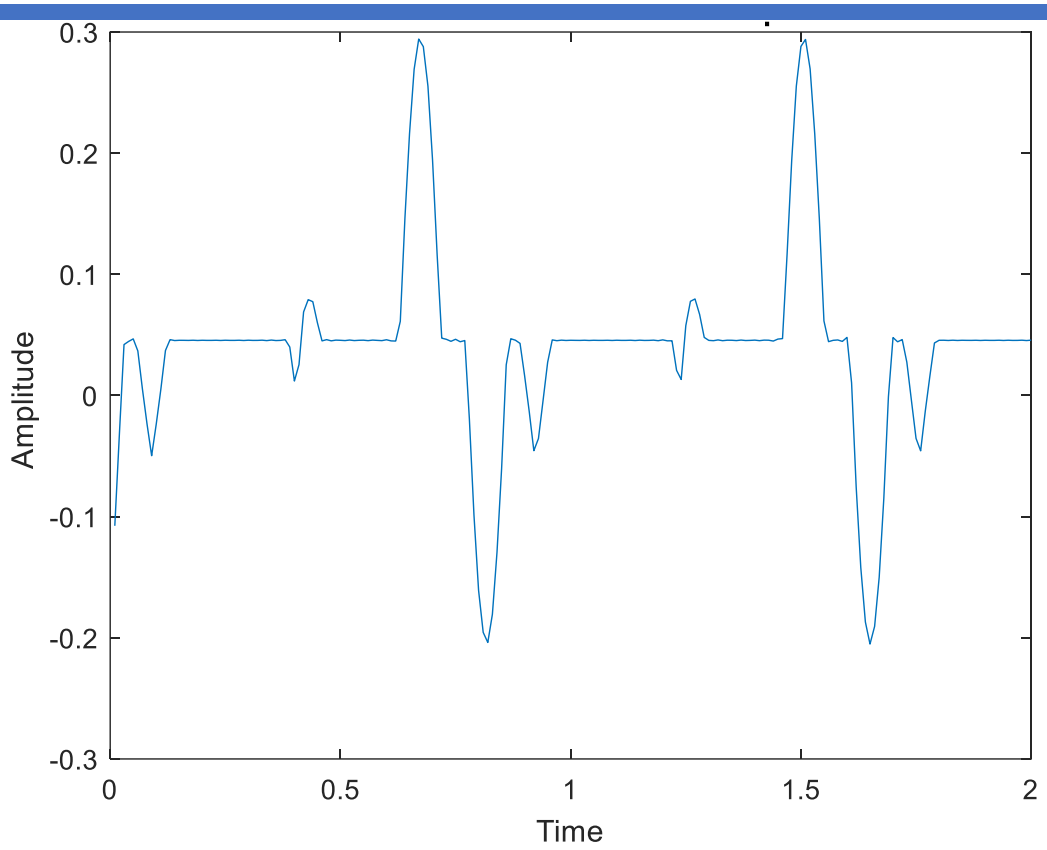


Figure 4.9: error between the model and the process (patient 1)

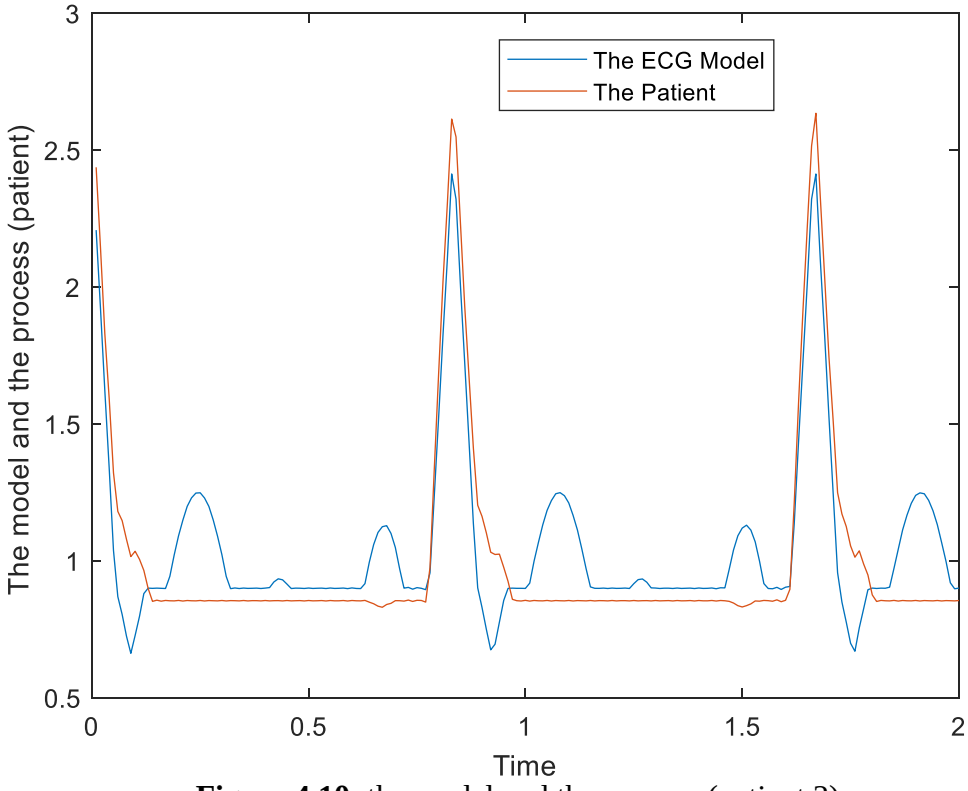


Figure 4.10: the model and the process (patient 2)

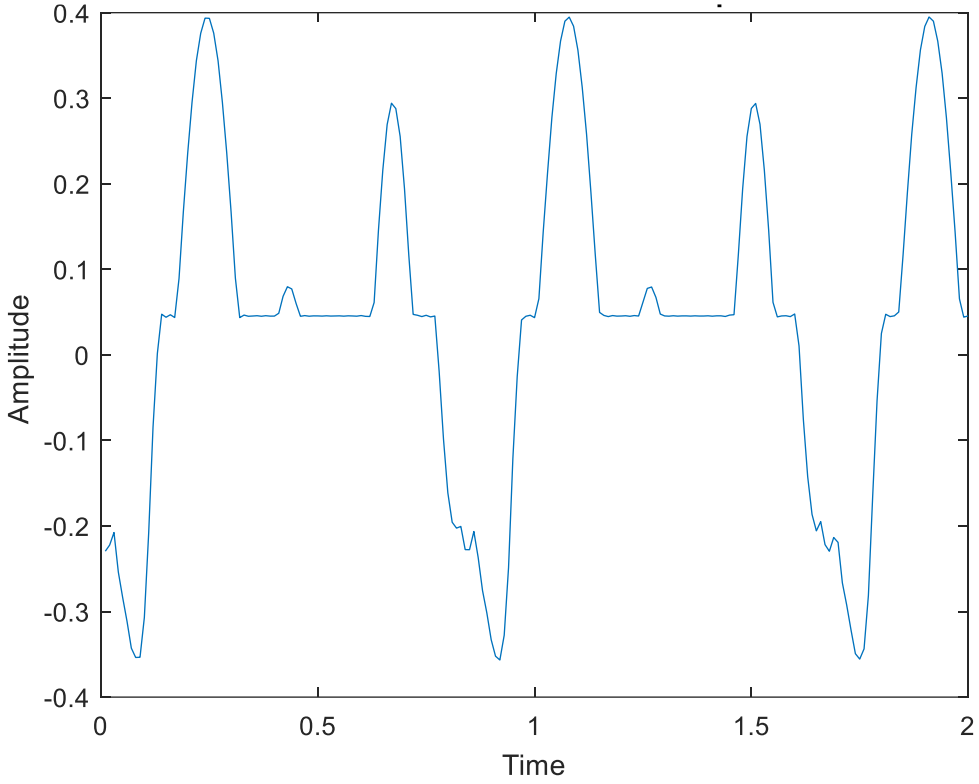


Figure 4.11: error between the model and the process (patient 2)

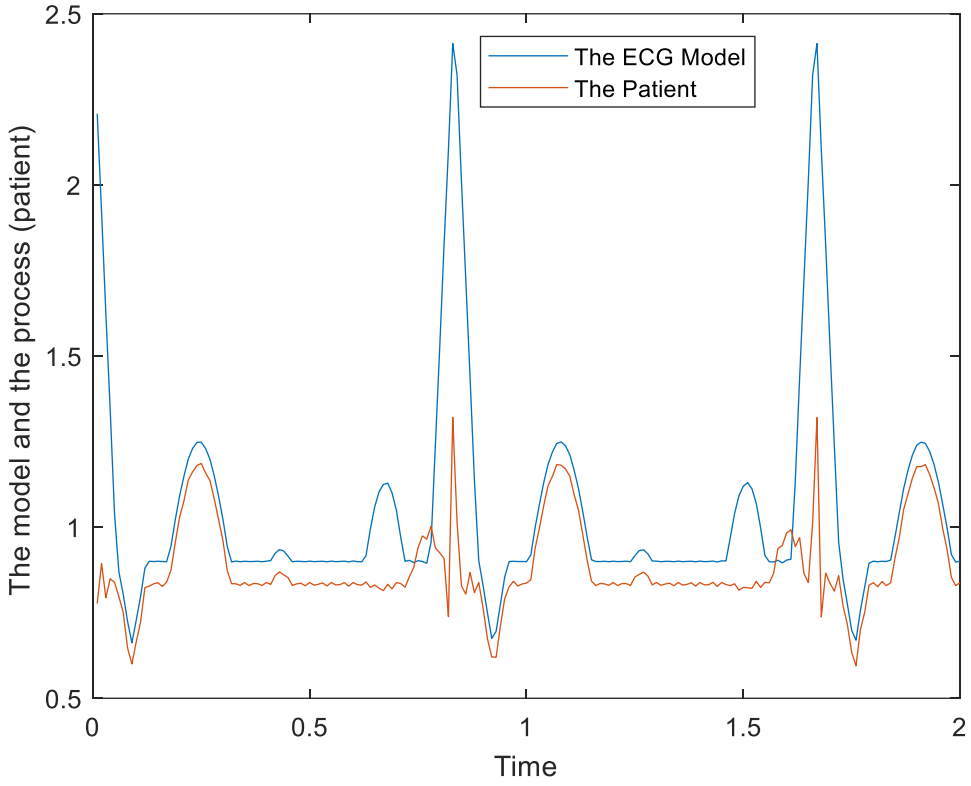


Figure 4.12: the model and the process (patient 3)

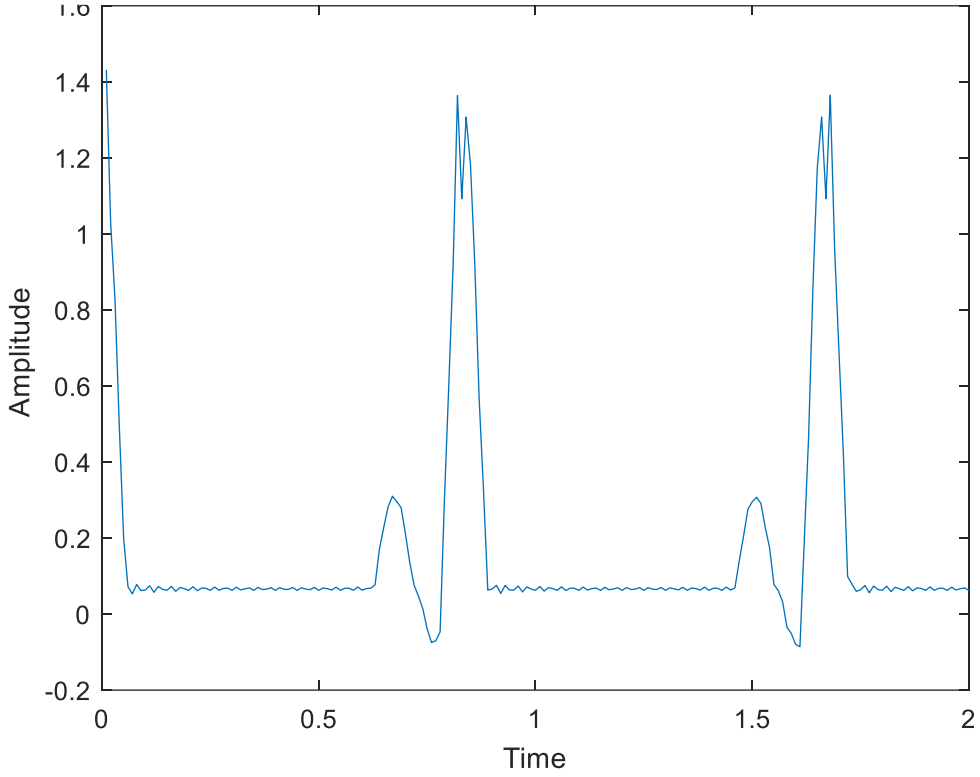


Figure 4.13: error between the model and the process (patient 3)

4.Interpretation of results:

In this work we use the Fuzzy model for identification of 3 nonlinear signals and Classification of ECG signal.

First, we studied the identification of nonlinear signals according to their complexity. Signal 1, which has two inputs $[y(k), y(k-1)]$ and one output,signal 2,which has 5 inputs

$$[y(k), y(k-1), y(k-2), u(k), u(k-1)]$$

And one output and ECG signal its inputs ar showen in the **table(10)**

To evaluate the robustness of fuzzy model identification, we assess three key performance criteria: training time, number of iterations, and the Mean Square Error (MSE) between the estimated models and the desired outputs.

We see from **(Fig 4.1)** that our FIS identifier with two inputs follow the desired signal 1, For excitation, we used a random input. The computation time for the identification process was 59.56(s) over 6001 iterations, as shown in (**Fig 4.2**) the Mean Square Error (MSE) between our model and the original signal equal to 3.7832e-005. as shown in **(fig 4.3)** the FIS identifier

withe 5 inputs follow the desired signal 2, we again used a random input. The computation time for the identification process was 159.86(s) over 3001 iterations, the Mean Square Error (MSE) between our model and the original signal shown in **(fig 4.4)** equal to 2.7417e-004. for ECG signal identification as shown in **(fig 4.6)** the FIS identifier follows the original ECG signal,1, For excitation, we used a 7 randoms inputs.

The computation time for the identification process was 9422 (s) over 18000 iterations.the Mean Square Error (MSE) between our model and the original signal shown in **(fig4.7)** equal to 0.8818

Next, we classified the ECG signal by comparing three ECGs for three patients with ECG model identifier:

- Patient 1: by comparison shown in the **(fig 4.8)** the classification based on the absence of the P wave and sum change in the shape of the U wave the amplitude of the QRS complex. **(fig 4.9)** represents the error between the model and the patient's 1 ECG.
- Patient 2: by comparison shown in the **(fig 4.10)** the classification based on the absence of the P wave and S wave. **(fig 4.11)** represents the error between the model and the patient's 2 ECG.
- Patient 3:by comparison shown in the **(fig 4.12)** the classification is based on a distinct abnormality in the QRS complex, indicating that the patient is in a highly critical condition. **(Fig 4.13)** represents the error between the model and the patient's 3 ECG.

The following table represents the diseases associated with each patient:

Patient	diseases
1	Atrial fibrillation (AFib)
2	Myocardial Infarction (MI) Atrial fibrillation (AFib)
3	Atrial flutter (AFL) Left Ventricular Hypertrophy (LVH)

Table (13): diseases associated with each patient.[122]

5.Conclusion:

In this chapter we illustrated the identification by fuzzy logic, we took some particular signals of different levels of complexity, and we classified the ECG signal, and finally we discussed the results.



General conclusion

General conclusion:

This thesis sought to elucidate a major challenge in modern cardiology: the automated and accurate interpretation of the electrocardiogram (ECG), a complex biological signal characterized by its nonlinearity, interindividual variability, and sensitivity to artifacts. Given the limitations of traditional approaches, this research rigorously explored the contribution of Fuzzy Inference Systems (FIS), demonstrating their relevance for modeling the uncertainty inherent in cardiac signals while preserving clinical interpretability .

The methodological architecture, structured into four chapters, allowed a logical progression from theoretical foundations to practical validation :

1. In-depth analysis of nonlinear signals established the conceptual framework necessary to understand the dynamic complexity of the ECG .
2. Physiological and electrical modeling of the heart clarified the link between microscopic phenomena (cardiomyocyte depolarization) and their macroscopic manifestation (ECG morphology) .
3. The comparative study of classification methods highlighted the decisive advantage of FIS in processing imprecise data, unlike conventional transforms .
4. MATLAB simulations experimentally validated the robustness of the proposed model, with competitive classification rates for target pathologies.

The prospects for this work are numerous :

- Several recent methods, such as fuzzy logic, can be applied to identification and classification .
- From a physical perspective, the ECG signal can be studied with disturbances and a technique designed to filter out distortions, especially those from the instrument.
- From a disease perspective, the classification of ECG signals by disease can be studied based on signal parameters, particularly considering the most common types of cardiovascular diseases.



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