PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA

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Preface

The course offered in this course is an introductory course in physics 2 and electricity and magnetism. Specifically addressed to first-year science and technology students. Our concern while writing this book has been to refer to the knowledge that students have acquired in high school classes in order to ensure the most continuous transition as possible.

Certainly the main difficulty we encountered during this course was mathematical. Physics is a science that requires precision and concepts gained during high school learning are dealt with here in a more formal and rigorous manner. Therefore, we focused a lot on the mathematical tools that we believe are necessary for a good understanding of the physics course.

Chapter I: Mathematical notions

1. Elements of length, surface area, volume.

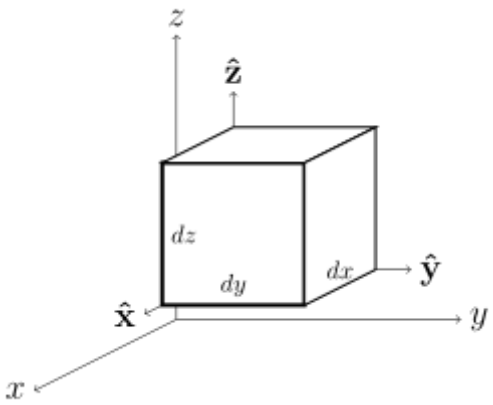
1.1 Cartesian coordinates

We define the position vector $\overrightarrow{OM}$ in Cartesian coordinates as follows

$$\overrightarrow{OM} = \vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

The expression for the differential element $d\vec{r}$ is

$$d\vec{r} = \frac{\partial \vec{r}}{\partial x}dx + \frac{\partial \vec{r}}{\partial y}dy + \frac{\partial \vec{r}}{\partial z}dz$$



Where

$$\frac{\partial \vec{r}}{\partial x} = \vec{\imath}$$

$$\frac{\partial \vec{r}}{\partial y} = \vec{\jmath}$$

$$\frac{\partial \vec{r}}{\partial z} = \vec{k}$$

We have the differential element $d\vec{r}$ in Cartesian coordinates as

$$d\vec{r} = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$$

Differential normal area dS is

$$dS = dydz\vec{\imath} + dxdz\vec{\jmath} + dxdy\vec{k}$$

and the volume element dV is

$$dV = dxdydz$$

1.2 Polar coordinates

We define the position vector $\overrightarrow{OM}$ in Polar coordinates as follows

$$\overrightarrow{OM} = \vec{r} = r\vec{u}_r$$

Where

$$\begin{cases} x = rcos(\theta) \\ y = rsin(\theta) \\ r = \sqrt{x^2 + y^2} \end{cases}$$

The expression for the differential element $d\vec{r}$ is

$$d\vec{r} = \frac{\partial \vec{r}}{\partial r}dr + \frac{\partial \vec{r}}{\partial \theta}d\theta$$

Where

$$\frac{\partial \vec{r}}{\partial r} = \vec{u}_r$$

$$\frac{\partial \vec{r}}{\partial \theta} = r\vec{u}_\theta$$

We have the differential element $d\vec{r}$ in Polar coordinates as

$$d\vec{r} = dr\vec{u}_r + rd\theta\vec{u}_\theta$$

Differential normal area dS is

$$dS = rdrd\theta$$

1.3 Cylindrical coordinates

We define the position vector $\overrightarrow{OM}$ in Cylindrical coordinates as follows

$$\overrightarrow{OM} = \vec{r} = \rho\vec{e}_\rho + z\vec{k}$$

Where

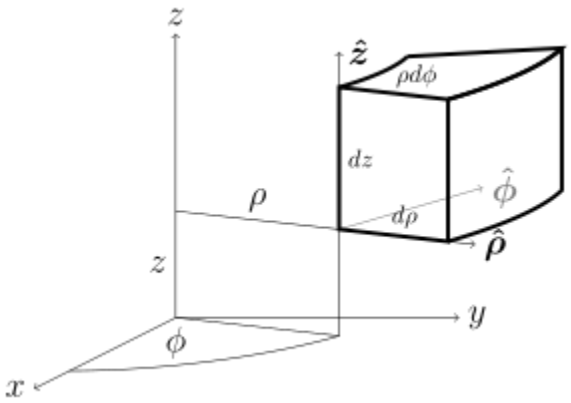
$$\begin{cases} x = \rho cos(\theta) \\ y = \rho sin(\theta) \\ z = z \end{cases}$$

And

$$\rho = \sqrt{x^2 + y^2}$$

The expression for the differential element $d\vec{r}$ is

$$d\vec{r} = \frac{\partial \vec{r}}{\partial \rho} d\rho + \frac{\partial \vec{r}}{\partial \theta} d\theta + \frac{\partial \vec{r}}{\partial z} dz$$



Where

$$\frac{\partial \vec{r}}{\partial \rho} = \vec{e}_\rho$$

$$\frac{\partial \vec{r}}{\partial \theta} = \rho \vec{e}_\theta$$

$$\frac{\partial \vec{r}}{\partial z} = \vec{k}$$

We have the differential element $d\vec{r}$ in Cylindrical coordinates as

$$d\vec{r} = d\rho \vec{e}_\rho + \rho d\theta \vec{e}_\theta + dz\vec{k}$$

Differential normal area dS is

$$dS = \rho d\theta dz \vec{e}_\rho + d\rho dz \vec{e}_\theta + \rho d\rho d\theta \vec{k}$$

and the volume element dV is

$$dV = \rho d\rho d\theta dz$$

1.4 Spherical coordinates

We define the position vector $\overrightarrow{OM}$ in Spherical coordinates as follows

$$\overrightarrow{OM} = \vec{r} = r\vec{e}_r$$

Where

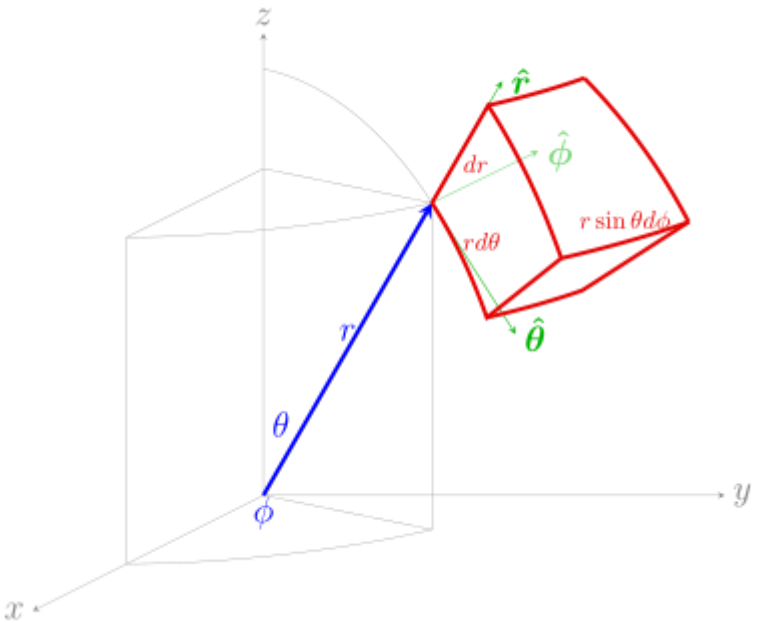
$$\begin{cases} x = rsin(\theta)cos(\phi) \\ y = rsin(\theta)sin(\phi) \\ z = rcos(\theta) \end{cases}$$

and

$$r = \sqrt{x^2 + y^2 + z^2}$$

The expression for the differential element $d\vec{r}$ is

$$d\vec{r} = \frac{\partial \vec{r}}{\partial r}dr + \frac{\partial \vec{r}}{\partial \theta}d\theta + \frac{\partial \vec{r}}{\partial \phi}d\phi$$



Where

$$\frac{\partial \vec{r}}{\partial r} = \vec{e}_r$$

$$\frac{\partial \vec{r}}{\partial \theta} = r\vec{e}_\theta$$

$$\frac{\partial \vec{r}}{\partial \phi} = rsin\theta\vec{e}_\phi$$

We have the differential element $d\vec{r}$ in Spherical coordinates as

$$d\vec{r} = dr\vec{e}_\rho + rd\theta\vec{e}_\theta + rsin\theta d\phi\vec{e}_\phi$$

Differential normal area dS is

$$dS = r^2 sin\theta d\theta d\phi\vec{e}_\rho + rsin\theta dr d\phi\vec{e}_\theta + rdrd\theta\vec{e}_\phi$$

and the volume element dV is

$$dV = r^2 sin\theta dr d\theta d\phi$$

2. The operators

2.1. Cartesian coordinates

The unit vectors of the Cartesian basis are $(\vec{i}, \vec{j}, \vec{k})$. we define the scalar function $f(x, y, z)$ and the vector $\vec{A}$

$$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$$

2.1.1 Gradient

from which we deduce the expression of the gradient of the function $f(x, y, z)$

$$\nabla f = \frac{\partial f}{\partial x}\vec{\imath} + \frac{\partial f}{\partial y}\vec{\jmath} + \frac{\partial f}{\partial z}\vec{k}$$

and therefore of the operator ∇ (called **del**)

$$\nabla = \vec{\imath}\frac{\partial}{\partial x} + \vec{\jmath}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

We also deduce the **Laplacian** ($\Delta = \nabla^2$) form of the function $f(x, y, z)$

$$\Delta = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

2.1.2 The Divergence

The divergence of the vector $\vec{A}$ is a scalar that can be represented as

$$\nabla . \vec{A} = \left(\frac{\partial}{\partial x}\vec{\imath} + \frac{\partial}{\partial y}\vec{\jmath} + \frac{\partial}{\partial z}\vec{k}\right) . (A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k})$$

$$\nabla . \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

2.1.3 The Curl

The Curl of the vector $\vec{A}$ is a vector that can be represented as

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{\imath} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{k}$$

2.2 Cylindrical coordinates

The unit vectors of the Cylindrical basis are $(\vec{e}_\rho, \vec{e}_\theta, \vec{k})$. we define the scalar function f and the vector $\vec{A}$

$$\vec{A} = A_\rho \vec{e}_\rho + A_\theta \vec{e}_\theta + A_z \vec{k}$$

2.2.1 Gradient

from which we deduce the expression of the gradient of the function f is

$$\nabla f = \frac{\partial f}{\partial \rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{k}$$

We also deduce the **Laplacian** ($\Delta = \nabla^2$) form of the function f is

$$\Delta = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

2.2.2 The Divergence

The divergence of the vector $\vec{A}$ is a scalar that can be represented as

$$\nabla . \vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

2.2.3 The Curl

The Curl of the vector $\vec{A}$ is a vector that can be represented as

$$\nabla \times \vec{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\vec{e}_\theta + \frac{1}{\rho}\left(\frac{\partial(\rho A_\theta)}{\partial \rho} - \frac{\partial A_\rho}{\partial \theta}\right)\vec{k}$$

2.3 Spherical coordinates

The unit vectors of the Spherical basis are $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi)$. we define the scalar function f and the vector $\vec{A}$

$$\vec{A} = A_r\vec{e}_r + A_\theta\vec{e}_\theta + A_\varphi\vec{e}_\varphi$$

2.3.1 Gradient

from which we deduce the expression of the gradient of the function f is

$$\nabla f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\vec{e}_\varphi$$

We also deduce the **Laplacian** ($\Delta = \nabla^2$) form of the function f is

$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin\theta^2}\frac{\partial^2 f}{\partial \varphi^2}$$

2.3.2 The Divergence

The divergence of the vector $\vec{A}$ is a scalar that can be represented as

$$\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

2.3.3 The Curl

The Curl of the vector $\vec{A}$ is a vector that can be represented as

$$\nabla\times\vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right)\vec{e}_r$$

$$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}(rA_\varphi)\right)\vec{e}_\theta+\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right)\vec{e}_\varphi$$

Example

We have the position vector $\vec{r}$

$$\vec{r}=x\vec{\imath}+y\vec{\jmath}+z\vec{k}$$

and the magnitude of the position vector $\vec{r}$

$$r=\sqrt{x^2+y^2+z^2}$$

1- Find the gradient of $\vec{r}$.

2- Calculate the divergence of $\vec{r}$.

3- Calculate the divergence of $\vec{u}_r$.

4- Calculate the curl of $\vec{r}$.

5- Calculate the curl of $\vec{u}_r$.

Solution

1- The gradient

$$\nabla r=\frac{\partial r}{\partial x}\vec{\imath}+\frac{\partial r}{\partial y}\vec{\jmath}+\frac{\partial r}{\partial z}\vec{k}$$

Where

$$\frac{\partial r}{\partial x}=\frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right)=\frac{2x}{2\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial r}{\partial y}=\frac{\partial}{\partial y}\left(\sqrt{x^2+y^2+z^2}\right)=\frac{2y}{2\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial r}{\partial z}=\frac{\partial}{\partial z}\left(\sqrt{x^2+y^2+z^2}\right)=\frac{2z}{2\sqrt{x^2+y^2+z^2}}$$

So,

$$\nabla r=\frac{x}{\sqrt{x^2+y^2+z^2}}\vec{\imath}+\frac{y}{\sqrt{x^2+y^2+z^2}}\vec{\jmath}+\frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}$$

$$\nabla r=\frac{x\vec{\imath}+y\vec{\jmath}+z\vec{k}}{\sqrt{x^2+y^2+z^2}}$$

And

$$\nabla r=\frac{\vec{r}}{r}=\vec{u}_r$$

2- The divergence

$$\frac{\partial x}{\partial x}=\frac{\partial y}{\partial y}=\frac{\partial z}{\partial z}=1$$

We have

$$\nabla.\vec{r}=1+1+1=3$$

3- The divergence of $\vec{u}_r$

We have

$$\vec{u}_r=\frac{\vec{r}}{r}=\frac{x\vec{\imath}+y\vec{\jmath}+z\vec{k}}{\sqrt{x^2+y^2+z^2}}$$

And

$$\frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)=\frac{\sqrt{x^2+y^2+z^2}-\frac{2x^2}{2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$\frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)=\frac{y^2+z^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

And

$$\frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)=\frac{x^2+z^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=\frac{x^2+y^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

So,

$$\nabla.\vec{u}_r=\frac{y^2+z^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

$$+\frac{x^2+z^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}+\frac{x^2+y^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

And

$$\nabla.\vec{u}_r=\frac{2(x^2+y^2+z^2)}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

$$\nabla.\vec{u}_r=\frac{2}{\sqrt{x^2+y^2+z^2}}=\frac{2}{r}$$

4- The curl of $\vec{r}$.

$$\nabla \times \vec{r} = \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right)\vec{\imath} + \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right)\vec{k}$$

We have

$$\nabla \times \vec{r} = (0)\vec{\imath} + (0)\vec{\jmath} + (0)\vec{k}$$

And

$$\nabla \times \vec{r} = \vec{0}$$

5- The curl of $\vec{u}_r$.

$$\nabla \times \vec{u}_r = \left(\frac{\partial}{\partial y}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) - \frac{\partial}{\partial z}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)\right)\vec{\imath}$$

$$+\left(\frac{\partial}{\partial z}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) - \frac{\partial}{\partial x}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)\right)\vec{\jmath}$$

$$+\left(\frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) - \frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)\right)\vec{k}$$

And

$$\nabla \times \vec{u}_r = \left(-\frac{\frac{yz}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} + \frac{\frac{yz}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}\right)\vec{\imath}$$

$$+\left(-\frac{\frac{xz}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}+\frac{\frac{xz}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}\right)\vec{j}$$

$$+\left(-\frac{\frac{xy}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}+\frac{\frac{xy}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}\right)\vec{k}$$

We have

$$\nabla \times \vec{u}_r = \vec{0}$$

3. Derivatives and multiple integrals

3.1. Derivatives multiple

Example 01

We have

$$f(x,y) = 7x^2 - x^3y^4 + 5x^4y^3$$

So,

$$\frac{\partial f(x,y)}{\partial x} = 14x - 3x^2y^4 + 20x^3y^3$$

$$\frac{\partial f(x,y)}{\partial y} = -4x^3y^3 + 15x^4y^2$$

Example 02

We have

$$f(x,y) = 3x^2y^4 - 5x^7 + 4y^8$$

So,

$$\frac{\partial f(x,y)}{\partial x} = 6xy^4 - 35x^6$$

$$\frac{\partial f(x,y)}{\partial y} = 12x^2y^3 + 32y^7$$

Example 03

We have

$$f(x,y) = e^{x^2y^3}$$

So,

$$\frac{\partial f(x,y)}{\partial x} = 2xy^3e^{x^2y^3}$$

$$\frac{\partial f(x,y)}{\partial y} = 3x^2y^2e^{x^2y^3}$$

Example 04

We have

$$f(x,y) = x^4e^{y^5}$$

So,

$$\frac{\partial f(x,y)}{\partial x} = 4x^3e^{y^5}$$

$$\frac{\partial f(x,y)}{\partial y} = 5x^4y^4e^{y^5}$$

Example 05

$$f(x,y) = \ln(x^2 + y^2)$$

So,

$$\frac{\partial f(x,y)}{\partial x} = \frac{2x}{x^2+y^2}$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{2y}{x^2+y^2}$$

Example 06

We have

$$f(x,y) = \sqrt{x^2+y^2}$$

So,

$$\frac{\partial f(x,y)}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}}$$

Example 07

We have

$$f(x,y,z) = 4x^2y^3z + 6xy^2z^2$$

So,

$$\frac{\partial f(x,y,z)}{\partial x} = 8xy^3z + 6y^2z^2$$

$$\frac{\partial f(x,y,z)}{\partial y} = 12x^2y^2z + 12xyz^2$$

$$\frac{\partial f(x,y,z)}{\partial z} = 4x^2y^3 + 12xy^2z$$

3.2. Multiple integrals

Example 01

$$\int_0^2 \int_1^3 xy^2 dydx = \int_0^2 \left[\int_1^3 xy^2 dy\right] dx$$

$$= \int_0^2 \left[\frac{xy^3}{3}\right]_{y=1}^{y=3} dx = \int_0^2 x\left(\frac{27}{3} - \frac{1}{3}\right) dx = \frac{26}{3}\int_0^2 x\, dx$$

$$= \frac{26}{3}\left[\frac{x^2}{2}\right]_{x=0}^{x=2} = \frac{26}{3}\left(\frac{4}{2} - 0\right) = \frac{52}{2}$$

Example 02

$$\int_0^2 \int_0^1 (2y - 3x^2y^2)\, dxdy = \int_0^2 \left[\int_0^1 (2y - 3x^2y^2) dx\right] dy$$

$$= \int_0^2 [2xy - x^3y^2]_{x=0}^{x=1}\, dy = \int_0^2 (2y - y^2)\, dy$$

$$= \left[y^2 - \frac{y^3}{3}\right]_{y=0}^{y=2} = \left(4 - \frac{8}{3}\right) = \frac{4}{3}$$

Example 03

$$\int_0^2 \int_0^{2y} (4 + 2x - y^2)\, dxdy = \int_0^1 \left[\int_0^{2y} (4 + 2x - y^2) dx\right] dy$$

$$= \int_0^1 [4x + x^2 - xy^2]_{x=0}^{x=2y}\, dy = \int_0^1 (8y + 4y^2 - 2y^3)\, dy$$

$$= \left[4y^2 + \frac{4}{3}y^3 - \frac{2}{4}y^4\right]_{y=0}^{y=1} = \left(4 + \frac{4}{3} - \frac{1}{2}\right) = \frac{29}{6}$$

Chapter II: Electrostatics

Brief

The science of electricity and magnetism arose gradually, starting from simple observations of natural phenomena. Perhaps the compass is the first application, and the first observations date back to 600 years BC.

At the end of the eighteenth century, the French physicist Charles Coulomb discovered that the attraction between two charges is subject to a law such as Newton's law of general attraction. The difference between them is that the effect between two charges can be repulsion or attraction, depending on the sign of the charge.

Research into electricity and magnetism continued independently until 1820, when Ørsted noticed that passing an electric current through a conductive wire near a compass causes its needle to deflect. That is, the passage of an electric current generates a magnetic field around it, and this discovery is what allowed electricity and magnetism to be combined into one science, which is electromagnetism.

1. Electrostatic charges and fields

Many experiments have proven that when objects are rubbed, they acquire a new property called electrification. This property generates a reaction called the electrical effect.

To find out the properties of electric charge, we can carry out some basic experiments using plastic, glass and metal rods fitted with a wooden handle. We also need pieces of wool and silk as well as string.

Experiment 1: A plastic rod is brought close to another suspended plastic rod. The stems have not been rubbed with wool or silk. The stems remain immobile. There is therefore no force between the two rods.

Now, two plastic rods are rubbed with wool. When we approach the first stem of the hanging stem, there is a repulsion between the stems.

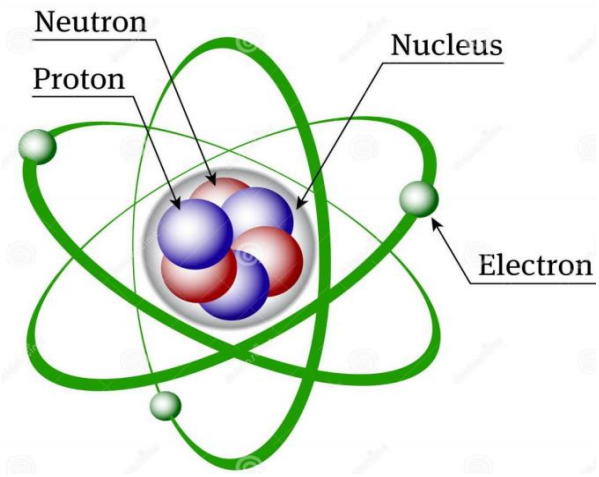
Experiment 2: Two glass rods are rubbed with silk. When the first stem is approached from the hanging stem, there is a repulsion between the stems.

We rub a glass rod with silk and a plastic rod with wool. We hang the plastic rod. When we approach the glass rod, there is an attraction between the rods.

It was the American physicist Benjamin Franklin (1706-1790) who stated the convention used today: the charge of the glass rod rubbed with silk is positive, and that of the plastic rod rubbed with wool is negative. When glass is rubbed with silk, the glass becomes positively charged and the silk becomes negatively charged. Similarly, if you rub plastic with wool, the plastic becomes negatively charged and the wool becomes positively charged.

Structure of matter

Matter is made up of atoms, the atom is composed of a massive and compact nucleus (of the order of $10^{-14}\ m$) surrounded by electrons (see figure). The nucleus is made up of protons and neutrons. The electrons do not have well-defined trajectories, but they form an electronic cloud whose diameter is of the order of $10^{-10}\ m$.



Chapter II: Electrostatics

A chemical element is represented in the periodic table by the symbol X_Z^A where A is the mass number (sum of protons and neutrons). And Z is the atomic number (Number of protons).

The total charge of the nucleus

$$\mathrm{Q} = +Ze$$

Charge of Electronic cloud

$$\mathrm{Q} = -Ze$$

The following table shows us the values of the mass charges of the particles that make up the nucleus in the international system

Particle	Charge	Mass
Electron	$\mathrm{q}_e = -e = -1.602 \times 10^{-19} C$	$\mathrm{m}_e = 9.109 \times 10^{-31} kg$
Proton	$\mathrm{q}_p = +e = +1.602 \times 10^{-19} C$	$\mathrm{m}_p = 1.672 \times 10^{-27} kg$
Neutron	$\mathrm{q}_n = 0$	$\mathrm{m}_n = 1.674 \times 10^{-27} kg$

Charge Is Quantized and Conserved

It was done after Millikan's experiments 1909 (The charge of any particle is equal to an integer multiple of the charge elementary).

$$\mathrm{Q} = \pm Ne$$

N is natural number

Electric charge cannot be created or destroyed. The net load of a closed system must remain constant.

$$e = 1.602 \times 10^{-19} C$$

Where

$$1uC = 10^{-6}C$$

$$1nC = 10^{-9}C$$

$$1pC = 10^{-12}C$$

Insulators and conductors

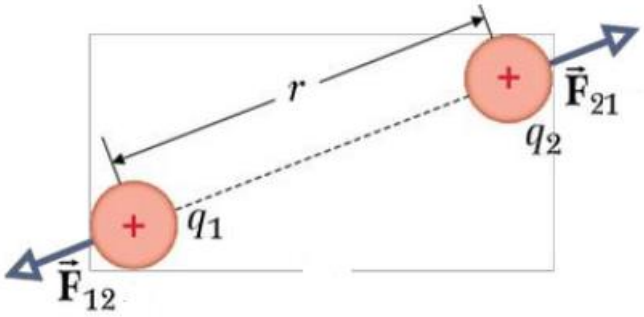
In many materials like copper and other metals, some electrons are free to move throughout the entire material. These materials are known as conductors. On the other hand, in materials like wood or glass, all electrons are tightly bound to nearby atoms and cannot move freely. These materials are referred to as insulators.

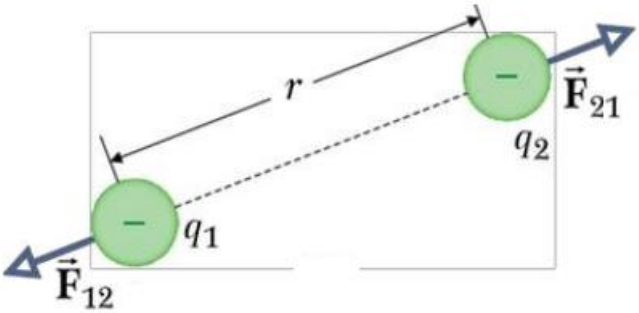
2. Electrostatic interaction force

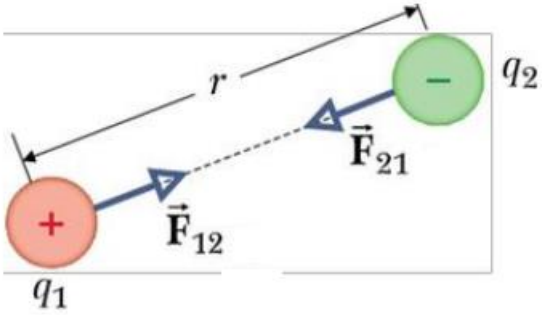
Previous rubbing experiments showed that charged objects exert an electric force on each other. This force is attractive when the charges are of opposite signs, and it is repulsive when the charges are of the same sign. In addition, the modulus of this force increases if we increase the value of the charges or if we decrease the distance between the objects.

Coulomb's law

The relationship between the quantity of charge, distance, and force was discovered by the French physicist and engineer Charles Coulomb (1736–1806) . Coulomb discovered that, for two immobile point charges, the modulus of the force is proportional to the product of the charges and inversely proportional to the square of the distance between the charges.







According to Coulomb's discovery, the electrostatic force is written as follows

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\vec{u}$$

or

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\frac{\vec{r}}{r}$$

The constant of proportionality k is called Coulomb's constant and its value is

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}$$

ε_0 is called the electric constant or permittivity of the vacuum.

$$\varepsilon_0 = \frac{1}{4\pi c^2} = 8{,}85 \times 10^{-12} C^2N^{-1}m^{-2}$$

$c = 3 \times 10^8 ms^{-1}$ is the velocity of light.

For calculation of the magnitude of the electrostatic force, we insert an absolute value, because the modulus of a force is always positive while charges can be positive or negative.

$$\left|\vec{F}_{12}\right| = F_{12} = F_{21} = k\frac{|q_1||q_2|}{r^2}$$

We note that Coulomb's law is similar to Newton's law of general gravitation between two bodies of different masses m_1 and m_2

$$\vec{F} = G\frac{m_1m_2}{r^2}\vec{u}$$

The constant of proportionality G is called Newton's constant and its value is

$$G = 6{,}67 \times 10^{-11} Nm^2kg^{-2}$$

Example

Comparison between electric force and gravitational force.

The electron and proton of a hydrogen atom are separated by a distance $5{,}3 \times 10^{-11}m$ Find the magnitude of the electric force and gravitational force between the electron and proton.

1- The electric force.

We use the Coulomb's law, we have

$$F_e = k\frac{|e||e|}{\mathrm{r}^2}$$

and

$$F_e = (9 \times 10^9 Nm^2C^{-2})\frac{(1.602 \times 10^{-19}C)^2}{(5{,}3 \times 10^{-11}m)^2}$$

$$F_e = 0{,}82 \times 10^{-7}N$$

2- The gravitational force.

From the Newton's law, we have

$$F_g = G\frac{m_1 m_2}{r^2}$$

And

$$F_g = (6{,}67 \times 10^{-11} Nm^2kg^{-2})\frac{(9.11 \times 10^{-31}kg)(1.67 \times 10^{-27}kg)}{(5{,}3 \times 10^{-11}m)^2}$$

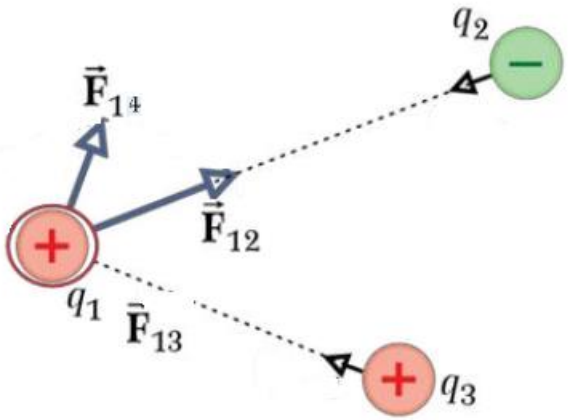
$$F_e = 3{,}62 \times 10^{-47}N$$

Note: We notice that the gravitational force is negligible compared to the electric force.

$$F_g \ll F_e$$

The principle of superposition

When several charges exert a force on a charge q_1, the resulting electric force is obtained using the principle of superposition, we calculate the vector sum of the forces exerted on q_1.



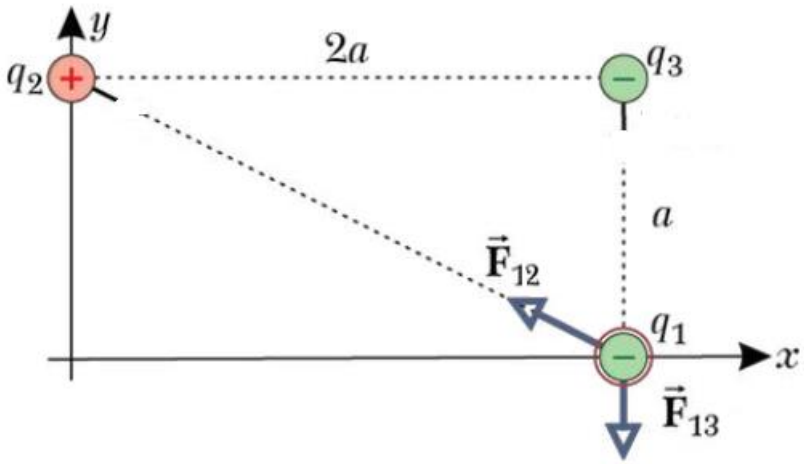
We write

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots$$

Example

Three charges are located in a plane, as shown in the figure opposite. The charges are $q_1 = -1.8uC$, $q_2 = 3.5uC$, and $q_3 = -1.4uC$. Furthermore, $a = 5.2cm$.

- Calculate the resulting electric force exerted on the load q_1.



Solution

Using the principle of superposition, we have

$$\vec{F}_{q_1} = \vec{F}_{12} + \vec{F}_{13}$$

Where

$$\vec{F}_{12} = -F_{12}cos\theta\vec{\imath} + F_{12}sin\theta\vec{\jmath}$$

and

$$\vec{F}_{13} = -F_{13}\vec{\jmath}$$

F_{12} and F_{13} are the magnitude of the vectors $\vec{F}_{12}$ and $\vec{F}_{13}$.

$$F_{12} = k\frac{|q_1||q_2|}{{r_{12}}^2}$$

$$F_{13} = k\frac{|q_1||q_3|}{{r_{13}}^2}$$

and

$$cos\theta = \frac{2a}{r_{12}}$$

$$sin\theta = \frac{a}{r_{12}}$$

Where

$$r_{12} = \sqrt{a^2 + (2a)^2} = \sqrt{5a^2}$$

$$r_{13} = a$$

So, we have

$$\vec{F}_{q_1} = -F_{12}cos\theta\vec{\imath} + F_{12}sin\theta\vec{\jmath} - F_{13}\vec{\jmath}$$

And

$$\vec{F}_{q_1} = -k\frac{|q_1||q_2|}{5a^2}\frac{2a}{\sqrt{5a^2}}\vec{\imath} + k\frac{|q_1||q_2|}{5a^2}\frac{a}{\sqrt{5a^2}}\vec{\jmath} - k\frac{|q_1||q_3|}{a^2}\vec{\jmath}$$

And

$$\vec{F}_{q_1} = -k\frac{|q_1||q_2|}{5a^2}\frac{2}{\sqrt{5}}\vec{\imath} + \left(k\frac{|q_1||q_2|}{5a^2}\frac{1}{\sqrt{5}} - k\frac{|q_1||q_3|}{a^2}\right)\vec{\jmath}$$

And

$$\vec{F}_{q_1} = -9\times10^9\frac{1.8\times10^{-6}\times3.5\times10^{-6}}{5(5.2\times10^{-2})^2}\frac{2}{\sqrt{5}}\vec{\imath}$$
$$+\frac{1.8\times10^{-6}\times9\times10^9}{(5.2\times10^{-2})^2}\left(\frac{3.5\times10^{-6}}{5\sqrt{5}} - 1.4\times10^{-6}\right)\vec{\jmath}$$

we have electric force

$$\vec{F}_{q_1} = (-3.75\vec{\imath} - 6.51\vec{\jmath})N$$

And the magnitude of the electric force vector, we have

$$F_{q_1} = \sqrt{(-3.75)^2 + (-6.51)^2}$$

and

$$F_{q_1} = 7.51N$$

3. The electric field $\vec{E}$

We have seen previously that electric charges exert an electric force on each other. This force is a force at a distance: the charged objects do not need to touch each other for the force to be exerted. We now approach the mediator of the electric force called the electric field. Each electric charge produces a field around it. Additionally, when a charge is in a field, it experiences an electric force.

The electric field $\vec{E}$ is the area of space surrounding the charge q in which this charge can affect another charge according to Coulomb's law. Where

$$\vec{F} = q\vec{E}$$

On the other hand we have

$$\vec{E} = \frac{\vec{F}}{q}$$

Finally, we write the expression for the electric field

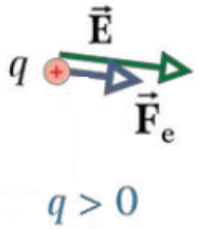
$$\vec{E} = k\frac{q'}{r^2}\vec{u}$$

The intensity of the electric field in the international system estimate NC^{-1} or Vm^{-1}.

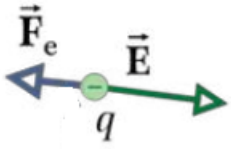
The orientation of the force depends on the orientation of the electric field and the sign of the charge q as

1- For a positive charge q, the electric force has the same direction as the electric field.

2- For a negative charge q, the electric force has a direction opposite to the electric field.

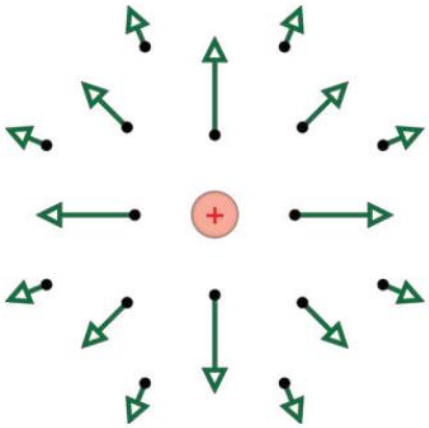


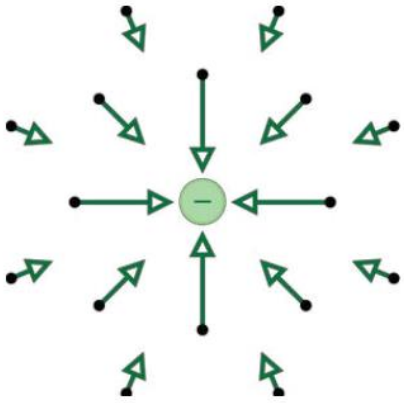
$q > 0$



$q < 0$

In the case of multiple charges, we draw the electric field as follows





To calculate the electric field at charge q resulting from the effect of a group of charges, we calculate the electric field resulting from each charge and then collect them as follows

$$\vec{E}_1 = \vec{E}_{12} + \vec{E}_{13} + \vec{E}_{14} + \cdots$$

Example 01

we have a negative charge $q = -10uC$. Find the electric field at the point P whose position is $\vec{r} = 4\vec{\imath} - 7\vec{\jmath}$.

Solution

The expression of the electric field is

$$\vec{E} = k\frac{q}{r^2}\vec{u}$$

Where $\vec{u}$ is the unit vector, we calculate it as the following

$$\vec{u} = \frac{\vec{r}}{r}$$

Where

$$r = \sqrt{(4)^2 + (-7)^2} = 8.06$$

So,

$$\vec{u} = \frac{4\vec{\imath} - 7\vec{\jmath}}{8.06}$$

And we have

$$\vec{E} = 9 \times 10^9 \frac{(-10 \times 10^{-6})}{(8.06)^2}\frac{(4\vec{\imath} - 7\vec{\jmath})}{8.06}$$

The electric field

$$\vec{E} = (-687.54\vec{\imath} + 1203.19\vec{\jmath})\frac{N}{C}$$

The magnitude of the electric field is

$$E = \sqrt{(-687.54)^2 + (1203.19)^2} = 1385.77\frac{N}{C}$$

Example 02

A point charge $q_1 = 2uC$ is placed at the coordinates $(2,-3,4)$ and another point charge $q_2 = -3uC$ is placed at the coordinates $(-1,3,1)$.

- Find the electric field and its magnitude at the point $(1,1,1)$.

Solution

We have

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Where

$$\vec{E}_1 = k\frac{q_1}{{r_1}^2}\vec{u}_1$$

$$\vec{E}_2 = k\frac{q_2}{{r_2}^2}\vec{u}_2$$

And

$$\vec{r}_1 = (1-2)\vec{\imath} + \big(1-(-3)\big)\vec{\jmath} + (1-4)\vec{k}$$

$$\vec{r}_2 = \big(1-(-1)\big)\vec{\imath} + (1-3)\vec{\jmath} + (1-1)\vec{k}$$

We have

$$\vec{r}_1 = -\vec{\imath} + 4\vec{\jmath} - 3\vec{k}$$

$$\vec{r}_2 = 2\vec{\imath} - 2\vec{\jmath}$$

So,

$$r_1 = \sqrt{(-1)^2 + (4)^2 + (-3)^2} = \sqrt{26}$$

$$r_2 = \sqrt{(2)^2 + (-2)^2} = \sqrt{4}$$

We have

$$\vec{E}_1 = 9\times 10^9 \frac{(2\times 10^{-6})}{\big(\sqrt{26}\big)^2}\frac{\big(-\vec{\imath} + 4\vec{\jmath} - 3\vec{k}\big)}{\sqrt{26}}$$

$$\vec{E}_2 = 9\times 10^9 \frac{(-3\times 10^{-6})}{\big(\sqrt{4}\big)^2}\frac{(2\vec{\imath} - 2\vec{\jmath})}{\sqrt{4}}$$

and

$$\vec{E}_1 = \left(-0.136\vec{\imath} + 0.544\vec{\jmath} - 0.408\vec{k}\right) \times 10^3 \frac{N}{C}$$

$$\vec{E}_2 = \left(-6.75\vec{\imath} + 6.75\vec{\jmath}\right) \times 10^3 \frac{N}{C}$$

For the electric field $\vec{E}$, we have

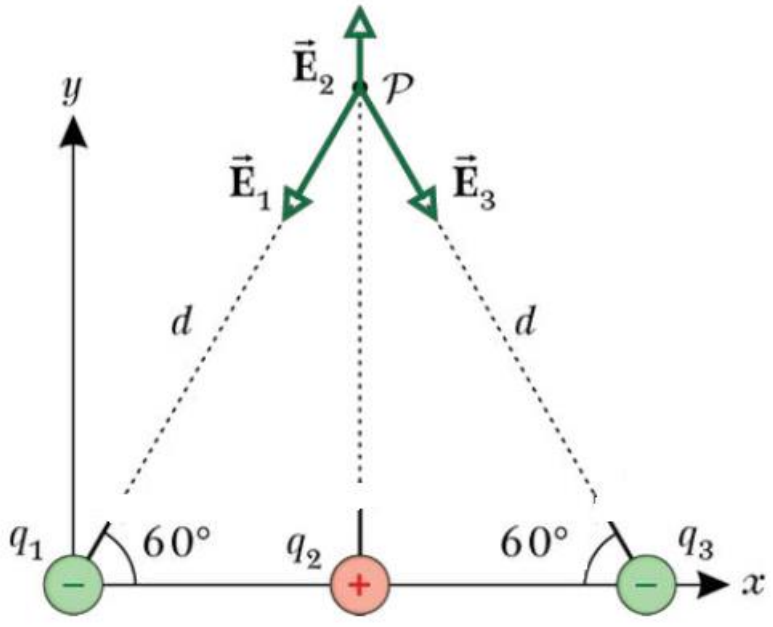
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

And

$$\vec{E} = \left(-6.886\vec{\imath} + 7.294\vec{\jmath} - 0.408\vec{k}\right) \times 10^3 \frac{N}{C}$$

Example 03

Three charges ($q_1 = q_3 = -2Q$ and $q_2 = Q$) are placed as shown in the figure opposite. Calculate the electric field at point P, such that the charges q_1, q_3 and point P are at the vertices of an equilateral triangle.



Solution

The electric field $\vec{E}$ is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Where

$$\vec{E}_1 = -E_1 cos(60^0)\vec{\imath} - E_1 sin(60^0)\vec{\jmath}$$

$$\vec{E}_2 = E_2\vec{\jmath}$$

$$\vec{E}_3 = E_3 cos(60^0)\vec{\imath} - E_3 sin(60^0)\vec{\jmath}$$

So,

$$\vec{E} = -(E_1 + E_3)cos(60^0)\vec{\imath} + \left(-E_1 sin(60^0) + E_2 + E_3 sin(60^0)\right)\vec{\jmath}$$

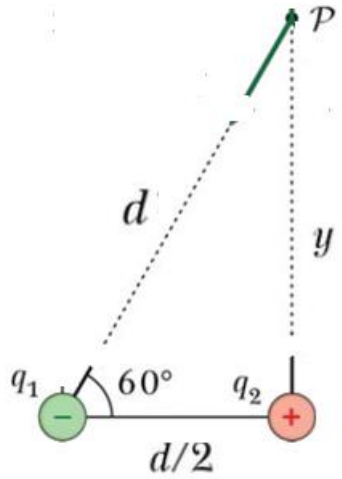
Where

$$E_1 = k\frac{|q_1|}{d^2} = k\frac{2Q}{d^2}$$

$$E_2 = k\frac{|q_2|}{y^2} = k\frac{Q}{y^2}$$

$$E_3 = k\frac{|q_3|}{d^2} = k\frac{2Q}{d^2}$$

Now we calculate y as the following.



From our figure, we have

$$y^2 + \left(\frac{d}{2}\right)^2 = d^2$$

And, we have

$$y^2 = d^2 - \left(\frac{d}{2}\right)^2 = \frac{3d^2}{4}$$

So,

$$\vec{E} = \frac{kQ}{d^2}\left(\frac{4}{3} - 2\sqrt{3}\right)\vec{j}$$

4. The electric field of a dipole

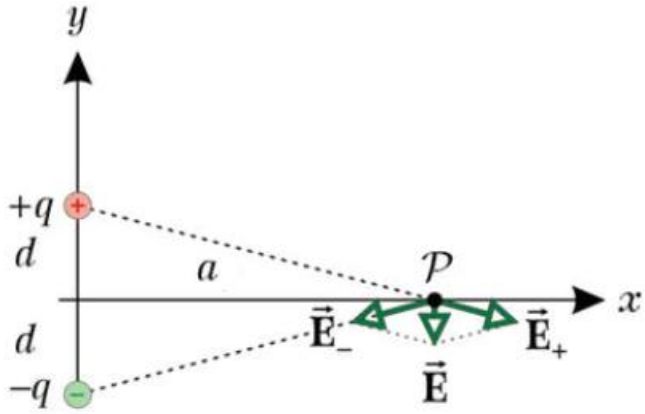
An electric dipole consists of two opposite charges $+q$ and $-q$ separated by a distance d. Several atoms become dipoles when placed in an electric field. Some molecules, for example HCl, CO, H_2O, CO_2, are intrinsically electric dipoles.

Example

We have a system of two point charges $+q$ and $-q$ placed in the y axis at a distance $+d$ and $-d$ so that they are an electric dipole as in the figure

1 - Determine the electric field produced at the position P by the other two charges.

2 - if we have a third point charge $q_3 = -q$ at the position P. What is the electric force exerted on the third point charge.



Solution

1- The electric field $\vec{E}$ is

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

Where

$$\vec{E}_+ = E_+ cos(\theta)\vec{\imath} - E_+ sin(\theta)\vec{\jmath}$$

$$\vec{E}_- = -E_- cos(\theta)\vec{\imath} - E_- sin(\theta)\vec{\jmath}$$

So,

$$\vec{E} = E_+ cos(\theta)\vec{\imath} - E_+ sin(\theta)\vec{\jmath} - E_- cos(\theta)\vec{\imath} - E_- sin(\theta)\vec{\jmath}$$

And

$$\vec{E} = (E_+ - E_-)cos(\theta)\vec{\imath} - (E_+ + E_-)sin(\theta)\vec{\jmath}$$

Where

$$E_+ = k\frac{|q|}{{r_+}^2} = k\frac{q}{{r_+}^2}$$

$$E_- = k\frac{|-q|}{{r_-}^2} = k\frac{q}{{r_-}^2}$$

And

$${r_+}^2 = a^2 + d^2$$

$${r_-}^2 = a^2 + d^2$$

And, we have the electric field of the electric dipole

$$\vec{E} = -\frac{2kq}{(a^2 + d^2)} sin(\theta)\vec{\jmath}$$

2- The electric force $\vec{F}$ is

$$\vec{F} = q_3\vec{E}$$

We have

$$\vec{F} = \frac{2kq^2}{(a^2 + d^2)} sin(\theta)\vec{\jmath}$$

5. Continuous charge distributions

There are different types of continuous charge distributions:

- Linear Charge Density (λ): This refers to charge Q distributed along a line l.

$$\lambda = \frac{Q}{l}$$

The linear charge density (λ) is measured in coulombs per meter (C/m).

$$dQ = \lambda dl$$

- Surface Charge Density (σ): This is the charge Q distributed over a surface A.

$$\sigma = \frac{Q}{A}$$

Surface charge density (σ) is expressed in coulombs per square meter (C/m^2).

$$dQ = Ad\sigma$$

- Volume Charge Density (ρ): In cases where charge Q is distributed within a volume V

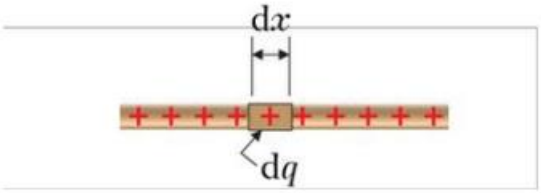
$$\rho = \frac{Q}{V}$$

volume charge density (ρ) is used. It is measured in coulombs per cubic meter (C/m^3).

$$dQ = \rho dV$$

The previous definitions are valid when charge densities are uniform, meaning they do not change with the position of the object. Otherwise, one must use differential calculus. For the rod illustrated in Figure, we consider an element Δx with charge Δq. Linear charge density is defined as the ratio of charge to length as the length approaches zero:

$$\lambda = \lim_{\Delta x \to 0} \frac{\Delta q}{\Delta x} = \frac{dq}{dx}$$



To calculate the total charge of the rod, one must sum the charges of all the elements of the rod. Each element has a charge $\Delta q = \lambda \Delta x$. Therefore, the total charge is obtained by taking the limit as $\Delta x \to 0$.

$$Q = \lim_{\Delta x \to 0} \sum \lambda \, \Delta x = \int \lambda dx$$

When the charge is distributed over a surface in a non-uniform manner, we consider an infinitesimal area element dA, on which there is a charge dq. Then,

$$Q = \int \sigma dA$$

If the charge is distributed within a volume in a non-uniform manner, we divide it into infinitesimal volume elements dV, each carrying a charge dq.

$$Q = \int \rho dV$$

6. The electric potential

To move a charge q from position a to position b in an electric field $\vec{E}$, work w_{ab} is done against the electric field force , and this work is given by the formula

$$w_{ab} = \int_a^b \vec{F}.d\vec{r} = -\int_a^b \vec{F}_{electric}.d\vec{r} = -q\int_a^b \vec{E}.d\vec{r}$$

The electric potential difference V_{ab} (also known as voltage) between two points a and b in an electric field $\vec{E}$ is a measure of the work w_{ab} done per unit charge in moving a positive test charge from one point to the other. Mathematically, it is defined as

$$V_{ab} = \frac{w_{ab}}{q} = -\int_a^b \vec{E}.d\vec{r}$$

Example

What is the work w_{ab} done by the electric field $\vec{E}$ created by an electric charge $q' = 20uC$. In order to move a static electric charge $q = 3uC$ from a distant place ($r_a = \infty$) to a position b at a distance $50cm$ from the source q'?

Solution

The work

$$w_{ab} = \int_a^b \vec{F}.d\vec{r}$$

We have

$$w_{ab} = kqq' \int_{\infty}^{r} \frac{\vec{r}}{r^3}.d\vec{r}$$

Where

$$\vec{r}.d\vec{r} = rdr$$

We have

$$w_{ab} = kqq' \int_{\infty}^{r} \frac{r}{r^3}.dr = kqq' \int_{\infty}^{r} \frac{dr}{r^2}$$

$$w_{ab} = kqq'(-\frac{1}{r} + \frac{1}{\infty})$$

And

$$w_{ab} = 9 \times 10^9(3 \times 10^{-6})(20 \times 10^{-6})\left(-\frac{1}{(50 \times 10^{-2})}\right) = -1.08 \text{ Joule}$$

If the potential difference between the two positions V_{ab} of the electric field is very small $d\emptyset$. Permission

$$d\emptyset = \frac{dw}{q} = -\vec{E}.d\vec{r}$$

Using the properties of scalar product, we can write

$$d\emptyset = -\vec{E}.d\vec{r} = -Edrcos(\theta)$$

And

$$d\emptyset = -Ecos(\theta)dr = -E_r dr$$

Where E_r is the projection of vector $\vec{E}$ on the support $d\vec{r}$

$$E_r = -\frac{d\emptyset}{dr}$$

In Cartesian coordinates we write

$$E_r = -\frac{\partial\emptyset}{\partial x} - \frac{\partial\emptyset}{\partial y} - \frac{\partial\emptyset}{\partial z}$$

And

$$\vec{E} = -\nabla\emptyset$$

To obtain the potential of the point charge q', we perform the addition process as follows

$$V_{ab} = \int_a^b d\emptyset$$

We have

$$\emptyset_b - \emptyset_a = -kq' \int_a^b \frac{dr}{r^2}$$

$$\emptyset_b - \emptyset_a = -kq'(-\frac{1}{r_b} + \frac{1}{r_a})$$

we write

$$\emptyset_b = kq'\left(\frac{1}{r_b} - \frac{1}{r_a}\right) + \emptyset_a$$

If we consider the potential at point a to be zero $\emptyset_a = 0$. And $r_b = r$ We find the expression for **Coulomb potential.**

$$\emptyset(r) = \frac{kq'}{r}$$

7. Electrical potential energy E_p.

The work w_{ab} done by the electric field $\vec{E}$ on the electric charge q is transformed into potential energy E_p.

$$w_{ab} = \Delta E_p = -q \int_a^b \vec{E}.d\vec{r}$$

Where ΔE_p is the change in electrical potential energy within the electric field. where

$$\Delta E_p = -\Delta E_c$$

So, we write

$$E_{p.b} - E_{p.a} = q\emptyset_b - q\emptyset_a$$

We have

$$E_p(r) = q\emptyset(r)$$

Returning to the expression of the electric field

$$\vec{E} = -\nabla\emptyset$$

Multiply both sides by q, we have

$$q\vec{E} = -\nabla(q\emptyset)$$

And

$$\vec{F} = -\nabla(E_p)$$

It is the expression that represents the relationship between the electrical force and the electrical energy potential.

Example 01

Derive an expression for the electric field beam from the following expression for the electric potential

$$V(x,y,z) = 3x^2y + z^2$$

Then calculate the intensity of the electric field at the point $A(1,2,-1)$.

Solution

We have

$$\vec{E} = -\nabla V$$

And

$$\vec{E} = -\frac{\partial V}{\partial x}\vec{\imath} - \frac{\partial V}{\partial y}\vec{\jmath} - \frac{\partial V}{\partial z}\vec{k}$$

Where

$$\frac{\partial V}{\partial x} = \frac{\partial(3x^2y + z^2)}{\partial x} = 6xy$$

$$\frac{\partial V}{\partial y} = \frac{\partial(3x^2y + z^2)}{\partial y} = 3x^2$$

$$\frac{\partial V}{\partial z} = \frac{\partial(3x^2y + z^2)}{\partial z} = 2z$$

So the electric field $\vec{E}$ vector expression becomes as follows

$$\vec{E} = -\left(6xy\vec{\imath} + 3x^2\vec{\jmath} + 2z\vec{k}\right)$$

At the point $A(1,2,-1)$

$$\vec{E} = -12\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$$

And

$$E = \sqrt{(-12)^2 + (-3)^2 + (2)^2} = \sqrt{157}\frac{N}{C}$$

Example 02

A ring with center O and radius R carries a uniformly distributed charge q with linear density λ.

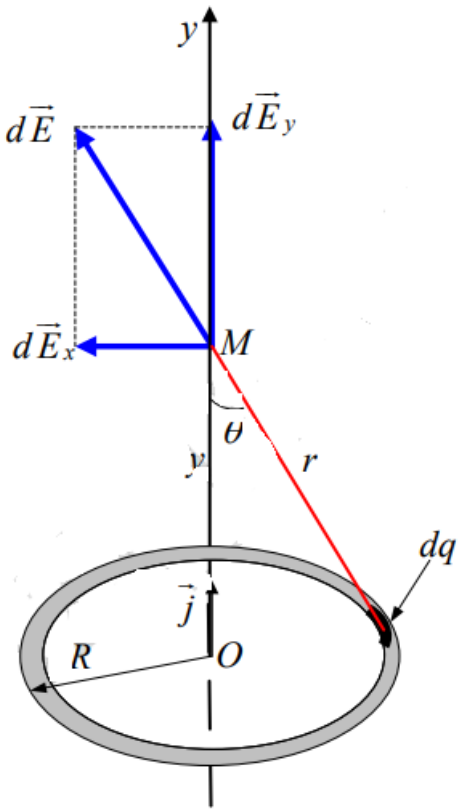
1- Calculate the electrical potential V generated at point M.

2- Deduce the electric field vector $\vec{E}$ at point M.

Solution

1- we have

$$dV = \frac{kdq}{r}$$



Where

$$r = \sqrt{y^2 + R^2}$$

And

$$\lambda = \frac{q}{l} = \frac{q}{2\pi R}$$

So,

$$q = 2\pi\lambda R$$

And the electrical potential V generated at point M is

$$V = k\frac{2\pi\lambda R}{\sqrt{y^2 + R^2}}$$

2- we have

$$E_x = -\frac{\partial V}{\partial x} = \frac{\partial(k\frac{2\pi\lambda R}{\sqrt{y^2+R^2}})}{\partial x} = 0$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{\partial(k\frac{2\pi\lambda R}{\sqrt{y^2+R^2}})}{\partial y} = k\frac{2\pi\lambda R y}{(y^2 + R^2)^{\frac{3}{2}}}$$

$$E_z = -\frac{\partial V}{\partial z} = \frac{\partial(k\frac{2\pi\lambda R}{\sqrt{y^2+R^2}})}{\partial z} = 0$$

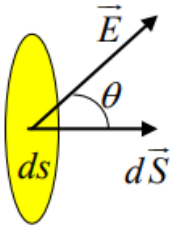
and

$$\vec{E} = k\frac{2\pi\lambda R y}{(y^2 + R^2)^{\frac{3}{2}}}\vec{j}$$

8. The electrical flow

We symbolize it as Φ, which is measured by the number of lines of force that penetrate a hypothetical surface. If M is a point on surface element $d\vec{S}$ and $\vec{E}$ is an electric field at point M, then we write the flow Φ of the electric field across this surface as follows

$$d\Phi = \vec{E}.d\vec{S} = EdScos(\theta)$$



Depending on the value of the angle θ, the flow of the electric field can be positive, negative, or null. So we know the total flow Φ of the electric field $\vec{E}$ across the surface S qualitatively according to the following relationship

$$\Phi = \oint_S \vec{E}.d\vec{S}$$

9. Gauss's theorem

Gauss's theorem expresses the relationship between the electric flow Φ across a closed surface S and the number of charges present within this surface.

We write Gauss's theorem as follows

$$\Phi = \oint_S \vec{E}.d\vec{S} = \frac{\sum Q_i}{\varepsilon_0}$$

Where

S is Gauss's surface. ε_0 is Permittivity in vacuum.

We also know the electric induction beam or flux density as follows

$$\vec{D}(\vec{r}) = \varepsilon_0 \vec{E}(\vec{r})$$

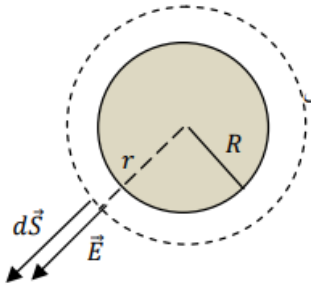
Example 01 Study of the electric field resulting from a spherical distribution of charges.

We have a sphere of radius R carrying a charge Q distributed with a uniform surface density σ. The electric field at each point in space is diagonal depending only on r which allows us to choose a Gauss's surface.

Solution

we have

$$\Phi = \oint_S \vec{E}.d\vec{S} = \frac{\sum Q_i}{\varepsilon_0}$$



Where

$$\vec{E}.d\vec{S} = E.dScos(0^0) = EdS$$

We find

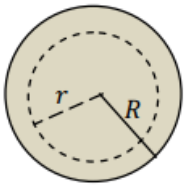
$$\Phi = \int EdS = E\int dS = E.S$$

$S = 4\pi r^2$ is Gauss's surface (Is a surface of a sphere with radius r). So,

$$\Phi = 4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

Case 01: $r < R$

The charge inside a Gaussian surface is zero $Q = 0$.



We have

$$4\pi r^2 E_1 = 0$$

So,

$$E_1 = 0$$

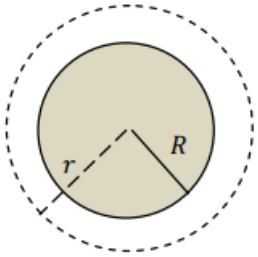
We calculate the electrical potential V_1

$$V_1 = C_1$$

Case 02: $r > R$

The charge distributed with a uniform surface density $\sigma = \frac{Q}{4\pi R^2}$.

$$Q = 4\pi\sigma R^2$$



We have

$$4\pi r^2 E_2 = \frac{Q}{\varepsilon_0} = \frac{4\pi\sigma R^2}{\varepsilon_0}$$

So,

$$E_2 = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r^2}$$

We calculate the electrical potential V_2

$$E_2 = -\frac{\partial V_2}{\partial r}$$

And

$$V_2 = -\int E_2 dr$$

We have

$$V_2 = -\int \frac{\sigma R^2}{\varepsilon_0}\frac{dr}{r^2}$$

$$V_2 = -\frac{\sigma R^2}{\varepsilon_0}\int \frac{dr}{r^2}$$

Where

$$\int \frac{dr}{r^2} = -\frac{1}{r}$$

We have

$$V_2(r) = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r} + C_2$$

To find the value of the constants C_1 and C_2, we use the boundary conditions

$$V_2(\infty) = 0$$

So,

$$C_2 = 0$$

According to the continuity of electrical potential. At $r = R$

$$V_1(R) = V_2(R)$$

We have

$$C_1 = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{R}$$

Finally, we write

$$E_1 = 0$$

$$V_1 = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{R}$$

And

$$E_2 = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r^2}$$

$$V_2(r) = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r}$$

Example 02

We have a sphere of radius R carrying a charge Q distributed with a uniform volume density ρ.

Solution

we have

$$\Phi = \oint_S \vec{E}.d\vec{S} = \frac{\sum Q_i}{\varepsilon_0}$$

And

$$\Phi = 4\pi r^2 E = \frac{\sum Q_i}{\varepsilon_0}$$

Case 01: $r < R$

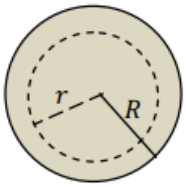
$$\rho = \frac{Q}{V}$$

Where

$$V = \frac{4}{3}\pi r^3$$

And

$$Q = \frac{4}{3}\pi\rho r^3$$



we have

$$4\pi r^2 E_1 = \frac{\frac{4}{3}\pi\rho r^3}{\varepsilon_0}$$

And

$$E_1 = \frac{\rho}{3\varepsilon_0} r$$

The electrical potential V_1

$$V_1 = -\int E_1 dr$$

And

$$V_1 = -\frac{\rho}{3\varepsilon_0}\int r dr$$

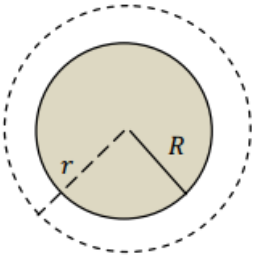
where

$$\int r dr = \frac{r^2}{2}$$

We find

$$V_1(r) = -\frac{\rho}{6\varepsilon_0} r^2 + C_1$$

Case 02: $r > R$



$$V = \frac{4}{3}\pi R^3$$

We have

$$4\pi r^2 E_2 = \frac{Q}{\varepsilon_0} = \frac{\frac{4}{3}\pi\rho R^3}{\varepsilon_0}$$

And

$$E_2 = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r^2}$$

The electrical potential V_2

$$V_2 = -\int E_2 dr$$

And

$$V_2 = -\frac{\rho R^3}{3\varepsilon_0}\int \frac{dr}{r^2}$$

$$V_2 = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r} + C_2$$

To find the value of the constants C_2, we use the boundary conditions

$$V_2(\infty) = 0$$

We find

$$C_2 = 0$$

According to the continuity of electrical potential. At $r = R$

$$V_1(R) = V_2(R)$$

We have

$$-\frac{\rho}{6\varepsilon_0}R^2 + C_1 = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{R}$$

And

$$C_1 = \frac{\rho}{3\varepsilon_0}R^2 + \frac{\rho}{6\varepsilon_0}R^2$$

So,

$$C_1 = \frac{\rho}{2\varepsilon_0}R^2$$

Finally, we write

$$E_1 = \frac{\rho}{3\varepsilon_0}r$$

$$V_1(r) = -\frac{\rho}{6\varepsilon_0}r^2 + \frac{\rho}{2\varepsilon_0}R^2$$

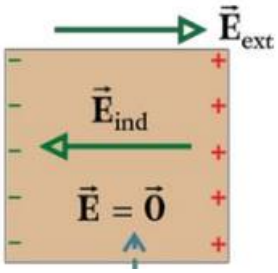
And

$$E_2 = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r^2}$$

$$V_2 = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r}$$

10. Conductors in electrostatic equilibrium

As we mentioned previously, a conducting body is a body in which charges (electrons) can move freely within the boundaries of this body. When there is no resultant movement of the charge within the conductor, this conductor is in a state of electrostatic equilibrium, and these charges are static.



Properties

1- The electric field inside the balanced conductor is zero $\vec{E} = \vec{0}$.

2- The electric charge (volume density of charge ρ) inside the balanced conductor is zero. $\sum Q_i = 0$. So,

$$\Phi = \oint_S \vec{E}.d\vec{S} = \frac{\sum Q_i}{\varepsilon_0} = 0$$

3- If the conductor is charged, the charge settles on its surface. The electric field on the surface of this conductor is perpendicular to the surface. Because if there is a tangential component, the charges will move on it.

4- The conductor forms a volume to equal the potential.

$$E = 0 \Rightarrow V = constant$$

The outer surface is the surface of equal potential and the field is perpendicular to this surface.

Let σ be the surface charge of the conductor at the point where the cylinder crosses the conductor. Then, according to Gauss' theorem,

$$\Phi = \oint_S \vec{E}.d\vec{S} = EA = \frac{Q}{\varepsilon_0}$$

Where

$$\sigma = \frac{Q}{A}$$

And

$$Q = \sigma A$$

We have

$$EA = \frac{\sigma A}{\varepsilon_0}$$

In outside the conductor the electric is

$$E = \frac{\sigma}{\varepsilon_0}$$

Or

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{n}$$

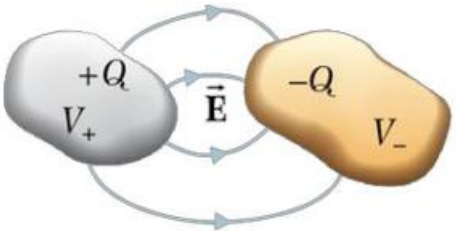
11. Electrostatic pressure

Since force per unit area is pressure, we can define the electrostatic pressure P that the charge exerts on the surface A of a conductor as the magnitude of the force per unit area

$$P = \frac{dF}{dS} = \frac{\sigma^2}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A^2}$$

The unit of measurement for pressure is the Pascal.

12. Capacity of a conductor and a capacitor



We define the capacitance C of the capacitor as follows

$$C = \frac{q}{\Delta V}$$

And

$$\Delta V = V_+ - V_-$$

In the international system, the unit of measurement for capacitance is the farad F.

where

$$1uF = 10^{-6}F$$

$$1nF = 10^{-9}F$$

$$1pF = 10^{-12}F$$

Example

We have a spherical conductor with radius R and a charge of Q.

So,

$$V = \frac{kQ}{R}$$

We have

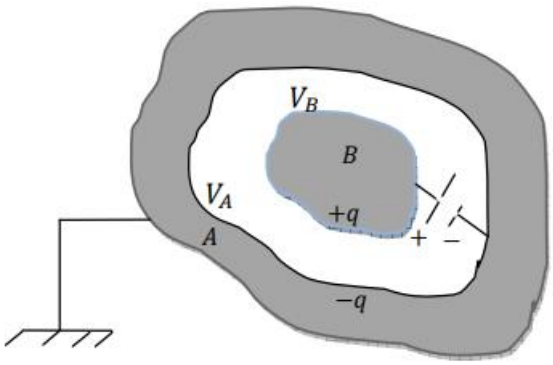
$$C = \frac{Q}{V} = \frac{QR}{kQ} = \frac{R}{k}$$

We note that the capacity of this conductor depends only on the radius of the spherical conductor, that is, on its geometric shape only.

The capacitor

A capacitor is a group of conductors A and B that are under electrostatic effect.

Such condensation can be achieved using two conductors A and B in the case of total mutual influence.



Where q_A and q_B are equal in value and different in sign. We call $q = |q_A| = |q_B|$ the condenser charge, if V potential difference between conductors $\Delta V = V_B - V_A$ We can prove

$$C = \frac{q}{\Delta V}$$

C is the capacitance of the capacitor, it's depends only on the shape of the conductors and the nature of the medium between them, and it increases as the two conductors get closer to each other.

How we calculate the capacity of a capacitor

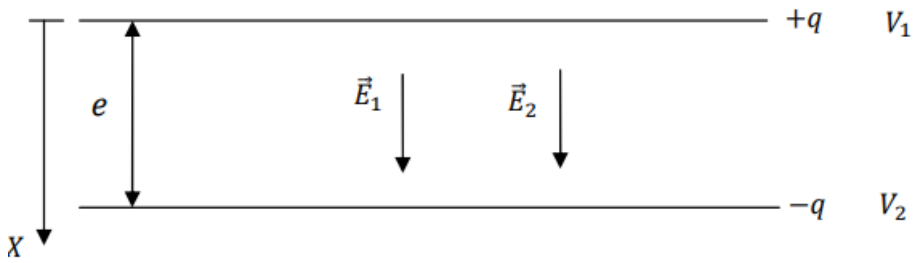
1- Calculate the electric field at each point inside the capacitor (we use Gauss theory for example).

2- Deducing the difference potential between the two conductors(we use $\vec{E} = -\nabla V$).

3- Finding the ratio $C = \frac{q}{\Delta V}$.

Example 01. Planar condenser

Calculate the capacity of a planar capacitor. The area of the two capacitors is S, and they are separated by a distance e .



Solution

For an infinite plane whose surface density σ what point is it equal to

$$E = \frac{\sigma}{2\varepsilon_0}$$

The total field between the two loops

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

And

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{\imath} + \frac{\sigma}{2\varepsilon_0}\vec{\imath} = \frac{\sigma}{\varepsilon_0}\vec{\imath}$$

Calculate the potential difference between the two ends of the capacitor

$$\vec{E} = -\nabla V \Rightarrow E = -\frac{dV}{dx}$$

We have

$$dV = -Edx$$

And

$$\int_{V_1}^{V_2} dV = -\int_{0}^{e} Edx$$

$$\int_{V_1}^{V_2} dV = -\int_{0}^{e} \frac{\sigma}{\varepsilon_0} dx$$

We have

$$V_2 - V_1 = -\frac{\sigma}{\varepsilon_0} e$$

And

$$V = V_1 - V_2 = \frac{\sigma}{\varepsilon_0} e$$

The capacitance V

$$C = \frac{q}{V} = \frac{\sigma S}{\frac{\sigma}{\varepsilon_0} e} = \frac{\varepsilon_0 S}{e}$$

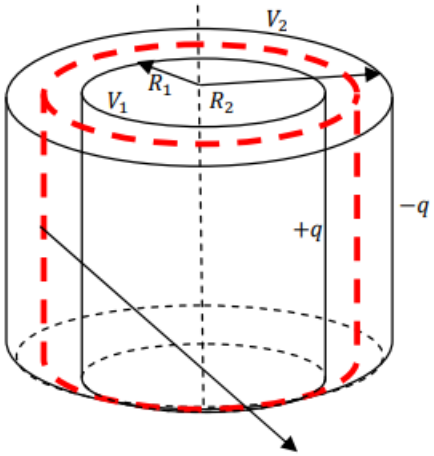
Note: As we saw earlier, the capacitance of a capacitor relates only to the geometric shape represented by S and e.

Example 02. Cylindrical condenser

Calculate the capacitance of a cylindrical condenser with radius respectively R_1 and R_2 and height h.

Solution

We calculate the electric field between the condensate by applying Gauss theory in the region where $R_1 < R_2$ and we choose a Gauss surface of a cylinder of radius r.



Gauss surface

So,

$$\oint_S \vec{E}.d\vec{S} = E(2\pi hr) = \frac{q}{\varepsilon_0}$$

We have the electric field

$$E = \frac{q}{2\pi h\varepsilon_0}\frac{1}{r}$$

Now, we calculate the potential difference between the two ends of the capacitor

$$E = -\frac{dV}{dr}$$

We have

$$dV = -Edr$$

And

$$\int_{V_1}^{V_2} dV = -\int_{R_1}^{R_2} Edr$$

$$\int_{V_1}^{V_2} dV = -\int_{R_1}^{R_2} \frac{q}{2\pi h\varepsilon_0}\frac{dr}{r}$$

We have

$$V_2 - V_1 = -\frac{q}{2\pi h\varepsilon_0}\int_{R_1}^{R_2} \frac{dr}{r}$$

Where

$$\int_{R_1}^{R_2} \frac{dr}{r} = lnR_2 - lnR_1 = ln\frac{R_2}{R_1}$$

And

$$V = V_1 - V_2 = \frac{q}{2\pi h\varepsilon_0} ln\frac{R_2}{R_1}$$

The capacitance V

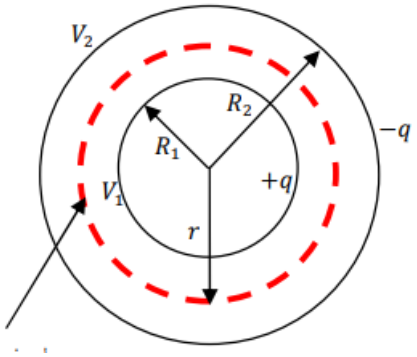
$$C = \frac{q}{V} = \frac{q}{\frac{q}{2\pi h\varepsilon_0} ln\frac{R_2}{R_1}} = \frac{2\pi h\varepsilon_0}{ln\frac{R_2}{R_1}}$$

Example 03. Spherical condenser

Calculate the capacitance of a Spherical condenser with radius respectively R_1 and R_2.

Solution

We calculate the electric field between the condensate by applying Gauss theory in the region where $R_1 < R_2$ and we choose a Gauss surface.



Gauss surface

So,

$$\oint_S \vec{E}.d\vec{S} = E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

We have the electric field

$$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

Now, we calculate the potential difference between the two ends of the capacitor

$$E = -\frac{dV}{dr}$$

We have

$$dV = -Edr$$

And

$$\int_{V_1}^{V_2} dV = -\int_{R_1}^{R_2} Edr$$

$$\int_{V_1}^{V_2} dV = -\int_{R_1}^{R_2} \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2}$$

We have

$$V_2 - V_1 = -\frac{q}{4\pi\varepsilon_0}\int_{R_1}^{R_2} \frac{dr}{r^2}$$

Where

$$\int_{R_1}^{R_2} \frac{dr}{r^2} = -\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

And

$$V = V_1 - V_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

The capacitance V

$$C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)} = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

13. Internal energy of a charged conductor

As we know that

$$E_p(r) = qV(r)$$

the variation in potential energy dE_p is calculated from the displaced charge and the potential difference. So,

$$dE_p(r) = dqV(r) = \frac{q}{C}dq$$

we obtain the potential energy of the fully charged capacitor by summing all the dE_p (therefore by integrating), when the charge on the capacitor varies from $q = 0$ to $q = Q$. We have

$$E_p = \int_0^Q \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}$$

E_p can take one of the following statements and it is always positive

$$E_p = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

14. The combination of capacitors

Electrical circuits can include several capacitors. To know the charge, the potential difference or the energy stored by a particular capacitor, the circuit must be simplified. In this section, we present a method that allows us to replace a circuit or part of a circuit with an equivalent circuit, simpler to analyze. To simplify the diagrams, standard symbols are used for the different circuit elements.

Capacitors are represented by the symbol showing two parallel plates connected to wires



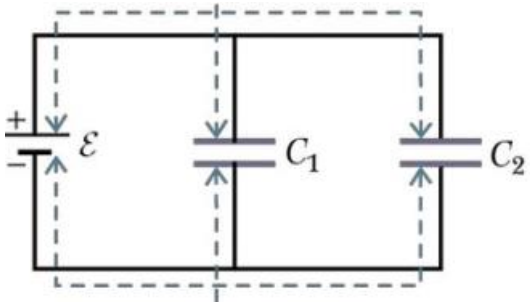
Capacitors in parallel

This implies that the potential difference between the terminals of the two capacitors is identical

$$\Delta V_1 = \Delta V_2 = \Delta V_C$$

The battery does work to charge the capacitors. At equilibrium, the potential difference of the capacitors is equal to that of the battery

$$\Delta V_C = \varepsilon$$



The capacitors C_1 and C_2 then has a charge

$$Q_1 = C_1 \Delta V_C$$

$$Q_2 = C_2 \Delta V_C$$

We find the equivalent capacity C_{eq}

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C} = C_1 + C_2$$

If we have several capacitors in parallel, we can replace them with an equivalent capacitor whose capacity is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

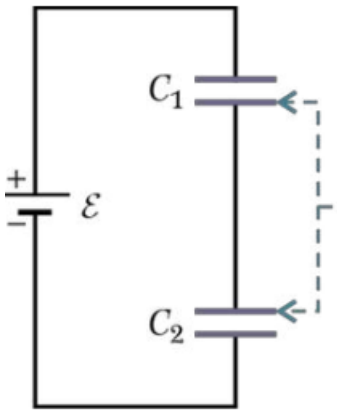
Series capacitors

The charge is the same on each capacitor connected in series

$$Q = Q_1 = Q_2$$

The potential difference

$$\Delta V_1 + \Delta V_2 = \Delta V_C$$



The potential difference across C_1 and C_2

$$\Delta V_1 = \frac{Q}{C_1}$$

$$\Delta V_2 = \frac{Q}{C_2}$$

we obtain

$$\frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q}$$

And

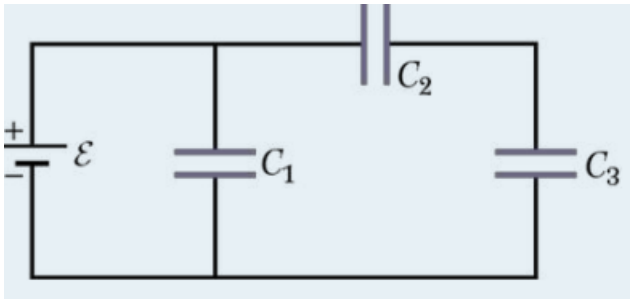
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

When several capacitors are connected in series, the equivalent capacitance is obtained by calculating the inverse of the addition of the reciprocals of the capacitances

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

Example

Three capacitors ($C_1 = 4nF, C_2 = 3nF$ and $C_3 = 6nF$) are connected in a circuit to a battery ($\varepsilon = 12\ V$), as shown the figure opposite.



1- How much energy is stored in the circuit?

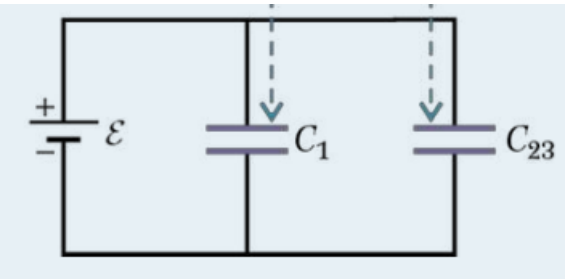
2- What is the charge and potential difference for each capacitor?

Solution

1- we have

$$E_p = \frac{1}{2} C_{eq} V^2$$

C_2 and C_3 are in series, so,



$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3} + \frac{1}{6}\right)^{-1} = 2\ nF$$

The equivalent capacity C_{eq} of the circuit is

$$C_{eq} = C_1 + C_{23}$$

$$C_{eq} = 4 + 2 = 6nF$$

The energy stored in the circuit is

$$E_p = \frac{1}{2}(6 \times 10^{-9})(12)^2 = 432\ nJ$$

2- we have

$$V_1 = 12\ V$$

$$V_2 + V_3 = V_{23} = 12\ V$$

And

$$Q = CV$$

we have

$$Q_1 = C_1 V_1 = 4 \times 12 = 48\ nC$$

$$Q_{23} = Q_2 = Q_3 = C_{23} V_{23} = 2 \times 12 = 24\ nC$$

Where

$$V_2 = \frac{Q_2}{C_2} = \frac{24}{3} = 8\ V$$

$$V_3 = \frac{Q_3}{C_3} = \frac{24}{6} = 4\ V$$

Chapter II: Electrostatics

Exercise 01: How much is the net charge?

1- 5×10^{14} Electron.

2- A mix of 4×10^{13} electron and 7×10^{13} proton.

Exercise 02: In a hydrogen atom, the distance between an electron and a proton $d = 5 \times 10^{14}$ where

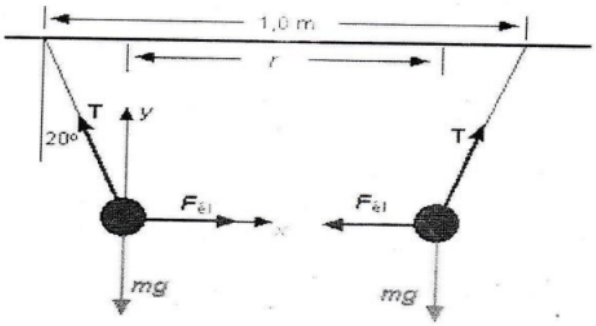
$$m_p = 1.67 \times 10^{-27} kg, \qquad m_e = 9.1 \times 10^{-31} kg$$

$$G = 6.67 \times 10^{-11} Nm/kg^2, \qquad K = 9 \times 10^9 Nm^2/C^2$$

1- Compare the strength of the electrostatic interaction F_e between the proton and the electron in the hydrogen atom with the strength of the general gravitational interaction F_g between them.

2- What is the distance between the proton and the electron for the strength of the electrostatic interaction F_e to be $1N$.

Exercise 03: The figure shows two balls suspended by two identical ropes, each ball 60 cm long. The mass of each ball is 1 g. After loading the two balls, the two ropes were separated from the plumb line at an angle of 20^0. If the total charge of the two balls reaches $0.5uC$. What is the charge of each of them?



Exercise 04: The electric charge $q_1 = 300\ uC$ is located at the position $M_1(3,-3,-2)$, and the electric charge q_2 is located at the position $M_2(1,-1,-3)$. The electric q_2 exerts a force $\vec{F}_{21}$ on electric charge q_1. Where $\vec{F}_{21} = 8\vec{\imath} - 8\vec{\jmath} + 4\vec{k}$.

1- Find the force exerted by the electric charge q_1 on the electric charge q_2.

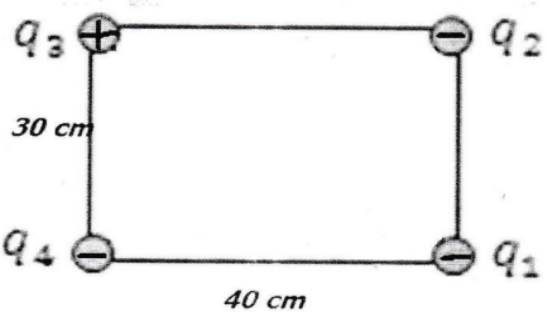
2- Find the value of the electric charge q_2.

3- We consider the system of the two charges as the previous positive points q_1 and q_2 and the distance between them d. The required is what is the possible to place a third electric charge q_3 such that the resultant of the electric forces acting on it by the two previous electric charges q_1 and q_1 equals zero. That is, the electric charge q_3 is in equilibrium.

Exercise 05: Based on the figure, calculate the electric force and it's magnitude that each of the charges q_2 , q_3 and q_4 produces on charge q_1.

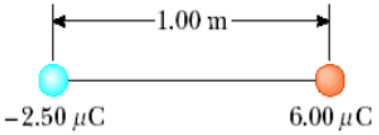
Where

$$q_1 = -5uC, \quad q_2 = -8uC$$
$$q_3 = 15uC, \quad q_4 = -16uC$$



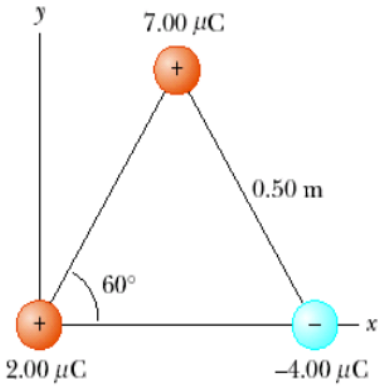
Chapter II: Electrostatics

Exercise 01: In the corresponding figure, find the point where the electric field is zero.



Exercise 02: We have two point charges placed on the x axis. The first is $+Q$ placed at a distance of $-a$ from the principle, while the second is an unknown charge placed at a distance of $+3a$ from the principle. In principle, these two charges generate an electric field with an intensity of $2\frac{kQ}{a^2}$. What then are the two possible values for the unknown charge?

Exercise 03: Three charges are distributed over an equilateral triangle, as shown in the figure below. Find the resultant electric field at the location of charge $2uC$.

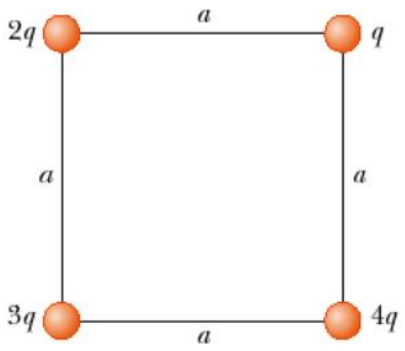


Exercise 04: An infinite number of charges, each of which has a value of q, are distributed along the positive axis and over the distances a, $2a$, $3a$,... in principle. What is the electric field in principle resulting from this distribution of charge? Use the relationship

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=\frac{\pi^2}{6}$$

Exercise 05: Four charges are distributed at the vertices of a square, as in the corresponding figure. Find the electric field (length and direction) at the location of the charge q.

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First year of science and technology Series 02 Physics 2



Exercise 06: A charge distributed with linear density over a ring of radius R.

1- At the point $M(x,0)$ Calculate the electric potential and the electric field.

2- Find the value of x when the electric field is maximum.

Exercise 07: We consider that there is a point charge $q = 6uC$ located in the axis of a disk with radius $R = 4cm$ at a distance $R = 3cm$ from the center of the disk. Calculate the flow of the electric field due to the point charge q across the surface of the disk.

Exercise 08: The point charge q is uniformly distributed over the volume of a sphere of radius R.

- Find the expression for the electric field in the regions $r \leq R$ and $r \geq R$.
- Find the expression for the electric potential in the regions $r \leq R$ and $r \geq R$.

Exercise 09: A cylinder of length h and radius R has a surface density σ.

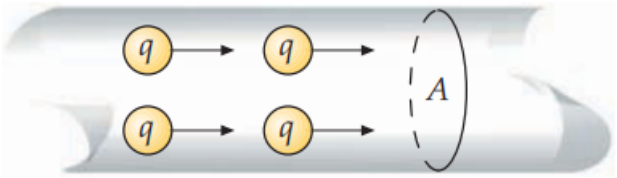
Find the expression for the electric field inside and outside the cylinder.

Chapter III :Electrokinetics

1. Current and the motion of charge

The electric current is quantified as the rate of charge flow across a surface, typically a cross-sectional area A of a conductive wire. The diagram depicts a segment of a wire with a current, illustrating the motion of charges. If " Q " denotes the amount of charge passing through area A within a specific period, then current is determined by this charge per unit time, expressed as

$$I = \frac{\Delta Q}{\Delta t}$$



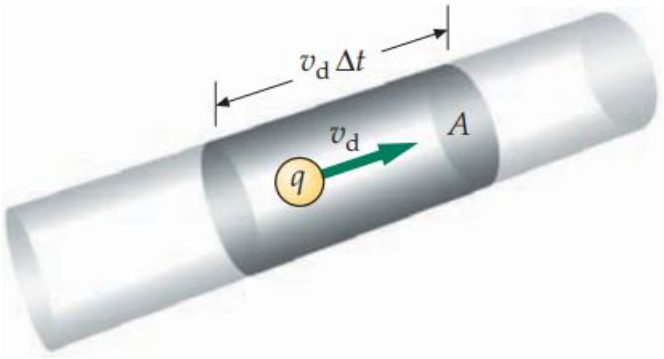
As the time interval Δt approaches zero, the SI unit of current is the ampere (A).

$$1A = 1\frac{C}{s}$$

Consider the number of particles mobile charge n, within a conducting wire of cross-sectional area A. This quantity is referred to as the number density of the charge carriers. Let's assume that each particle carries a charge and moves in the positive direction with a drift speed. Over a specific period, denoted as v_d, all particles within the volume depicted in Figure, highlighted as a shaded region, traverse the area element.

The total number of particles within this volume is, and as a result, the overall free charge within the volume is.

$$\Delta Q = qnAv_d\Delta t$$



The current

$$I = \frac{\Delta Q}{\Delta t} = qnAv_d$$

And the current per unit area, denoted by qnv_d , is obtained by dividing both sides of current by the area A . The **current density** vector, represented by $\vec{J}$, is defined as follows

$$\vec{J} = qn\vec{v}_d$$

The slow drift speed of mobile electrons in a wire, as observed, is due to the individual electrons moving at a relatively low speed, typically a few hundredths of a millimeter per second. However, the apparent contradiction with the instant illumination of an electric light when a switch is turned on can be explained by considering the analogy of water flow in a hose.

In the case of the water hose analogy, when you turn on a water faucet and the hose is already filled with water, the water emerges almost instantaneously from the nozzle. This rapid flow is attributed to the pressure wave that is transmitted through the water in the hose. The water pressure near the faucet pushes on the adjacent water segment, initiating a chain reaction where each segment successively pushes on the next, creating a pressure wave that travels down the hose. This wave moves at the speed of sound in water, allowing the water to quickly reach a steady flow rate.

Similarly, in an electric circuit, when the switch is turned on, an electric field is established instantly. This electric field exerts a force on the mobile electrons in the wire, causing them to move collectively and creating a flow of electric current. While individual electrons have low drift speeds, the electric field effect propagates rapidly through the conductor, leading to the nearly instantaneous illumination of the light bulb. This phenomenon is analogous to the pressure wave in the water hose, where the collective response of particles results in a quick and coordinated flow.

In contrast to a water hose, a metal wire is never truly empty. It consistently contains a vast number of conduction electrons distributed throughout its length. Consequently, when the light switch is turned on, electrons commence movement along the entire length of the wire, including the section within the light bulb, almost instantaneously.

The conveyance of a substantial quantity of electrons in a wire doesn't occur through the rapid movement of a few electrons down the wire. Instead, it is accomplished by the slow drifting of an immensely large number of electrons. As electrons move, surface charges are established on the wires, generating an electric field. This electric field, resulting from the surface charges, serves as the driving force propelling the conduction electrons through the wire, allowing for the rapid initiation of electric current and the prompt illumination of the light bulb.

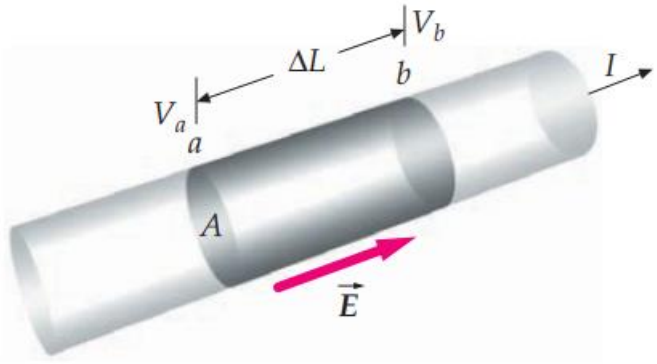
2. Resistance and Ohm's law

The flow of current in a conductor is propelled by an electric field that operates within the conductor, exerting force on the free charges present. It's important to note that in electrostatic equilibrium, the electric field inside a conductor must be zero. However, when a current flows through a conductor, the conductor is no longer in electrostatic equilibrium.

The free charges, typically negatively charged electrons in metals, undergo a drifting motion down the conductor, influenced by the forces imposed on them by the electric field. It's worth mentioning that these free charges move in a direction opposite to the electric field.

Chapter III :Electrokinetics

In the absence of other forces, the free charges would theoretically accelerate indefinitely under the influence of electric forces. However, this doesn't occur due to the interaction between free electrons and the lattice of ions constituting the metal. The interaction forces between free electrons and the lattice ions act to oppose the drifting motion of the electrons. This opposition prevents the electrons from gaining speed indefinitely and establishes a dynamic equilibrium in the conductor, allowing for a steady and controlled flow of current.



In the figure, a wire segment is illustrated with a length ΔL, cross-sectional area A, and a current I. Electric field $\vec{E}$ typically point in the direction of decreasing potential. In this is greater than the potential at point a scenario, the potential at point b.

If we model the current as the flow of positive charge carriers, these positive charge carriers will drift in the direction of decreasing potential. Assuming that the electric field is uniform throughout the segment, the **potential drop** V between points a and b can be expressed as

$$V = V_a - V_b = E\Delta L$$

The ratio of the potential drop V in the direction of the current to the current is defined as the resistance R of the segment. Mathematically, this relationship is expressed by Ohm's Law

$$R = \frac{V}{I}$$

In other words, resistance is a measure of how much the potential drops across a segment of a conductor per unit of current flowing through it. The unit of resistance is the ohm Ω. The higher the resistance, the more opposition the segment presents to the flow of electric current.

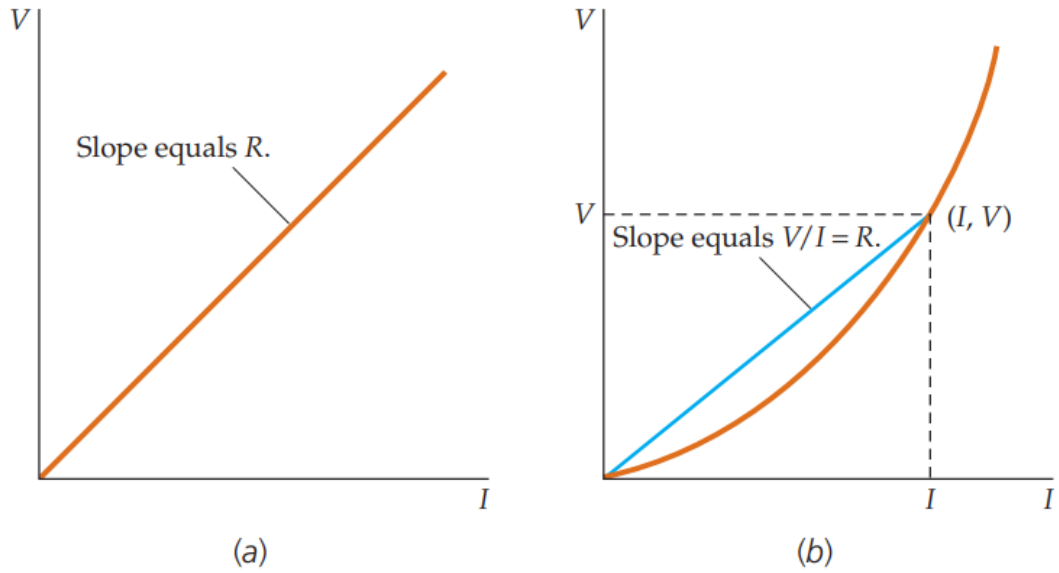
Indeed, the term "**ohmic materials**" refers to substances, with metals being a prominent example, whose resistance doesn't rely on the potential drop or the current. In the case of ohmic materials, resistance remains relatively constant over a broad range of conditions. This characteristic is often described as ohmic behavior.

For ohmic materials, the potential drop across a segment of the material is directly proportional to the current flowing through it. This proportionality is encapsulated by Ohm's Law $V = RI$. This linear relationship between potential drop and current is a defining feature of ohmic behavior and is a fundamental principle in understanding the electrical properties of many materials, particularly metals.

Figure illustrates the relationship between potential difference and current for two conductors. In the case of one conductor (Figure a), the relationship is linear, meaning the potential difference is directly proportional to the current. However, for the other conductor (Figure b), the relationship is not linear.

It's important to note that Ohm's Law is not considered a fundamental law of nature like Newton's laws or the laws of thermodynamics. Instead, Ohm's Law is an empirical description, meaning it is derived from observation and experimentation rather than being deduced from first principles. It describes a property shared by many materials under specific conditions.

The linear relationship depicted in Figure a is consistent with Ohm's Law (where the potential difference V is proportional to the current I with a constant of proportionality, the resistance R. However, the non-linear relationship in Figure b suggests that Ohm's Law does not hold for that particular conductor under the given conditions.



Moreover, it's noted that the resistance of a conductor can vary with temperature, indicating that the conditions, particularly the temperature, play a significant role in determining the electrical behavior of materials.

The resistance R of a conductive wire is determined to be directly proportional to its length L and inversely proportional to the cross-sectional area A of the wire.

$$R = \rho \frac{L}{A}$$

The resistance of a conductive wire is determined to be directly proportional to its length and inversely proportional to the cross-sectional area of the wire, with the proportionality constant referred to as the resistivity ρ of the conducting material.

The unit of resistivity ρ is the ohm-meter $\Omega.m$

For a segment of wire that has a length L a cross-sectional area A a current I and a resistance R the voltage drop V across the length of the segment is related to the current in the segment by

$$V = IR = I\rho \frac{L}{A}$$

Where $V = EL$, we have

$$EL = \rho \frac{I}{A} L$$

We have

$$EL = \rho JL$$

we obtain

$$\vec{E} = \rho \vec{J}$$

This equation presents an formulation of Ohm's law, asserting that the current density vector at a specific point within a current-carrying conductor is equivalent to the reciprocal of the resistivity, multiplied by the electric field vector at that identical point. Additionally, the equation incorporates the **temperature coefficient of resistivity** α, which is defined as the ratio of the fractional change in resistivity to the change in temperature.

$$\alpha = \frac{(\rho - \rho_0)/\rho_0}{(T - T_0)}$$

Where ρ_0 is the resistivity at temperature T_0 and is the resistivity ρ at temperature T.

3. Joule's effect

In the presence of an electric field within a conductor, free electrons undergo an increase in kinetic energy owing to the work exerted on them by the field. Nevertheless, a state of equilibrium is rapidly attained as the acquired kinetic energy is consistently transferred and dissipated into the thermal energy of the conductor. This dissipation occurs through interactions between the free electrons and the lattice ions of the conductor. The process responsible for elevating the thermal energy within a conductor is commonly referred to as **Joule heating**.

Certainly, the power delivered to a resistor can be expressed in various forms using Ohm's Law and the formulas for electrical power. Let's denote the following variables:

P: is the power delivered to the resistor (watts)

V: Voltage across the resistor (in volts)

I: Current flowing through the resistor (in amperes)

R: Resistance of the resistor (in ohms)

The power delivered to a resistor can be expressed using the formula

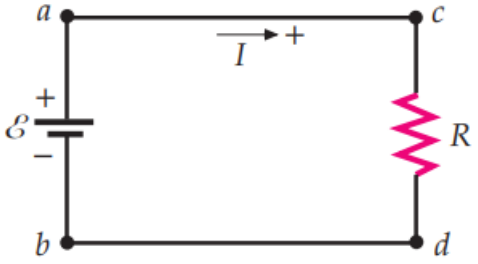
$$P = IV = I^2R = \frac{V^2}{R}$$

Emf of the battery

To sustain a constant current flow in a conductor, a continual supply of electrical energy is required. The apparatus responsible for providing electrical energy to a circuit is referred to as a source of **electromotive force** (emf). Despite the term "electromotive force," it is essential to note that emf is not a force; rather, it represents the work done per unit charge. Alternatively, a source of emf is sometimes termed a "seat of emf".

Examples of emf sources include batteries, which convert chemical energy into electrical energy, and generators, which transform mechanical energy into electrical energy. A source of emf performs nonconservative work on the passing charge, altering the potential energy of the charge, much like lifting a weight increases gravitational potential energy. The work done per unit charge is designated as the emf ($\mathcal{E}$) of the source, measured in volts— the same unit as potential difference.

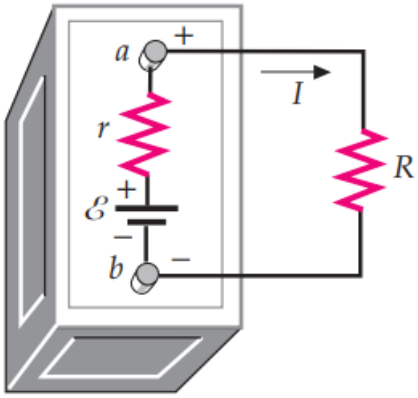
An ideal battery is an emf source that maintains a consistent potential difference between its terminals, irrespective of the current flowing through the battery. The potential difference across the terminals of an ideal battery equals the magnitude of the battery's emf.



In this circuit, the rate at which energy is supplied by the source of emf is the power output of the source

$$P = I\mathcal{E}$$

In the case of a real battery, the potential difference across its terminals, known as the terminal voltage, doesn't strictly equate to the electromotive force (emf) of the battery. When examining a circuit that includes a real battery and a variable resistor, and the current is adjusted by changing the resistance, it's observed that the terminal voltage decreases as the current increases (as depicted in the Figure). This phenomenon suggests the existence of an internal resistor within the battery.



Therefore, a practical model for a real battery involves considering it as comprising two components: an ideal source of emf ($\mathcal{E}$) and a resistor with a specific resistance termed the internal resistance of the battery. The internal resistance contributes to the decrease in terminal voltage with an increase in current. This internal resistance is a characteristic of the battery itself and is influenced by factors such as the battery's construction and materials.

In summary, the terminal voltage of a real battery is affected not only by its emf but also by the internal resistance, which leads to variations in voltage under different current conditions.

The potential at point a is related to the potential at point b by

$$V_a - V_b = \mathcal{E} - Ir = IR$$

Solving for the current I, we obtain

$$I = \frac{\mathcal{E}}{r + R}$$

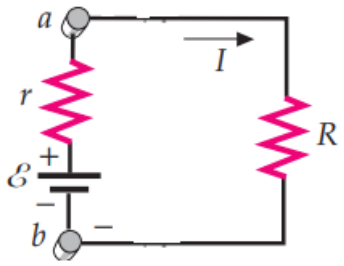
The energy stored in the battery is the product of the emf and the total charge it can deliver.

$$E_{stored} = Q\mathcal{E}$$

Example

An resistor $R = 11\Omega$ is connected across a battery $\mathcal{E} = 6V$ of emf and internal resistance $r = 1\Omega$. Find

The current, the terminal voltage of the battery, the power supplied by the chemical reactions within the battery, and the power delivered to the external resistor.



Solution

The current

$$I = \frac{\mathcal{E}}{r+R} = \frac{6}{1+11} = 0.5A$$

And

$$V_a - V_b = \mathcal{E} - Ir = 6 - 0.5 \times 1 = 5.5V$$

The power

$$P_{\mathcal{E}} = I\mathcal{E} = 0.5 \times 6 = 3\ W$$

$$P_R = I^2R = (0.5)^2 \times 11 = 2.75\ W$$

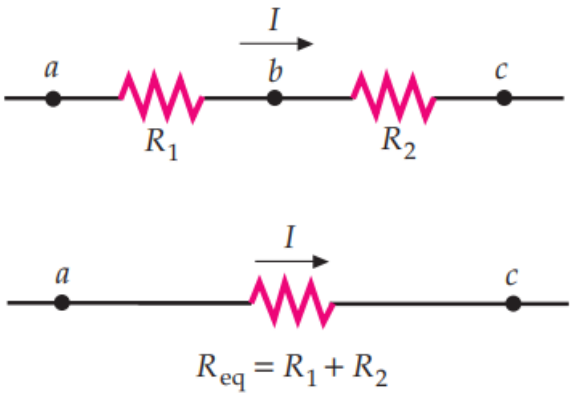
$$P_r = I^2r = (0.5)^2 \times 1 = 0.25\ W$$

4. The analysis of a circuit (Combinations of resistors**)**

Simplifying the analysis of a circuit is frequently achieved by substituting a combination of two or more resistors with a single equivalent resistor, possessing identical current flow and potential drop characteristics as the original resistor combination. This process of replacing resistor combinations with an equivalent resistor mirrors the analogous approach used in substituting combinations of capacitors with an equivalent capacitor.

Resistors in series

When two or more resistors are arranged in the configuration depicted in the given Figure, resulting in an identical current flowing through each resistor, this arrangement is referred to as a series connection. The potential drop across each resistor is denoted as V_1 and V_2, respectively, and the current passing through each resistor is represented by I. The total potential drop across the two resistors is the cumulative sum of the individual potential drops across each resistor.



So,

$$V = V_1 + V_2$$

Where

$$V = IR_{eq}$$

$$V_1 = IR_1$$

$$V_2 = IR_2$$

We have

$$IR_{eq} = IR_1 + IR_2$$

and

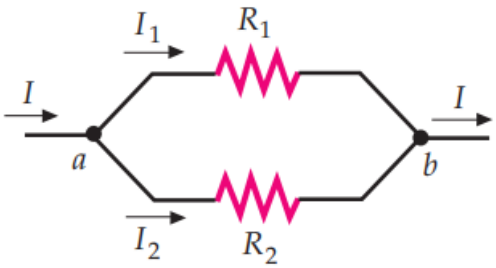
$$R_{eq} = R_1 + R_2$$

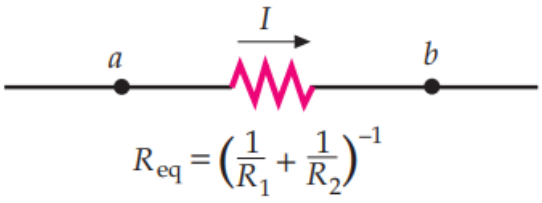
The equivalent resistance (R_{eq}) when there are more than two resistors connected in series, is determined by adding up the resistances of all the individual resistors. The formula for calculating the equivalent resistance in a series circuit with more than two resistors is

$$R_{eq} = R_1 + R_2 + R_3 + \ \ldots\ldots\ldots$$

Combinations in parallel

Two resistors are connected, as illustrated in the Figure, in a manner that results in an equal potential difference across them due to their wiring configuration are connected in parallel. It should be noted that, because of the circuit's arrangement, one terminal of each resistor aligns with the potential of a certain point, while the other terminal aligns with the potential of another point. Let I denote the current in the wire leading to point a. At point a, the circuit diverges into two branches, and the current splits into two components: the current flowing through the upper branch, containing resistor R_1, and the current flowing through the lower branch, containing resistor. The branch currents, denoted as I_1 and I_2, respectively, sum up to the total current in the wire leading into point a.





We write

$$I = I_1 + I_2$$

Where

$$V = IR_{eq} = I_1R_1 = I_2R_2$$

We have

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Dividing both sides by V gives

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

This principle can be extended to parallel combinations, as depicted in Figure, where we have more resistors are connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots$$

5. Application of Ohm's Law

Example 01

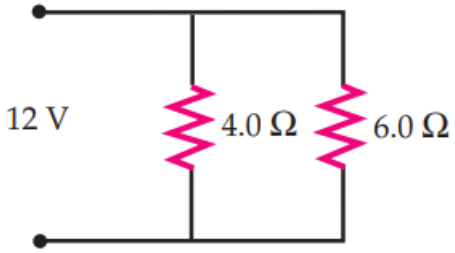
A battery has a potential difference V of across the parallel combination of R_1 and R_2 resistors shown in Figure. Find

1- R_{eq}.

2- The I current,

3- I_1 and I_2 of each resistor.

4- The power of each electric element.



Solution

1- Calculate the equivalent resistance

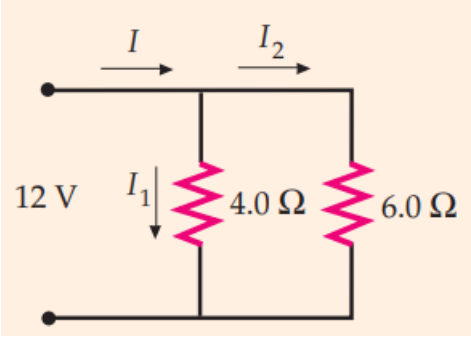
$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{6}$$

We have

$$R_{eq} = 2.4\ \Omega$$

2- The current

$$I = \frac{V}{R_{eq}} = \frac{12}{2.4} = 5\ A$$



3- The current I_1 and I_2

$$I_1 = \frac{V}{R_1} = \frac{12}{4} = 3\ A$$

$$I_2 = \frac{V}{R_2} = \frac{12}{6} = 2\ A$$

4- The power

$$P_{\mathcal{E}} = I\mathcal{E} = 5 \times 12 = 60\ W$$

$$P_{R_1} = {I_1}^2 R_1 = (3)^2 \times 6 = 36\ W$$

$$P_{R_2} = {I_1}^2 R_2 = (2)^2 \times 6 = 24\ W$$

Example 02

A resistor $R_1 = 4\ \Omega$ and a resistor $R_2 = 6\ \Omega$ are connected in series to a battery that has an emf equal $\mathcal{E} = 12\ V$ to and has a negligible internal resistance. Find

1- The equivalent resistance of the two resistors and the current in the circuit.
2- The potential drop across each resistor.
3- The power delivered to each resistor, and the total power delivered to the resistors.

Solution

1-

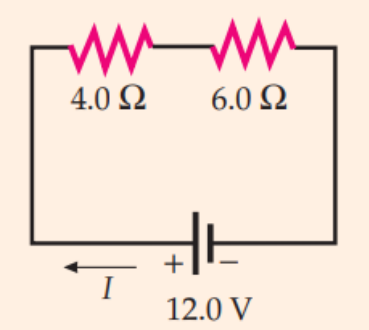
$$R_{eq} = R_1 + R_2 = 4 + 6 = 10\ \Omega$$

$$I = \frac{V}{R} = \frac{12}{10} = 1.2\ A$$

2-

$$V_1 = IR_1 = 1.2 \times 4 = 4.8\ V$$

$$V_2 = IR_2 = 1.2 \times 6 = 7.2\ V$$



3- The power

$$P_{\mathcal{E}} = I\mathcal{E} = 1.2 \times 12 = 14.4\ W$$

$$P_{R_1} = I^2 R_1 = (1.2)^2 \times 4 = 5.8\ W$$

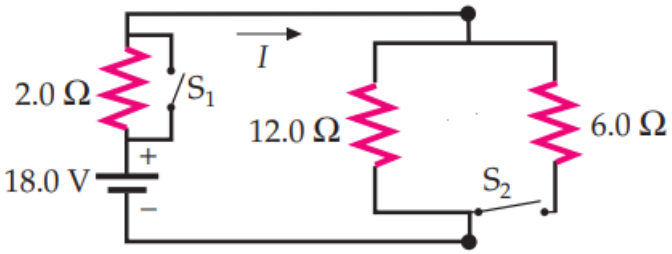
$$P_{R_2} = I^2 R_2 = (1.2)^2 \times 6 = 8.6\ W$$

Example 03

In the circuit, the switch S_1 is open and the switch S_2 is closed, find,

1- The equivalent resistance and the current in the source of emf.

2- The potential drop across each resistor and the current in each resistor.



Solution

1-

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6}$$

We have

$$R = 4\ \Omega$$

And

$$R_{eq} = 2 + 4 = 6\ \Omega$$

And

$$I = \frac{V}{R} = \frac{18}{6} = 3\ A$$

2-

$$V_{2\Omega} = R_{2\Omega} I = 2 \times 3 = 6\ V$$

$$V_{6\Omega} = V_{12\Omega} = RI = 4 \times 3 = 12\ V$$

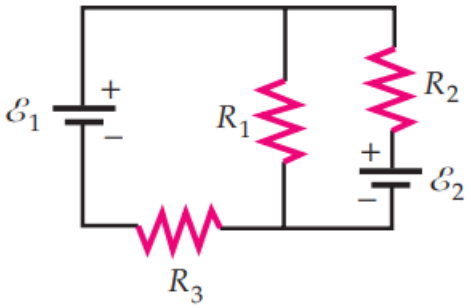
And

$$I_{6\Omega} = \frac{V_{6\Omega}}{6} = \frac{12}{6} = 2\ A$$

$$I_{12\Omega} = \frac{V_{12\Omega}}{12} = \frac{12}{12} = 1\ A$$

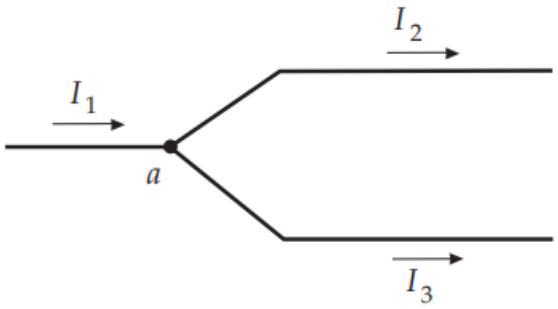
6. Kirchhoff's rules

There are many circuits, that cannot be analyzed by merely replacing combinations of resistors by an equivalent resistance.



An example of a circuit that cannot be analyzed by replacing combinations of resistors in series or parallel with their equivalent resistances. The potential drops across R_1 and R_2 are not equal because of the emf source $\mathcal{E}_2$, so these resistors are not in parallel. (Parallel resistors would be connected together at both ends.) The resistors do not have the same current, so they are not in series. Two rules, called **Kirchhoff's rules**, apply to this circuit and to any other circuit.

1- At any junction (branch point) in a circuit where the current can divide the sum of the currents into the junction must equal the sum of the currents out of the junction.

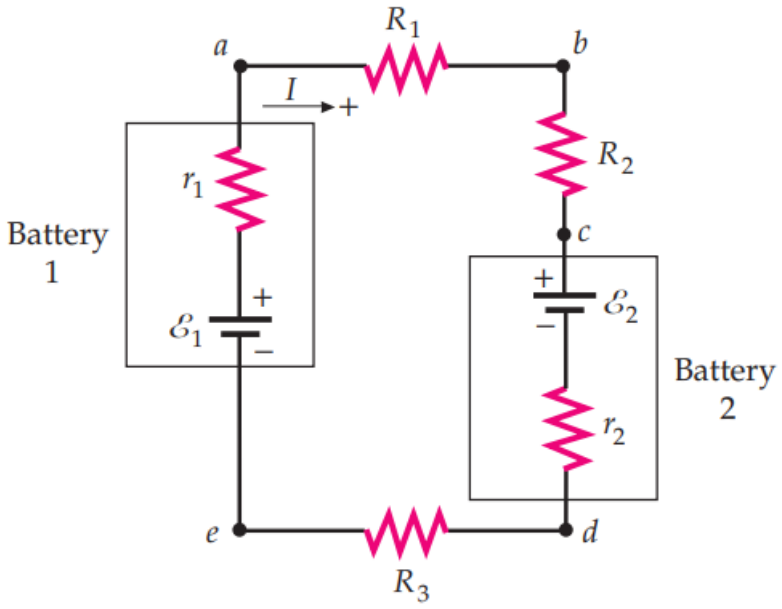


We write

$$I = I_1 + I_2$$

2- When any closed loop is traversed, the algebraic sum of the changes in potential around the loop must equal zero.

For example the changes in potential in this circuit



$$a \to b \quad -IR_1$$
$$b \to c \quad -IR_2$$
$$c \to d \quad -\mathcal{E}_2 - Ir_2$$
$$d \to e \quad -IR_3$$
$$e \to a \quad \mathcal{E}_1 - Ir_1$$

At point a we obtain

$$(V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_e - V_d) + (V_a - V_e) = 0$$

So,

$$(-IR_1) + (-IR_2) + (-\mathcal{E}_2 - Ir_2) + (-IR_3) + (\mathcal{E}_1 - Ir_1) = 0$$

we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + R_3 + r_1 + r_2}$$

Example 01

In the circuit have the values $\mathcal{E}_1 = 12\ V$ and $\mathcal{E}_2 = 4\ V$ a resistors $R_1 = R_2 = 5\ \Omega$ and $R_3 = 4\ \Omega, r_1 = r_2 = 1\ \Omega$. Find

1- The current in this circuit.

2- The power of each resistor, and each emf source.

Solution

1- The current

$$I = \frac{12 - 4}{1 + 5 + 5 + 1 + 4} = 0.5\ A$$

2- The power

$$P_{\mathcal{E}_1} = I\mathcal{E}_1 = 0.5 \times 12 = 6\ W$$

$$P_{\mathcal{E}_2} = I\mathcal{E}_2 = 0.5 \times 4 = 2\ W$$

$$P_{R_1} = I^2 R_1 = (0.5)^2 \times 5 = 1.25\ W$$

$$P_{R_2} = I^2 R_2 = (0.5)^2 \times 5 = 1.25\ W$$

$$P_{R_3} = I^2 R_3 = (0.5)^2 \times 4 = 1\ W$$

$$P_{r_1} = I^2 r_1 = (0.5)^2 \times 1 = 0.25\ W$$

$$P_{r_2} = I^2 r_2 = (0.5)^2 \times 1 = 0.25\ W$$

Example 02

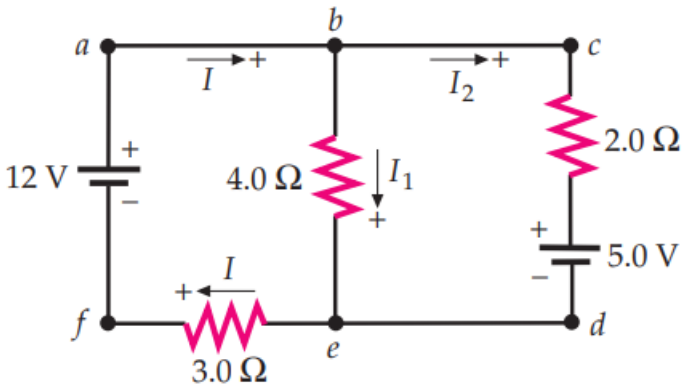
In this circuit, Find

1- The current in each branch of the circuit.

2- The energy dissipated in the resistor $4\ \Omega$ in $3s$.

Solution

1- The junction rule to point b, we have



$$I = I_1 + I_2$$

The loop rule to the outer loop $abcdefa$, we have

$$12 + (-2I_2) + (-5) + (-3)(I_1 + I_2) = 0$$

we obtain

$$7 - 3I_1 - 5I_2 = 0 \qquad (1)$$

The loop rule to the outer loop $bcdeb$, we have

$$(-2I_2) + (-5) + (4I_1) = 0$$

we obtain

$$-5 + 4I_1 - 2I_2 = 0 \qquad (2)$$

We use the equations (1) and (2), we have

$$I_1 = \frac{39}{26} = 1.5\ A$$

$$I_2 = \frac{2.5}{5} = 0.5\ A$$

$$I = 1.5 + 0.5 = 2\ A$$

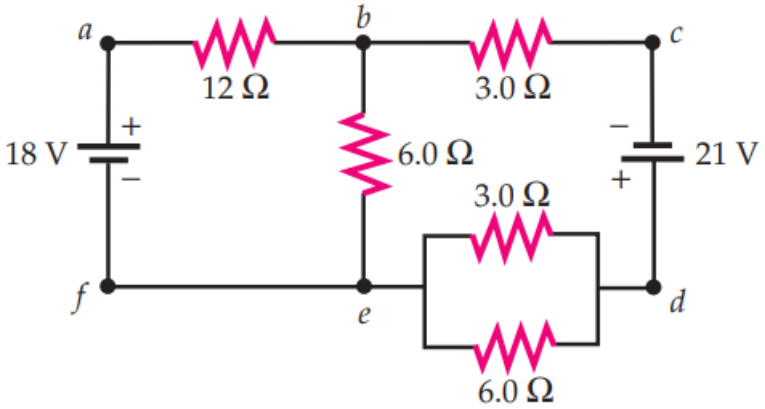
2-

$$P = 4 \times {I_1}^2 = 4 \times (1.5)^2 = 9\ W$$

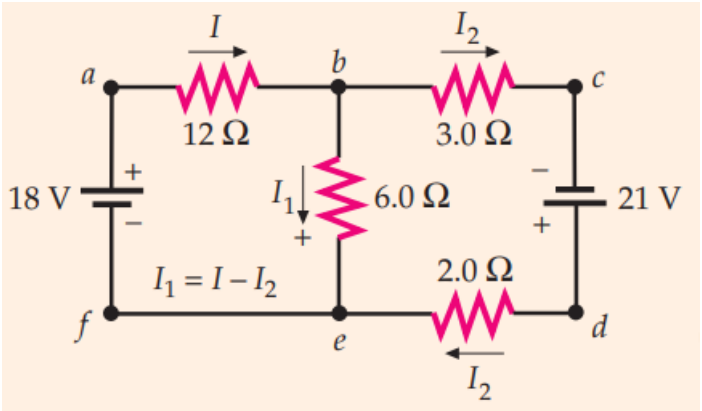
$$w = P(\Delta t) = 9 \times 3 = 27\ J$$

Example 03

In this circuit, Find the current in each branch of the circuit.



Solution



At point b, we have

$$I = I_1 + I_2$$

The loop rule to the outer loop $abcdefa$, we have

$$18 + (-12)(I_1 + I_2) + (-3I_2) + (21) + (-2)(I_2) = 0$$

we obtain

$$39 - 12I_1 - 17I_2 = 0 \qquad (1)$$

The loop rule to the outer loop $bcdeb$, we have

$$(-3I_2) + (21) + (-2)(I_2) + (6I_1) = 0$$

we obtain

$$21 + 6I_1 - 5I_2 = 0 \qquad (2)$$

We use the equations (1) and (2), we have

$$I_1 = \frac{39 - 51}{12} = -1\ A$$

$$I_2 = \frac{39 + 42}{27} = 3\ A$$

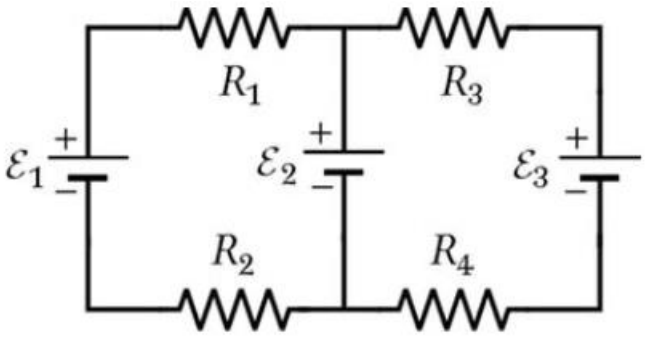
$$I = 3 - 1 = 2\ A$$

Example 04

In this circuit, Find the current in $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$. Where

$\mathcal{E}_1 = 3\ V, \mathcal{E}_2 = 8\ V$ and $\mathcal{E}_1 = 5\ V$.

$R_1 = 60\ \Omega, R_2 = 40\ \Omega, R_3 = 20\ \Omega$ and $R_4 = 80\ \Omega$.



Solution

In this circuit

I_1 Denote the current through $\mathcal{E}_1$ From top to bottom. Because $\mathcal{E}_2 > \mathcal{E}_1$.

I_2 Denote the current through $\mathcal{E}_2$ From bottom to top.

I_3 Denote the current through $\mathcal{E}_3$ From top to bottom. Because $\mathcal{E}_2 > \mathcal{E}_3$.

At point b, we have

$$I_2 = I_1 + I_3$$

The left loop, we have

$$\mathcal{E}_2 + (-R_1 I_1) - \mathcal{E}_1 + (-R_2 I_1) = 0$$

we obtain

$$I_1 = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_1 + R_2} = \frac{8 - 3}{100} = 0.05\ A$$

The right loop, we have

$$\mathcal{E}_2 + (-R_3 I_3) - \mathcal{E}_3 + (-R_4 I_3) = 0$$

we obtain

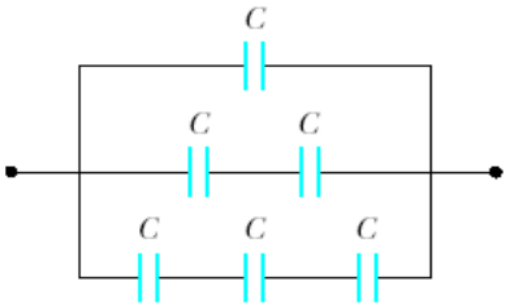
$$I_3 = \frac{\mathcal{E}_2 - \mathcal{E}_3}{R_3 + R_4} = \frac{8 - 5}{100} = 0.03\ A$$

So,

$$I_2 = 0.05 + 0.03 = 0.08\ A$$

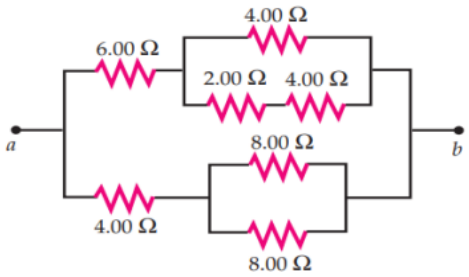
Chapter III :Electrokinetics

Exercise 01: Find the equivalent capacitance in the following figure



Exercise 02: 1- Find the equivalent resistance between point and point in the Figure .

2- If the potential drop between point and point is 12 V find the current in each resistor.

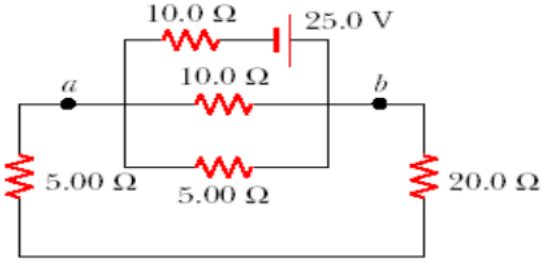


Exercise 03: When two capacitors are connected in parallel, their equivalent capacitance is $9\ pF$, and when they are connected in series, their equivalent capacitance is $2\ pF$. What is the capacity of each?

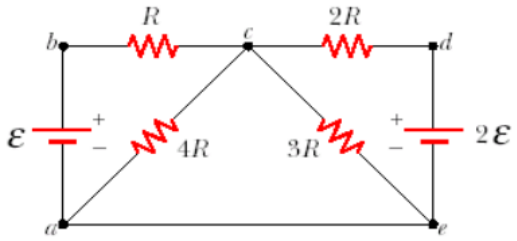
Exercise 04: In the circuit shown in the opposite figure, find:

1- The potential difference between points a and b.

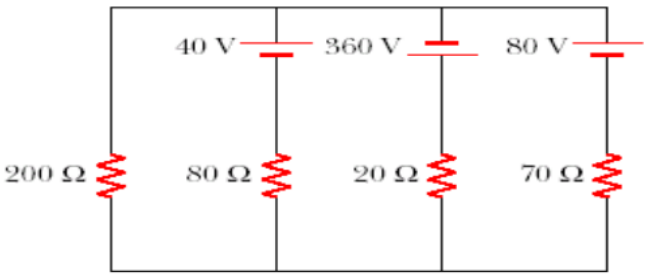
2- The current in the resistance whose value is $20\ \Omega$.



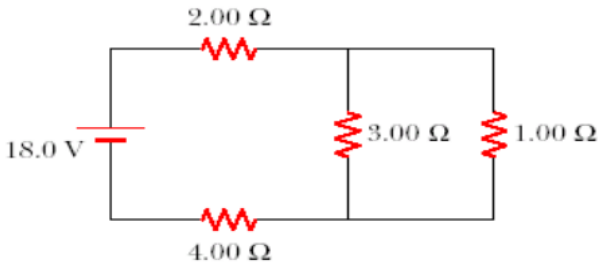
Exercise 05: Taking $R = 1\ k\Omega$ and $\varepsilon = 250$ V, find the direction and value of the current in the horizontal branch between points a and e.



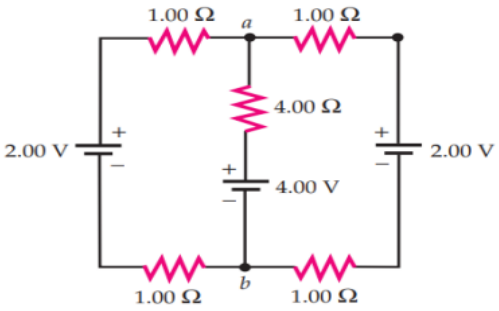
Exercise 06: In the corresponding circuit, find the electric current in all the resistors and the potential difference between the two ends of the resistor 200 Ω.



Exercise 07: In the corresponding circuit, calculate the power expended by each resistance.



Exercise 08: Find the potential difference between point a and point b.



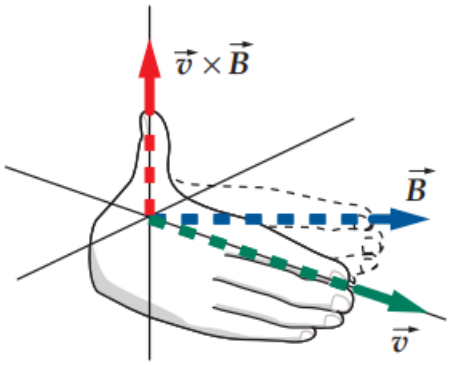
Chapter IV: Electromagnetism

1. The force of the magnetic field

In the presence of the magnetic field $\vec{B}$ in a specific area can be illustrated by observing the behavior of a compass needle, which will orient itself in alignment with the field's direction.

Experimental findings indicate that when a charged particle moves within a magnetic field, it experiences a force proportional to the magnitude of its charge q, velocity $\vec{v}$, and the sine of the angle between its velocity vector and the magnetic field vector. Interestingly, this force is perpendicular to both the particle's velocity and the magnetic field. To summarize these experimental observations: When a charged particle moves within a magnetic field, it experiences a magnetic force $\vec{F}$ that is perpendicular to both its velocity and the magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B}$$



The SI unit for magnetic field strength is the **tesla** (T). According to the relationship between magnetic field strength, charge, velocity, and force, a particle with a charge of one coulomb moving at a velocity of one meter per second perpendicular to a magnetic field of one tesla will experience a force of one newton.

$$1T = 1\frac{N}{C.m/s}$$

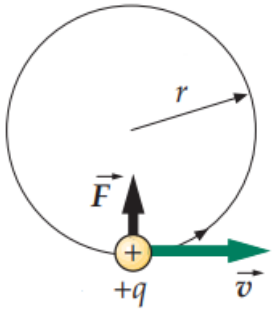
The magnetic fields are often given in **gauss**

$$1G = 10^{-4}T$$

Chapter IV: Electromagnetism

The motion of a point charge in the presence a magnetic field

The magnetic force exerted on a charged particle when it moves through a magnetic field always acts perpendicular to the particle's speed. As a result, this force changes the direction of the velocity without affecting its magnitude. Therefore, magnetic forces do not perform work on the particles nor do they change their kinetic energy.



In a particular scenario where the velocity of the particle is perpendicular to a magnetic field uniform, as depicted in the accompanying figure, the particle follows the circular path. The force acts as the centripetal force required for maintaining circular motion. we apply the Newton's second law, we can establish the relationship between the radius of the circle, the strength of the magnetic field, and the velocity of the particle. When the speed is perpendicular to the magnetic field, the force on a particle with charge q is expressed as a product of the particle's charge, speed, and the magnitude B of the magnetic field. the net force's magnitude equals the product of q, $\vec{v}$, and $\vec{B}$ due to their perpendicular orientation. We find

$$F = ma$$

And

$$qvB = m\frac{v^2}{r}$$

and

$$r = \frac{mv}{qB}$$

The period T is the amount of time that the object takes to complete one circular orbit of radius r.

The period is related to the speed by the following relationship

$$T = \frac{2\pi r}{v}$$

Or

$$T = \frac{2\pi(\frac{mv}{qB})}{v} = \frac{2\pi m}{qB}$$

The f frequency is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

And ω the angular frequency is

$$\omega = 2\pi f = \frac{qB}{m}$$

The K kinetic energy

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\frac{qB}{m}r)^2 = \frac{1}{2}\frac{q^2B^2}{m}r^2$$

When a charge is subject to an electric current and a magnetic field, the resultant is the **Lorentz** Force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

2. The magnetic field of a moving point charge

The magnetic field $\vec{B}$ at a point in space due to a moving point charge q with velocity $\vec{v}$ is given by

$$\vec{B} = \frac{u_0}{4\pi}\frac{q\vec{v} \times \vec{u}_r}{r^2}$$

Where $\vec{u}_r$ is a unit vector, $\vec{v}$ the speed, and u_0 is a constant called the magnetic constant or the **permeability of free space.**

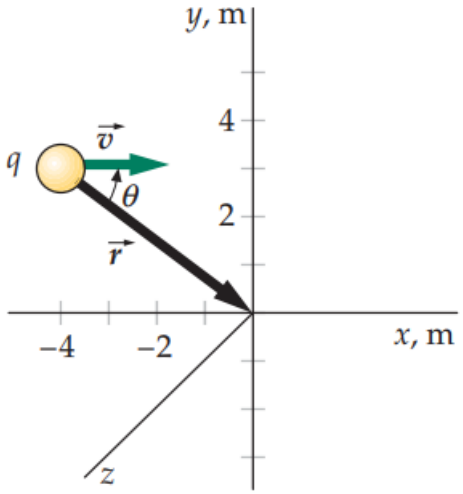
$$u_0 = 4\pi \times 10^{-7} Tm/A$$

Or

$$\vec{B} = \frac{u_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$$

Example

A point particle that has charge $q = 4.5uC$ is moving with velocity $\vec{v} = 3\vec{\imath}\, m/s$. Calculate the magnetic field $\vec{B}$ at the $(0,0)$ produced by the charge q when the charge is at the point $x = -4m, y = 3m$ with the mention of the figure.



Solution

The $\vec{B}$ magnetic field is given by

$$\vec{B} = \frac{u_0}{4\pi} \frac{q\vec{v} \times \vec{u}_r}{r^2}$$

Where

$$\vec{r} = (0+4)\vec{\imath} + (0-3)\vec{\jmath} = 4\vec{\imath} - 3\vec{\jmath}$$

$$r = \sqrt{(4)^2 + (-3)^2} = 5$$

and

$$\vec{u}_r = \frac{4\vec{\imath} - 3\vec{\jmath}}{5} = 0.8\vec{\imath} - 0.6\vec{\jmath}$$

$$\vec{B} = \frac{u_0}{4\pi}\frac{q(3\vec{\imath}) \times (0.8\vec{\imath} - 0.6\vec{\jmath})}{r^2} = \frac{u_0}{4\pi}\frac{q(-1.8\vec{k})}{r^2}$$

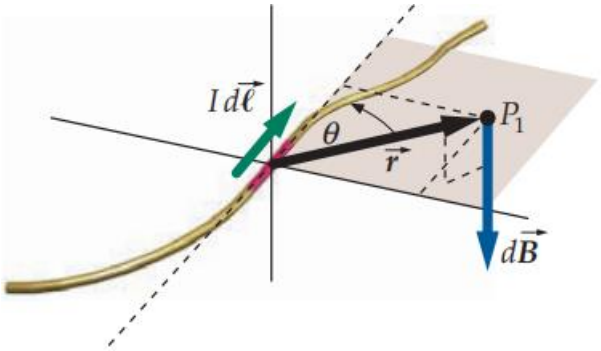
And

$$\vec{B} = -10^{-7}(4.5 \times 10^{-6})\frac{(1.8\vec{k})}{5^2} = (-0.324 \times 10^{-13}\vec{k}\,)T$$

3. The magnetic field of currents (The Biot –Savart law)

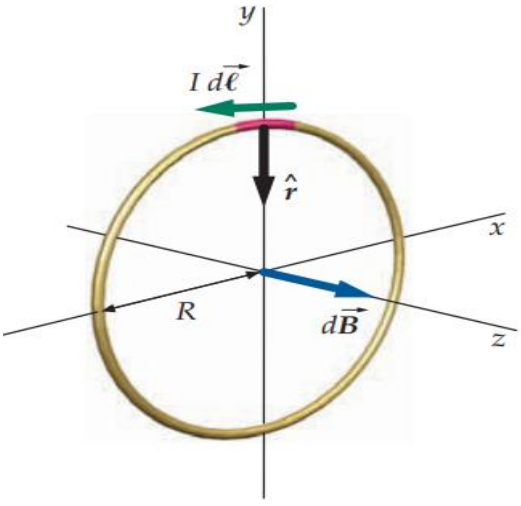
The magnetic field $d\vec{B}$ produced by a current element $Id\vec{l}$ is given by

$$d\vec{B} = \frac{u_0}{4\pi}\frac{Id\vec{l} \times \vec{u}_r}{r^2}$$



Example 01

a current loop of radius R



$$d\vec{B} = \frac{u_0}{4\pi}\frac{Id\vec{l} \times \vec{u}_r}{R^2}$$

Which $\theta = 90^0$ is for each current element, so The resultant magnetic field due to all the current elements in the loop is found by

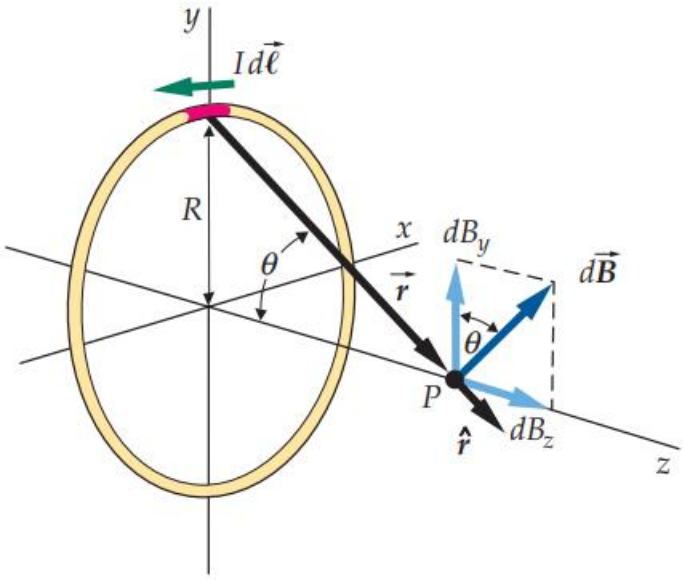
$$dB = \frac{u_0}{4\pi}\frac{Idl\,\sin(\theta)}{R^2} = \frac{u_0}{4\pi}\frac{Idl}{R^2}$$

Because I and R are the same for all elements, we obtain

$$B = \frac{u_0}{4\pi}\frac{I}{R^2}\oint dl = \frac{u_0}{4\pi}\frac{I(2\pi R)}{R^2} = \frac{u_0 I}{2R}$$

Example 02

For calculating the magnetic field $\vec{B}$ at a point on the axis of a circular current loop a distance from the circular loop's center. We use the geometry in the figure



$d\vec{B}$ is in the direction shown in the figure, perpendicular to $\vec{r}$ and to $Id\vec{l}$.

The magnitude of $d\vec{B}$ is

$$\left|d\vec{B}\right| = \frac{u_0}{4\pi}\frac{I\left|d\vec{l}\times\vec{u}_r\right|}{r^2} = \frac{u_0}{4\pi}\frac{Idl}{r^2}$$

and

$$r^2 = R^2 + z^2$$

We find

$$dB = \frac{u_0}{4\pi}\frac{Idl}{(R^2+z^2)}$$

And

$$dB_z = dB\sin(\theta)$$

$$dB_z = \frac{u_0}{4\pi}\frac{Idl}{(R^2+z^2)}\frac{R}{\sqrt{R^2+z^2}} = \frac{u_0}{4\pi}\frac{IRdl}{(R^2+z^2)^{\frac{3}{2}}}$$

To find B_z, we integrate dB_z around the circle

$$B_z = \oint dB_z = \oint \frac{u_0}{4\pi}\frac{IRdl}{(R^2+z^2)^{\frac{3}{2}}}$$

And

$$B_z = \frac{u_0}{4\pi}\frac{IR}{(R^2+z^2)^{\frac{3}{2}}}\oint dl$$

Where

$$\oint dl = 2\pi R$$

We have

$$B_z = \frac{u_0}{4\pi}\frac{2I\pi R^2}{(R^2+z^2)^{\frac{3}{2}}} = \frac{u_0}{2}\frac{IR^2}{(R^2+z^2)^{\frac{3}{2}}}$$

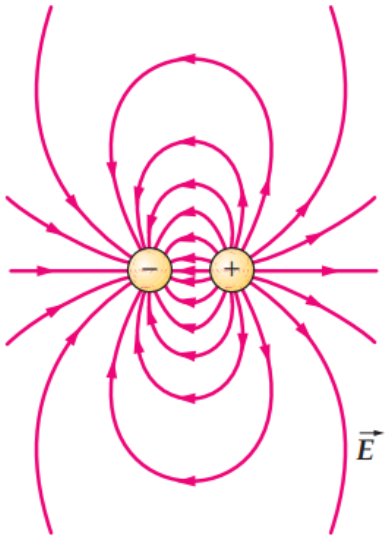
If $z = 0$ We have

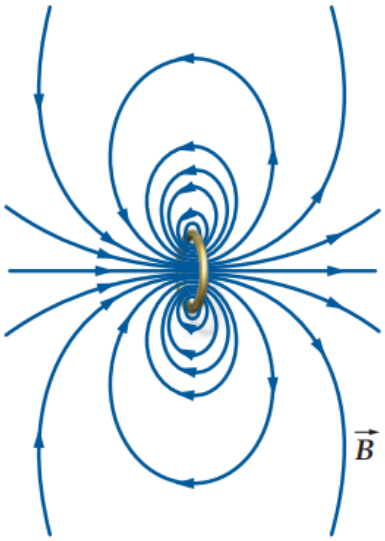
$$B = \frac{u_0 I}{2R}$$

4. Gauss's law for magnetism

In the definition of the electric flux replacing $\vec{E}$ by the $\vec{B}$, the magnetic flux Φ_m is called Gauss's law for magnetism

$$\Phi_m = \oint_S \vec{B}.d\vec{S} = 0$$



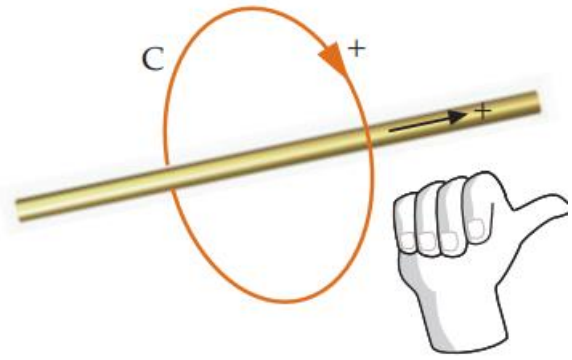


5. Ampère's law

In mathematical form, the Ampère's law is

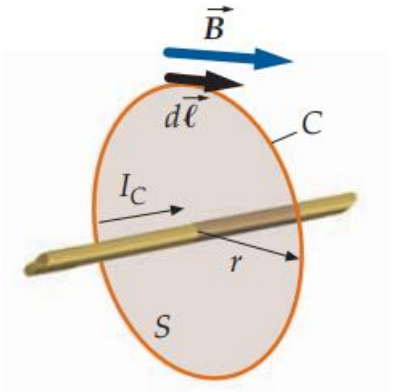
$$\oint_C \vec{B}.d\vec{l} = u_0 I_C$$

C is a closed curve.



then gives

$$B \oint_C dl = u_0 I_C$$



We thus obtain

$$B(2\pi R) = u_0 I$$

Or

$$B = \frac{u_0 I}{2\pi R}$$

6.The magnetic flux Φ_m

The magnetic flux Φ_m through S is

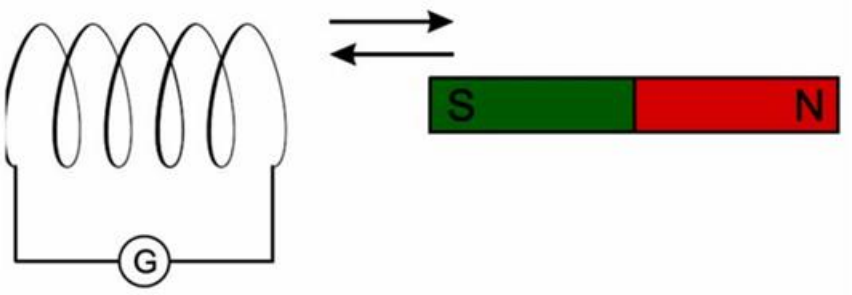
$$\Phi_m = \oint_S \vec{B}.d\vec{S}$$

The unit of magnetic flux Φ_m is the **tesla-meter-squared**, or we called a **weber** (Wb)

$$1\ \text{Wb} = 1\ T.m^2$$

7. Emf induced or Faraday's law

Let's take a permanent magnet and place it near a loop made of a conducting wire connected to a galvanometer. When the magnet is static, there is no measurable current in the wire. On the other hand, when we move the magnet, we see a current appear whose sign varies depending on whether we approach or move away from the magnet. In addition, this current is all the more important as the movement is rapid. These two types of experiments led Faraday to write: (When the flow through a closed circuit changes, we have an electric current or appears.)



An induced current can be obtained by rotating the electric coil in front of the stationary magnet.

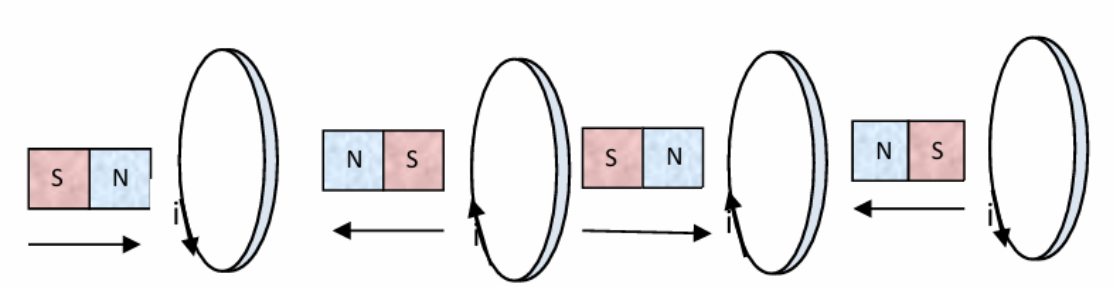
The reason for the appearance of induction current is the change in magnetic flux across the surface of the coil. The inductive current only lasts as long as the flux change lasts.

The appearance of an inductive current indicates the presence of an inductive electromotive force originating from the coil

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

8. Lenz's law

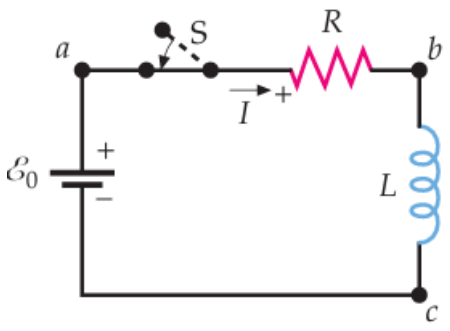
Lenz's law predicts the direction of the induced current. **(** The induced emf is in such a direction as to oppose, or tend to oppose, the change that produces it. **)**



The alternative statement of **Lenz's law (** When a magnetic flux through a surface changes, the magnetic field due to any induced current produces a flux of its own—through the same surface and opposite in sign to the initial change in flux. **)**

9. Application

The current begins to increase and a back emf of magnitude $L\frac{dI}{dt}$ is induced in the inductor just after the switch S is closed in this circuit. The potential drop across the inductor $L\frac{dI}{dt}$ plus the potential drop across resistor IR equals the emf of the battery $\mathcal{E}_0$.



This circuit gives:

$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$$

multiply by the current I, we obtain

$$\mathcal{E}_0 I = I^2 R + LI\frac{dI}{dt}$$

The term $\mathcal{E}_0 I$ is the rate at which electrical potential energy is supplied by the battery.

The term $I^2 R$ is the rate at which potential energy is delivered to the resistor.

The term $LI\frac{dI}{dt}$ is the rate at which potential energy is delivered to the inductor.

So if U_m is the energy stored in the inductor, then

$$\frac{dU_m}{dt} = LI\frac{dI}{dt}$$

which implies

$$dU_m = LIdI$$

Integrating this equation, we obtain the energy stored in an inductor

$$U_m = \frac{1}{2}LI^2$$

References

[1] J.-P. Perez, R. Carles, R. Fleckinger ; Electromagnétisme Fondements et Applications, Ed. Dunod, 2011.

[2] H. Djelouah ; Electromagnétisme ; Office des Publications Universitaires, 2011.

[3] P. Fishbane et al.; Physics for Scientists and Engineers with Modern Physics, 3rd ed.; 2005.

[4] P. A. Tipler, G. Mosca ; Physics For Scientists and Engineers, 6th ed., W. H. Freeman Company, 2008.