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Texture features using machine learning techniques for offline facial expression recognition

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ملخص:

تهدف مذكرة التخرج إلى دراسة وتطوير نظام للتعرف على تعابير الوجه بواسطة تعلم الآلة. تستخدم تقنية التعلم الآلي لتطوير نماذج وخوارزميات تحليل البيانات واكتشاف الأنماط والتنبؤ التلقائي، دون الحاجة إلى برمجة مباشرة.

في سياق تعرف الوجه وتعابيره، يمكن استخدام تعلم الآلة لتطوير نماذج قادرة على تصنيف التعابير الوجهية المختلفة. يتم تدريب هذه النماذج على مجموعة كبيرة من البيانات التي تحتوي على تعابير وجوه معروفة سلفًا ومصنّفة بشكل صحيح. بعد تدريب النموذج على هذه البيانات، يصبح قادرًا على التعرف على الأنماط الوجهية وتصنيف التعابير الجديدة التي يتم تقديمها له.

لقد تمت مشاركة نتائج عدة تجارب تجريبية في المذكرة باستخدام خوارزميات التصنيف المختلفة مثل SVM و KNN على مجموعة بيانات .FER2013 وقد أظهرت النتائج تحقيق نتائج مرضية تمامًا باستخدام هذه الخوارزميات.

ولكي يتم تحسين هذه النتائج، تم تطبيق مجموعة من الفلاتر على مجموعة البيانات، وتم أيضًا زيادة حجم البيانات المتاحة. تم إجراء دراسات متعددة لهذه العمليات وتم الحصول على نتائج أفضل بعد تطبيق التحسينات.

بشكل عام، يمكن استنتاج أن استخدام خوارزميات SVM و KNN في تعرف التعابير الوجهية قد أظهر نتائج مرضية. وقد أظهرت العمليات الإضافية مثل تطبيق الفلاتر وزيادة البيانات تحسينًا في الأداء. ويمكن أن يكون هذا النهج واعدًا في تطوير نظام فعال للتعرف على تعابير الوجه باستخدام تعلم الآلة. تمت مناقشة هذه النتائج والتحسينات المقترحة في المذكرة بشكل شامل.

يهدف البحث في المذكرة توسيع المعرفة وفهم أفضل لتطبيقات تعلم الآلة في مجال التعرف على تعابير الوجه، ويسعى لتحقيق تطورات وتحسينات أكبر في الأداء والدقة. يعتبر هذا النهج واعدًا وقد يسهم في تطوير نظم تعرف على تعابير الوجه أكثر فاعلية وتطبيقها في مختلف المجالات.

Abstract:

The graduation thesis aims to study and develop a facial expression recognition system using machine learning. Machine learning techniques are utilized to develop models and algorithms for data analysis, pattern recognition, and automatic prediction, without the need for direct programming.

In the context of facial recognition and expression analysis, machine learning can be used to develop models capable of classifying different facial expressions. These models are trained on a large dataset containing pre-labeled facial expressions. After training on this data, the model becomes capable of recognizing facial patterns and classifying new expressions presented to it.

Several experimental results have been shared in the thesis, using various classification algorithms such as SVM and KNN on the FER2013 dataset. The results have demonstrated satisfactory performance using these algorithms.

To improve these results, a set of filters has been applied to the dataset, and the available data volume has been increased. Multiple studies have been conducted on these processes, and better results have been achieved after implementing the enhancements.

In general, it can be concluded that the use of SVM and KNN algorithms in facial expression recognition has shown satisfactory results. Additional processes such as filter application and data augmentation have improved performance. This approach holds promise in developing an effective facial expression recognition system using machine learning. These results and proposed enhancements have been comprehensively discussed in the thesis.

The research in the thesis aims to expand knowledge and gain a better understanding of machine learning applications in facial expression recognition. It seeks to achieve further advancements and improvements in performance and accuracy. This approach is promising and may contribute to the development of more efficient facial expression recognition systems applicable in various fields.

Keywords: SVM(Support Vector Machine), KNN(k-Nearest Neighbors), FER(Facial Expression Recognition), HOG(Histogram of Oriented Gradients), LBP(Local Binary Patterns).

Résumé :

Le mémoire de fin d'études vise à étudier et développer un système de reconnaissance des expressions faciales à l'aide de l'apprentissage automatique. La technologie de l'apprentissage automatique est utilisée pour développer des modèles et des algorithmes d'analyse de données, de détection de motifs et de prédiction automatique, sans nécessiter de programmation directe.

Dans le contexte de la reconnaissance faciale et des expressions faciales, l'apprentissage automatique peut être utilisé pour développer des modèles capables de classer différentes expressions faciales. Ces modèles sont entraînés sur un ensemble de données volumineux contenant des expressions faciales préalablement connues et correctement classifiées. Après l'entraînement du modèle sur ces données, il devient capable de reconnaître les motifs faciaux et de classifier de nouvelles expressions qui lui sont présentées.

Plusieurs résultats d'expériences expérimentales ont été partagés dans le mémoire en utilisant différentes algorithmes de classification tels que SVM (machine à vecteurs de support) et KNN (plus proches voisins k). Ces résultats ont montré des performances tout à fait satisfaisantes en utilisant ces algorithmes.

Pour améliorer ces résultats, divers filtres ont été appliqués aux ensembles de données, et la disponibilité des données a également été augmentée. Plusieurs études ont été menées sur ces processus et des améliorations ont été obtenues après l'application des améliorations.

En général, on peut conclure que l'utilisation des algorithmes SVM et KNN dans la reconnaissance des expressions faciales a montré des résultats satisfaisants. Les opérations supplémentaires telles que l'application de filtres et l'augmentation des données ont amélioré les performances. Cette approche peut être prometteuse pour le développement d'un système efficace de reconnaissance des expressions faciales à l'aide de l'apprentissage automatique. Ces résultats et les améliorations proposées ont été discutés de manière exhaustive dans le mémoire.

La recherche dans le mémoire vise à étendre les connaissances et à mieux comprendre les applications de l'apprentissage automatique dans le domaine de la reconnaissance des expressions faciales. Elle vise à réaliser des avancées et des améliorations plus importantes en termes de performances et de précision. Cette approche est prometteuse et pourrait contribuer au développement de systèmes de reconnaissance des expressions faciales plus efficaces, applicables dans divers domaines.

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General Introduction

Machine learning is a branch of artificial intelligence that focuses on developing models and algorithms that enable systems to automatically extract knowledge, analyze data, and modify their behavior based on experiences and ongoing training. Machine learning relies on the use of data to create models capable of predicting outcomes and making decisions without explicit programming.

In recent years, facial recognition technology has made significant advancements. Facial recognition systems and facial expression analysis are important applications of machine learning. These systems aim to analyze images and videos to detect facial features and distinguish various facial expressions such as joy, sadness, anger, and surprise. This technology enables individual identification and understanding of their emotional state, finding applications in various fields such as security, emotion recognition, and human communication.

Machine learning plays a crucial role in the development of facial expression recognition systems. By utilizing machine learning techniques, models can be developed that accurately recognize facial patterns and classify facial expressions. These models are trained using large datasets containing pre-known and properly classified facial expressions.

The thesis consists of four main chapters, where different topics are detailed in each chapter as follows:

Chapter 1: Theoretical concepts

This chapter presents facial recognition systems and how machine learning is used in their development. The importance of these systems in recognizing facial patterns and classifying different facial expressions is highlighted.

Chapter 2: State of the art

This chapter reviews ongoing studies and research related to facial expression recognition systems and machine learning. The significant research conducted in this field, along with the techniques and methodologies used, are highlighted.

Chapter 3: Our approach

This chapter provides a detailed discussion of the methods used in experimental studies related to facial expression recognition and machine learning. The employed algorithms such as SVM and KNN, and other techniques used for the analysis and classification of facial expressions, are reviewed.

Chapter 4: The experimental results

Chapter 4 presents the experimental results obtained by applying the methods and techniques discussed in the previous chapters. The results are analyzed and discussed in detail, focusing on the accuracy and effectiveness achieved in facial expression recognition using machine learning.

Finally, in the conclusion, a comprehensive summary of the thesis is provided. The main findings and information discussed in the research and experiments are summarized. Additionally, the scientific and practical contribution and value of the thesis in the field of facial expression recognition and machine learning are highlighted.

Through this thesis, the research aims to deepen knowledge and better understand the applications of machine learning in facial expression recognition. Furthermore, the research aims to achieve new advancements and improvements in the performance and accuracy of facial expression recognition systems using machine learning. This approach is promising and could contribute to the development of more efficient facial expression recognition systems applied in various domains such as emotion recognition and human communication.

Chapter 01:

Theoretical concepts

1. Introduction

The first chapter of this note discusses facial recognition systems and the use of machine learning in their development. It highlights the importance of these systems in recognizing facial patterns and classifying facial expressions. Machine learning plays a key role in enabling the creation of models capable of accurately recognizing and classifying different facial expressions. This chapter explores these concepts in detail and emphasizes the potential benefits of these systems in various domains such as security and communication.

2. Fundamental Concepts of Facial Recognition:

2.1. Facial Recognition:

Facial recognition is the process of verifying or identifying a person by analyzing and comparing unique facial features from acquired facial images. Facial recognition systems use algorithms that measure and analyze the proportions, patterns, and contours of the face to create a digital representation of facial features. These representations are then compared to faces stored in a database to determine a match. [1]

2.2. History and Evolution of Facial Recognition Systems:

The earliest foundational work in the field of facial expression analysis dates back to the 19th century and was conducted by researchers such as Charles Darwin, Guillaume Duchenne De Boulogne, and Paul Ekman [2].

Charles Darwin, known for his theory of evolution, was the first to address the universality of facial expressions and propose an evolutionary theory on the formation of these expressions. According to Darwin, facial expressions found in children and infants also exist in adults, suggesting that they have a biological origin and are related to communication. He emphasized

the importance of non-verbal expressions in our daily lives, considering them as crucial as verbal interactions in the process of human communication [2].

In the 19th century, Guillaume Duchenne De Boulogne conducted innovative research in the field of facial expression analysis. He used electrical stimulation to individually activate different facial muscles, allowing him to precisely identify the muscles responsible for each expression. The photographs he obtained represented the first visual evidence of specific activation of facial muscles during the expression of emotions [2].

Paul Ekman, a renowned psychologist, conducted research in the 1960s on the categorization of human expressions and emotions. His work led to the discovery that there are a limited number of universally recognized facial expressions, regardless of culture. He identified seven basic emotions that are universally expressed through facial expressions: neutrality, joy, sadness, surprise, fear, anger, and disgust. These findings confirmed the existence of a common biological basis in the expression of human emotions [2].

Thus, the work of these researchers laid the foundation for facial expression analysis and contributed to the understanding of their universality and their role in human communication. [2].

2.3.Key Components of Facial Recognition Systems

Facial recognition systems rely on several key components that work together to identify and analyze faces. Here are some of the most important components [3] :

Face Detection: This is the initial step where the system locates and identifies the regions of the face in an image or video. This detection can be done using algorithms based on features such as contours and contrasts [3] .

Feature Extraction: Once the face is detected, relevant facial features are extracted. This can include elements such as the position of the eyes, nose, mouth, as well as the distances and proportions between these elements [3] .

Normalization and Preprocessing: The extracted features can be normalized to reduce variations due to lighting, camera angles, facial expressions, etc. Preprocessing techniques can also be used to enhance image quality or reduce noise [3] .

Learning and Classification: In this step, machine learning algorithms are used to learn how to recognize and classify faces based on the extracted features. Techniques such as neural networks, support vector machines (SVM), or deep learning-based recognition methods may be employed [3] .

Database and Comparison: The faces stored in a database are compared with newly captured faces. This comparison can be done by calculating similarity measures or using pattern matching techniques to determine if a positive match is found [3] .

Decision Making and Identification: Based on the comparison results, the facial recognition system makes a decision, such as identifying a specific person, verifying the authenticity of a face, or detecting similarities among multiple faces [3] .

It is worth noting that these components may vary depending on different facial recognition systems and the techniques used. Technological advancements and ongoing research in this field can also bring new improvements and functionalities [3] .

2.4.Face Detection:

Face detection involves identifying the presence of faces in an image and determining the locations and scales of the faces. The accuracy of face detection is particularly important in realistic conditions where the presence and overall location of the face in a scene are not known beforehand [5].

Generally, the main challenges in face detection are robustness to different identities, facial poses, facial expressions, and lighting variations. Figure 1.1 represents an example of a face detection system in group photos [5].



Figure 1: Face Detection [4]

Face detection is a necessary preliminary task for most facial recognition techniques. The techniques used are typically derived from the field of pattern recognition. For reliable and valid face detection, the system must ensure the following aspects [5]:

Robustness: It should be able to handle variations in appearance due to pose, size, lighting, occlusion, complex backgrounds, facial expressions, and resolution [5].

Speed: It should be fast enough to perform real-time processing [5].

Simplicity: It should be simple. For example, the training time should be short, the number of parameters should be small, and the training samples should be collectible at a low cost [5].

There are several techniques for face detection, some utilizing skin color, head shape, facial appearance, while others combine multiple of these characteristics. Among them, we mention the Viola-Jones (V&J) face detection technique. It is a classical and widely used implementation for face detection, which is robust and computationally simple for detecting nearly frontal faces. This algorithm explores Haar-like features using the Cascade classifier, which can efficiently combine multiple features and determine different filters in a resulting classifier [5].

2.5.Feature Extraction:

Once the face is detected, the step of facial feature detection usually begins with a rectangular bounding box returned by a face detector that localizes the face. Obtaining effective facial expression features is crucial for successful facial expression recognition. Facial landmark points are mainly located around facial components such as eyes, mouth, eyebrows, and nose. Therefore, feature extraction methods for expression analysis can be categorized into three types of approaches: geometric-based methods, appearance-based methods, and hybrid-based methods [5].

Geometric-based features describe the shape of facial components, such as the shape of the mouth, eyes, eyebrows, nose, and their locations (corners of the eyes, corners of the mouth, etc.). These features are extracted to form a feature vector representing the geometry of the face [5].

Appearance-based features can be extracted over the entire face or specific regions of the face. They describe changes in appearance (skin texture) of the face, such as wrinkles and creases [5].

Hybrid-based features are described by features based on both appearance and geometry. It is known that geometry-based and appearance-based features have their respective advantages and limitations. For example, geometric features are efficient in computation but sensitive to noise, while appearance features are robust to image misalignment but computationally time-consuming. Several researchers use hybrid features to improve expression decoding accuracy [5].

2.6.The classes of facial emotions:

Emotions and facial expressions are translated through a set of muscular movements that appear in the permanent features of the face. In this thesis, we study the description of facial expressions and associated deformations that occur on the face for the most frequently used Ekman emotions [6].

a. Joy: It is characterized by a person's state of intense satisfaction. It is related to desire, achievement, well-being, and fulfillment [6].

b. Sadness: It is the state of a person who suffers emotionally due to dissatisfaction and concerns. It is often caused by loss, mourning, or obstacles [6].

c. Anger: It is the state of a person in a violent and aggressive reaction to frustration. It is often caused by injustice, harm, or a violation of values [6].

d. Disgust: It is the state of a person who has an aversion to certain foods or a lack of appetite. It is often caused by rejection, someone or something repulsive, or an aversion [6].

e. Fear: It is the state of a person threatened by real or imaginary danger. It is often caused by a threat, danger, or unfamiliarity [6].

f. Surprise: It is the state of a person amazed by something unexpected. It is often caused by immediate danger, an unforeseen event, or encountering strangers [6].

We present the main facial muscle changes associated with different classes of emotions. The illustrative images in this table are extracted from the FER2013 database [6].

emotions	Facial analysis	*Image*
Joy	• The eyes are slightly squinted, i.e. the lower eyelid partially covers the eye. • The corners of the lips pulled up. • The mouth may be open and the teeth may be visible. • Cheeks can be lifted.	
The fear	• The eyes are open and the pupils become large. • Eyebrows are lifted and pulled inward. • The upper eyelids are lifted. • The muscles of the mouth tense and often lower the lower lip.	

The surprise	• The eyes are wide open. • Horizontal wrinkles that may appear on the forehead. • The mouth is open vertically. • Eyebrows are raised.	
Sadness	• The inner parts of the eyebrows are lowered. • The eyelids cover part of the field of vision. • The mouth is tight but it goes down slightly, the corners will be stretched down.	
Anger	• The eyelid covers part of the eye so the eyes will be almost closed. • As for the mouth, it remains closed but quite tight, otherwise it opens vertically. • The eyebrows tend to meet, they are frowning, wrinkled. In addition, their inner part is lowered slightly.	
Disgust	• The upper lip is raised, the greater the disgust, the more the upper dentition is visible. • Wrinkles appear on the nose.	
Neutral	• No change in facial muscles.	

Table 1: The classes of facial emotions

3. Machine Learning in Facial Recognition:

Machine learning, also known as ML, is a field of artificial intelligence that enables computer systems to learn from data without being explicitly programmed. It relies on algorithms that can detect patterns and make decisions based on these patterns. [7].

Machine learning is widely used in facial recognition systems to improve the accuracy and performance of face recognition. Here are some of the key concepts and techniques used in machine learning for facial recognition:

3.1. Supervised Learning:

Supervised learning is a machine learning technique where a model is trained using a labeled dataset, where each data example is associated with a predefined label or class. The model learns from these labeled data to be able to predict or classify new unlabeled examples. [8]

- K-Nearest Neighbors (KNN)
- Bayesian classification.
- Support Vector Machine (SVM).
- Artificial Neural Networks (ANN).
- Decision tree.

3.2. Unsupervised Learning:

Unsupervised learning is a branch of machine learning where a model is trained using an unlabeled dataset, meaning there is no information about the classes or labels of the data examples. The goal of unsupervised learning is to discover intrinsic structures, patterns, or clusters within the data. [8].

- Artificial Neural Networks "ANN"
- K-means “KMeans”
- Fuzzy K-Means
- Hierarchical clustering

3.3. Reinforcement Learning:

Reinforcement learning is a machine learning method where an agent learns to make decisions by interacting with an environment and receiving rewards or punishments based on its actions. The agent learns to maximize rewards over time by adjusting its action policy. [9].

3.4. Classification:

The classification stage is the final step in an automated facial expression recognition system. It relies on the feature vectors obtained in the preceding step. Various machine learning algorithms have been utilized in facial expression recognition, with the most commonly used ones being:

- Support Vector Machines (SVM)
- Hidden Markov Models (HMM)
- Linear Discriminant Analysis (LDA)
- K Nearest Neighbors (KNN)
- Bayesian Networks (BN)
- Neural Networks (NN)

The recognition phase of the FER system is grounded in machine learning theory. The feature vector is constructed to describe the facial expression, making the learning process the initial part of the classifier. Classifier training involves assigning labels to the detected images. Once the classifier is trained, it can identify input images by assigning them a specific expression class label. Classification methods can be categorized into two groups:

- Static data-based recognition, which solely involves images.
- Dynamic data-based recognition, which encompasses image or video sequences.

This paper specifically focuses on facial expression recognition systems that rely on static data, namely images.

4. Role of Machine Learning in Improving Facial Recognition Accuracy:

Machine learning plays a crucial role in improving the accuracy of facial recognition systems. It enables systems to recognize and classify faces based on patterns and algorithms capable of extracting knowledge from data. Machine learning is used in facial feature extraction, where algorithms identify distinctive facial traits such as facial contours, eyes, nose, and mouth. It is also employed in facial expression classification, allowing systems to recognize different emotions such as joy, sadness, anger, etc. [10]

One of the main advantages of machine learning in facial recognition is its ability to adapt and improve performance based on training data. Systems using machine learning can be trained on large datasets containing a diversity of faces and facial expressions, enabling them to learn to generalize and accurately recognize different faces and expressions even under varying conditions. [10]

In summary, machine learning plays a crucial role in improving the accuracy of facial recognition systems. It enables feature extraction, facial expression classification, and adaptation to data variations, making facial recognition systems more effective and reliable in various applications such as security, person identification, and human-machine interfaces. [10]

5. Applications and Domains of Facial Recognition Systems:

Facial recognition systems are used in a variety of applications and domains, providing significant benefits in different contexts. Here are some examples of applications and domains where facial recognition systems are used [11]:

Security and Surveillance: Facial recognition systems are widely used to enhance security and surveillance in public places, airports, train stations, government institutions, etc. They can detect and identify suspicious individuals, wanted persons, or control access to secure areas. [11]

Identity Management: Facial recognition systems are used for identity management, including citizen identification programs, biometric passports, building access control systems, border control systems, etc. [11]

Retail: In the retail industry, facial recognition is used for personalized customer experiences, such as identifying regular customers, providing targeted product recommendations, and analyzing customer emotions to assess their satisfaction. [11]

Marketing and Advertising: Facial recognition systems can be used to analyze consumer reactions and emotions towards advertisements or products. This allows marketers to adapt their strategies in real-time to maximize impact and customer engagement. [11]

Health and Well-being: In the healthcare field, facial recognition is used for monitoring patients' emotions and facial expressions, particularly in mental health. It can also help detect signs of fatigue, stress, or potential illnesses. [11]

Human-Machine Interfaces: Facial recognition systems are used to create more intuitive and interactive human-machine interfaces. For example, they can be used to unlock electronic devices, control video games, or enable gesture-based interactions in virtual reality. [11]

It is worth noting that the use of facial recognition systems also raises privacy and ethical concerns. It is important to establish appropriate policies and regulations to govern their use. [11]

6. Advantages and Challenges of Facial Recognition Systems:

Facial recognition systems offer several advantages, but they also face significant challenges. Here is a list of the main advantages and challenges associated with facial recognition systems [12]:

6.1. Advantage:

Quick and Accurate Identification: Facial recognition systems enable quick and accurate identification of individuals based on their unique facial features. This can be useful in applications such as security, identity management, and facility access. [12]

Convenience and User-Friendliness: Facial recognition offers a contactless method of identification, making it convenient and easy to use. Users do not need to carry or manipulate additional devices, simplifying the user experience. [12]

Wide Adoption: Facial recognition systems are increasingly being used in various fields, promoting their widespread adoption. This helps improve system accuracy through a larger amount of data available for training and validation. [12]

Adaptability to Changes: Facial recognition systems can adapt to natural variations in the face, such as changes in expression, hairstyle, beards, etc. This allows for more reliable and robust recognition under variable conditions. [12]

6.2.Challenges:

Privacy Protection: The use of facial recognition raises privacy concerns. Biometric data, such as facial images, can be sensitive and require proper management to prevent abuse or unauthorized use. [12]

Bias and Discrimination: Facial recognition systems can be subject to bias and classification errors, especially when used in diverse populations. This can result in unfair discrimination or inequitable outcomes, particularly based on race, gender, or age. [12]

Reliability under Variations: Facial recognition systems may be less reliable when faced with challenging conditions for capturing facial images, such as poor lighting, unusual viewing angles, or partial face occlusions. This can lead to higher error rates. [12]

Counterfeiting and Fraud: Facial recognition systems can be bypassed using counterfeiting or falsification techniques, such as the use of masks or printed images of the face. This poses challenges in terms of security and authenticity of identifications. [12]

It is essential to consider these advantages and challenges when designing, implementing, and using facial recognition systems to maximize their benefits while minimizing potential risks.

7. Datasets

Datasets refer to collections of structured or unstructured data that are used for training and evaluating machine learning models. They play a crucial role in the development and advancement of various AI applications.

7.1.LFW dataset:

The Labeled Faces in the Wild (LFW) database is a widely used benchmark dataset in the field of face recognition. It was created to evaluate and compare the performance of face recognition algorithms on real-world, unconstrained face images. The LFW database consists of more than 13,000 images of faces collected from the web, representing over 5,700 different individuals [13].

The images in the LFW database depict significant variations in pose, lighting conditions, facial expressions, and image quality, making it a challenging dataset for face recognition algorithms. Each image in the database is accompanied by the identity of the person depicted, allowing for evaluation of the algorithm's ability to correctly identify individuals.

Researchers and developers use the LFW database to assess the performance and accuracy of face recognition algorithms, as well as to compare different approaches in the field. It serves as a standard benchmark for evaluating the state-of-the-art in face recognition technology and has contributed to significant advancements in the field.



Figure 2: LFW dataset [14]

7.2.FER dataset:

The FER (Facial Expression Recognition) dataset is a commonly used dataset in the field of facial expression recognition. It consists of a collection of facial images labeled with corresponding emotion labels. The dataset is used for training and evaluating machine learning algorithms and models for facial expression recognition tasks [15].

The source of the FER dataset can vary depending on the specific version or variant of the dataset being referred to. However, one popular version of the FER dataset, known as the "FER2013" dataset, was introduced in the paper "Emotion Recognition in the Wild" by Goodfellow et al. (2013). This dataset was created by collecting facial images from the internet and crowdsourcing the annotation of emotion labels for each image.

The FER2013 dataset contains approximately 35,887 grayscale images of size 48x48 pixels. Each image is labeled with one of seven emotion categories: angry, disgust, fear, happy, sad, surprise, or neutral. The dataset is commonly split into training and testing subsets, with a standard split ratio used in research publications.

The FER dataset, including the FER2013 version, has been widely used in research and serves as a benchmark for evaluating the performance of facial expression recognition algorithms. It provides a diverse range of facial expressions and is a valuable resource for developing and testing models in this field.



Figure 3: FER dataset [16]

7.3. CK+ dataset:

The CK+ (Cohn-Kanade) dataset is a widely used benchmark dataset for facial expression analysis and emotion recognition. It was developed by researchers at the University of Pittsburgh and the Robotics Institute, Carnegie Mellon University. The dataset contains a collection of facial expression sequences captured from 123 subjects, including both posed and spontaneous expressions [17].

The CK+ dataset is notable for its focus on subtle facial expressions, which are often more challenging to recognize and analyze compared to more intense expressions. It includes seven basic facial expressions: anger, contempt, disgust, fear, happiness, sadness, and surprise. The dataset also provides information on the intensity and occurrence of each expression.

The CK+ dataset has been widely used in the research community for developing and evaluating facial expression recognition algorithms and related tasks. It has contributed to advancements in facial expression analysis, emotion recognition, and related fields.



Figure 4: CK+ dataset [17]

The JAFFE dataset, also known as the Japanese Female Facial Expression (JAFFE) database, is a widely used dataset in the field of facial expression analysis and recognition. It was created by the Japanese psychologist and researcher Miyuki Kamachi in collaboration with her colleagues [18].

The JAFFE dataset consists of 213 grayscale images of 7 Japanese female models, each displaying 7 different facial expressions. The facial expressions include anger, disgust, fear, happiness, sadness, surprise, and a neutral expression. The images were captured under controlled lighting conditions, and each image is labeled with the corresponding facial expression.

The JAFFE dataset has been a valuable resource for researchers and developers working on facial expression recognition algorithms. It has been used to train and evaluate the performance of various facial expression recognition models, including machine learning and deep learning approaches.

The JAFFE dataset can be accessed from various sources, including academic research websites, computer vision and pattern recognition repositories, and facial expression analysis datasets repositories. It is freely available for non-commercial use, and researchers are encouraged to cite the original paper when using the dataset in their work.



Figure 5: JAFFE dataset [19]

8. Dataset used:

In our graduation project, if we used the FER 2013 dataset, it has some distinguishing characteristics that set it apart from other facial expression recognition datasets. Here are a few notable features of the FER 2013 dataset:

Large and Diverse: The FER 2013 dataset consists of over 35,000 labeled facial images, making it one of the largest publicly available datasets for facial expression analysis. The dataset includes images of various individuals with diverse facial expressions, providing a wide range of examples for training and evaluation.

Seven Expression Categories: The FER 2013 dataset focuses on seven primary facial expressions: anger, disgust, fear, happiness, sadness, surprise, and a neutral expression. These categories cover a broad spectrum of emotions, allowing for comprehensive analysis and recognition of facial expressions.

Crowdsourced Annotations: The FER 2013 dataset was annotated using a crowdsourcing approach, where human annotators labeled the facial expressions in the images. This approach helps capture a collective understanding of the expressions and increases the dataset's diversity in terms of annotators' perspectives.

Real-World Images: The images in the FER 2013 dataset are collected from various sources, including the internet and social media platforms. As a result, the dataset reflects real-world conditions and captures natural variations in lighting, pose, and image quality, making it suitable for evaluating facial expression recognition algorithms in real-world scenarios.

Availability and Usage: The FER 2013 dataset is publicly available, allowing researchers and developers to access and utilize it for academic and non-commercial purposes. Its widespread use in the research community has facilitated benchmarking and comparison of different facial expression recognition methods.

These characteristics of the FER 2013 dataset make it a popular choice for training, testing, and evaluating facial expression recognition models, providing us with a rich and diverse set of images and annotations to support our research in the field.

9. Conclusion:

In conclusion of this chapter, we have examined the importance and role of facial recognition in modern systems, as well as the impact of machine learning on improving their accuracy and performance. We provided a definition of facial recognition and traced its historical evolution, while highlighting the key components of facial recognition systems. We also explored various machine learning techniques used in this field, such as supervised, unsupervised, and reinforcement learning, and emphasized their role in improving the accuracy of facial recognition. Furthermore, we highlighted the benefits of using machine learning in this context, such as accurately extracting facial features and classifying facial expressions, as well as its adaptability to data variations.

In the next chapter, we will delve into current research on facial expression recognition and machine learning in more detail. We will focus on recent advances aimed at enhancing our understanding of facial expressions and our ability to accurately recognize them. We will review recent studies and techniques used in this field and discuss challenges and future directions.

In summary, our first chapter has focused on the foundations of facial recognition and the importance of machine learning in improving the accuracy and efficiency of facial recognition systems. In the next chapter, we will explore current research focused on facial expression recognition and the use of machine learning.

Chapter 02: State of the art

1. Introduction:

In this chapter, we delve into the state-of-the-art advancements in facial expression recognition. We explore the cutting-edge techniques, methodologies, and research studies that have contributed to the current state of this field. By examining the latest developments, we gain valuable insights into the capabilities and limitations of facial expression recognition systems. This chapter provides a comprehensive overview of the state-of-the-art in facial expression recognition, setting the foundation for further exploration and understanding in subsequent sections of the thesis.

2. Related works

2.1. Improving Deep Learning Feature with Facial Texture Feature for Face Recognition:

In this paper [20], they propose a face recognition approach called Facial Texture Feature-Aided Deep Learning Feature (FTFA-DLF). The FTFA-DLF method combines the strengths of deep learning and hand-crafted features. Specifically, hand-crafted features extracted from key facial regions such as the eyes, nose, and mouth are utilized. These hand-crafted features are then incorporated into the objective function layer along with the deep learning features. This adaptive integration allows the deep learning features to better collaborate with the hand-crafted features, resulting in improved face recognition performance.

Experimental results demonstrate the efficacy of the proposed face recognition algorithm, which achieved an accuracy rate of 97.02% on the LFW face database. The integration of facial texture features and deep learning features in the FTFA-DLF method showcases the potential for enhanced performance in real-world face recognition tasks.

2.2. Hierarchical committee of deep convolutional neural networks for robust facial expression recognition:

This paper [21] presents an approach for robust facial expression recognition by training multiple deep convolutional neural networks as committee members and combining their decisions. Two strategies are used to improve this committee: varying network architecture, input normalization, and random weight initialization in training these deep models, and constructing a hierarchical architecture of the committee with exponentially-weighted decision fusion. The approach achieves a test accuracy of 61.6% for a seven-class problem of static FER in the wild for the EmotiW2015, and yields superior performance on other public FER databases, outperforming or competing with state-of-the-art results for these databases.

2.3. Extended deep neural network for facial emotion recognition:

In this paper [22], they present their approach, which is an extension of their previous work for facial emotion recognition. The aim of their work is to classify each image into one of six facial emotion classes. The proposed model is based on single Deep Convolutional Neural Networks (DNNs), which contain convolution layers and deep residual blocks.

In the proposed model, firstly the image label to all faces has been set for the training. Secondly, the images go through the proposed DNN model. This model trained on two datasets Extended Cohn–Kanade (CK+) and Japanese Female Facial Expression (JAFFE) Dataset. The overall results show that the proposed DNN model can outperform the recent state-of-the-art approaches for emotion recognition. Even the proposed model has accuracy improvement in comparison with their previous model.

2.4. Hybrid deep neural networks for face emotion recognition:

This study [23] introduces a novel approach called the Hybrid Convolution-Recurrent Neural Network (CNN-RNN) method for Facial Expression Recognition (FER) in Images. The network architecture comprises Convolutional layers followed by a Recurrent Neural Network (RNN). By integrating these two models, the proposed method effectively captures both the spatial relationships within facial images through the Convolutional layers and the temporal dependencies present in the images through the Recurrent Network. The performance of the

hybrid model is evaluated on two publicly available datasets, and the experimental results show promising improvements compared to existing state-of-the-art methods.

2.5.Facial emotion recognition using min-max similarity classifier:

In this paper [24], they propose an efficient and straightforward facial emotion recognition algorithm to reduce the problem of inter-class pixel mismatch during classification. The proposed method includes the application of pixel normalization to remove intensity offsets followed-up with a Min-Max metric in a nearest neighbor classifier that is capable of suppressing feature outliers. The results indicate an improvement of recognition performance from 92.85% to 98.57% for the proposed Min-Max classification method when tested on the JAFFE database. The proposed emotion recognition technique outperforms the existing template matching methods.

2.6.Facial expression recognition with Convolutional Neural Networks: Coping with few data and the training sample order:

In this study [25], a simple solution for facial expression recognition is proposed, utilizing a combination of Convolutional Neural Network (CNN) and specific image pre-processing techniques. Convolutional Neural Networks have shown improved accuracy with large datasets. However, the availability of such datasets for deep architectures in facial expression recognition is limited. To address this challenge, specific pre-processing techniques are applied to extract expression-specific features from facial images, while also investigating the order of sample presentation during training. The proposed technique is evaluated using three widely-used public databases (CK+, JAFFE, and BU-3DFE), with an analysis conducted to assess the impact of each image pre-processing operation on the accuracy rate. The results demonstrate competitive performance compared to other facial expression recognition methods, achieving an accuracy of 96.76% on the CK+ database. Additionally, the proposed method offers fast training times and enables real-time facial expression recognition on standard computers.

2.7.Face recognition using Ada-Boosted Gabor features:

The paper's [26] primary achievement can be summarized in two key aspects. First, it effectively employs AdaBoost for face recognition by incorporating the intra-face and extra-face difference

space within the Gabor feature space. Second, it adopts a suitable re-sampling approach to address the imbalance between positive and negative samples. Through this method, a mere selection of hundreds of Gabor features proves sufficient. Experimentation on the FERET database demonstrates that these selected features yield comparable performance to methods that utilize the complete set of Gabor features.

2.8. Face recognition based on convolutional neural network:

This research paper [27] suggests a modified Convolutional Neural Network (CNN) structure that includes two normalization operations in two of its layers. The addition of batch normalization as a form of normalization helped speed up the network. The CNN architecture was utilized to extract unique facial characteristics, while a Softmax classifier was employed to classify faces in the fully connected layer of the CNN. During the experimentation phase, the proposed method was tested on the Georgia Tech Database, and it demonstrated enhanced face recognition performance, yielding improved recognition outcomes.

2.9. Dynamic Texture Recognition Using Volume Local Binary Count Patterns With an Application to 2D Face Spoofing Detection:

This paper [28] introduces a new descriptor called the volume local binary count (VLBC) for representing and recognizing dynamic texture. It is similar to the volume local binary pattern (VLBP) but focuses on extracting histograms of thresholded local spatiotemporal volumes using appearance and motion features. Unlike VLBP, which uses binary encoding, VLBC counts the number of ones in the thresholded codes instead of exploiting local structure information. This allows VLBC to include more neighboring pixels without increasing the feature dimension exponentially like VLBP does. Additionally, a completed version of VLBC (CVLBC) is proposed, which incorporates information about local contrast and central pixel intensities to enhance dynamic texture recognition performance. Experimental results using three dynamic texture databases show that the proposed method achieves classification rates comparable to state-of-the-art approaches. Moreover, the paper suggests using CVLBC for 2-D face spoofing detection, where it effectively describes differences between facial videos of genuine users and impostors, leading to good performance in detecting face spoofing. The method is evaluated on

three face antispoofing databases (Print-Attack, Replay-Attack, and CAS Face Antispoofing), and the results demonstrate the effectiveness of CVLBC for this task.

2.10. Local binary patterns and its variants for face recognition:

This research paper [29] examines the performance assessment of various models based on Local Binary Pattern (LBP), including Multivariate Local Binary Pattern (MLBP), Center Symmetric Local Binary Pattern (CS-LBP), and Local Binary Pattern Variance (LBPV). The study focuses on extracting and comparing facial features using the K nearest neighbor classification algorithm, with the G-statistics distance measure utilized for classification. The experiments were conducted on the JAFFE female, CMU-PIE, and FRGC version 2 databases. The findings demonstrate that CS-LBP consistently outperforms the other models in terms of performance.

2.11. Gabor Ordinal Measures for Face Recognition:

This research paper [30] introduces a new technique called Gabor ordinal measures (GOM) for extracting facial features. GOM combines the uniqueness of Gabor features with the reliability of ordinal measures to address both the similarities between individuals and the variations within individuals in face images. The proposed method derives various types of ordinal measures from the magnitude, phase, real, and imaginary components of Gabor images. These measures are then encoded together as visual primitives in local regions. The statistical distributions of these visual primitives within blocks of face images are combined into a feature vector, which is further refined using linear discriminant analysis to create a concise and distinctive feature representation. To train a robust classifier for face recognition, a two-stage cascade learning method and a greedy block selection method are employed. The effectiveness of GOM is demonstrated through extensive experiments on well-known face image databases, including FERET, AR, and large scale FRGC v2.0, showcasing its superior performance compared to existing methods in face recognition.

3. State of the art Summary:

For illustration purposes, the following figure briefly summarizes all the works mentioned in this chapter.

	study	approach	dataset	accuracy
1	Li et al [20].	FTFA-DLF	LFW	97.02%
2	Kim et al [21].	CNN	FER	61.6%
3	Jain D.K. et al [22].	CNN	CK+	93.24%
			JAFFE	95.23%
4	N. Jain et al [23].	CNN	CK+	92.71%
			JAFFE	94.91%
5	O.Krestinskaya et al [24].	Proposed methods	JAFFE	98.57
6	A.T. Lopes [25]	CNN	CK+	96.76%
7	Peng Yang et al [26].	AdaBoost	FERET	92.8%
8	M. Coşkun et al [27].	CNN	GT	98.8%
9	X. Zhao et al [28].	CVLBC	UCLA	99.00%
10	K. Meena et al [29].	KNN	JAFFE	87%
			CMU-PIE	80%
			FRGC	82%
11	Z. Chai et al [30].	GOM	Fb	99.9%
			Fc	100%
			Dup1	95.7%
			Dup2	93.1%

Table 2: State of the art Summary

4. Conclusion:

In this chapter, an extensive review of recent studies in the field was conducted, encompassing research up until the writing of this dissertation in 2023. The analysis included an examination of the datasets employed, the methodologies utilized, and the outcomes achieved by each study. To provide a comprehensive overview, a summary table was compiled, encompassing all the papers referenced in this chapter.

Moving forward to the next chapter, our own approach will be presented in detail. This will involve a thorough explanation of the training methodology, the datasets utilized, and the preprocessing and augmentation techniques applied to the data. Additionally, the models employed to address the research questions will be discussed. This comprehensive discussion will offer a clear understanding of our research methodology and serve as a foundation for the subsequent chapters.

Chapter 03: Our approach

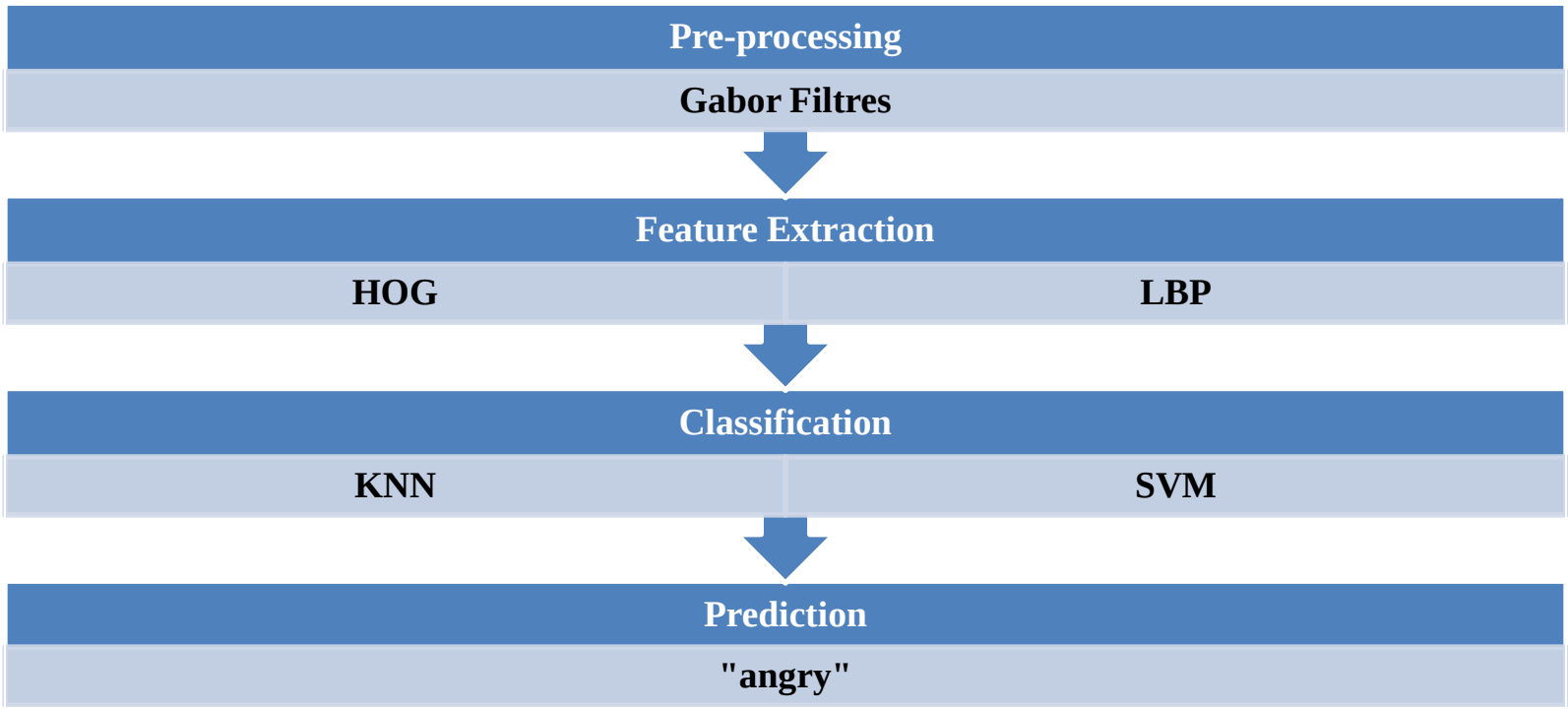
1. Introduction:

In this chapter, we provide an overview of the techniques, dataset, models, and processing methods employed in the project. We discuss the dataset utilized, describe the developed models, and demonstrate the methods employed for data processing. The chapter also includes the documentation of the technical details and tools utilized to accomplish the project's specific objectives.

2. Training method:

To develop a facial expression recognition system, we adopted a systematic approach that was outlined in a diagram. The diagram represents the sequential process we followed to attain the intended outcome:





This chapter will delve further into the process of creating a system that can identify facial expressions. The development of this system consisted of three crucial phases, sometimes merging certain stages together. We will provide a thorough explanation and discussion of each of these stages, emphasizing their significance in attaining the intended objective.

3. Pre-processing:

Pre-processing is of utmost importance in both facial expression recognition and overall facial recognition, as it serves as the crucial initial step. Its primary objective is to improve the performance of recognition algorithms by targeting specific areas.

In our study, we applied a group of image-enhancing filters to the prior stage to improve and facilitate the feature extraction process.

The applied filter group, we used the Gabor filter, modified its settings, and applied them to the data set every time we get a new data set that is saved, so that we do our experiments on these groups and follow up the results.

3.1. The Gabor filter:

The Gabor filter is a linear filter commonly used in image processing and computer vision applications for edge detection, texture analysis, and feature extraction. It is named after Dennis Gabor, who introduced the concept of Gabor functions in 1946. Gabor filters are widely employed in various image analysis tasks due to their ability to capture both spatial and frequency information simultaneously [31].

The Gabor filter is defined by a complex sinusoidal wave (or a harmonic function) modulated by a Gaussian envelope. Mathematically, the 2D Gabor filter function is given by:

$$g(x,y) = exp(-((x - x_0)^2 + (y - y_0)^2) / (2\sigma^2)) * exp(i(2\pi f_0 x + \theta_0))$$

Where:

- **(x, y)** represents the spatial coordinates of the image pixels,
- **(x_0, y_0)** represents the center of the Gaussian envelope,
- **σ** controls the spread (standard deviation) of the Gaussian envelope,
- **f_0** controls the frequency of the sinusoidal wave,
- **θ_0** represents the phase offset, and
- **i** is the imaginary unit.

The Gabor filter combines both spatial localization (determined by the Gaussian envelope) and frequency selectivity (determined by the sinusoidal wave). The Gaussian envelope determines the size and orientation of the filter, while the sinusoidal wave specifies the frequency content and orientation sensitivity [31].

When the Gabor filter is applied to an image, it convolves the filter function with the image to produce a filtered output. The filter response is high at locations where the image contains structures or textures that match the characteristics of the filter (e.g., edges or textures of specific frequency and orientation) [31].

The Gabor filter can be used to extract useful features from an image by convolving it with the image at multiple scales and orientations. By varying the parameters of the Gabor filter, such as the center position, scale, orientation, and frequency, it is possible to detect various types of edges, textures, and features [31].

A picture of some data sets after we applied the filter to them:

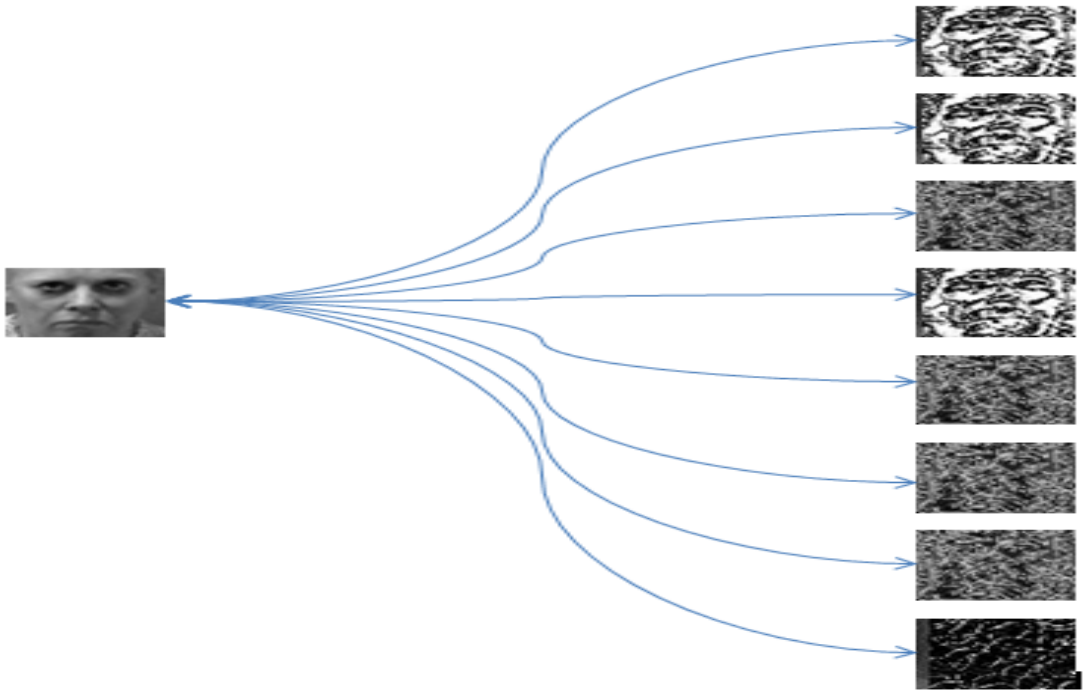


Figure 6: Gabor filter

4. Feature Extraction:

As stated earlier, feature extraction is a crucial step in image processing where raw image data is transformed into a meaningful set of representative features. This process occurs prior to the classification or training phase, and it aims to provide a suitable input for analysis and modeling purposes.

4.1.HOG:

HOG (Histogram of Oriented Gradients) is a feature extraction technique widely used in computer vision for object detection and recognition tasks. It was introduced by Navneet Dalal and Bill Triggs in their seminal paper "Histograms of Oriented Gradients for Human Detection" in 2005 [32].

The HOG algorithm calculates and represents the distribution of gradient orientations in an image. It captures the local shape and texture information of an object by analyzing the intensity gradients of the image pixels [32]. The key steps involved in the HOG feature extraction process are as follows:

Preprocessing: The input image is typically preprocessed to enhance contrast and reduce noise. Common preprocessing steps may include resizing, grayscale conversion, and applying normalization or equalization techniques [32].

Gradient Computation: The image gradients are computed using techniques such as the Sobel operator. The gradients capture the local intensity variations and provide information about the edges and texture of the image [32].

Cell Division: The image is divided into small cells, typically of square shape. Each cell contains a group of pixels [32].

Orientation Binning: Within each cell, a histogram of gradient orientations is constructed. The gradient orientations are quantized into predefined angular bins (e.g., 0-180 degrees divided into 9 bins), and the contribution of each gradient to the corresponding bin is accumulated [32].

Block Normalization: To enhance robustness to lighting and contrast variations, neighboring cells are grouped together into blocks. The histograms within each block are then normalized, either by L1 normalization (sum of absolute values) or L2 normalization (Euclidean norm). This normalization step helps to alleviate the effect of variations in illumination [32].

Descriptor Formation: The final HOG descriptor is formed by concatenating the normalized histograms from all the blocks. The resulting feature vector represents the object or region of interest and captures its shape and texture information [32].

4.2.LBP:

Local Binary Patterns (LBP) is a texture descriptor commonly used in computer vision and image processing. It was first introduced by T. Ojala, M. Pietikäinen, and D. Harwood in their paper "A Comparative Study of Texture Measures with Classification Based on Feature Distributions" published in 1994 [33].

The LBP operator calculates a binary pattern for each pixel in an image by comparing the pixel's intensity value with its neighboring pixels. The operator works as follows:

- Given an image, a target pixel is selected.
- The intensity value of the target pixel is compared with its surrounding pixels.
- For each neighboring pixel, if its intensity is greater than or equal to the intensity of the target pixel, a binary value of 1 is assigned; otherwise, a binary value of 0 is assigned.
- These binary values are concatenated clockwise to form a binary pattern.
- The resulting binary pattern is converted into a decimal value, which represents the texture information of the target pixel.

The LBP operator can be applied to grayscale as well as color images. For color images, the LBP operator is applied separately to each color channel, and the resulting patterns are concatenated to form a combined feature vector [33].

LBP has gained popularity in various computer vision tasks due to its simplicity, efficiency, and robustness to image variations such as illumination changes. It captures local texture information by encoding the spatial relationships between pixels within a small neighborhood. The resulting

LBP histograms or feature vectors can be used for tasks such as texture classification, face recognition, object detection, and image retrieval [33].

4. Classification

As stated earlier, the classification procedure relies on the feature vectors acquired during the feature extraction phase. Subsequently, these data are categorized using a classification method of choice.

4.3.SVM:

Support Vector Machines (SVMs) are a popular supervised learning algorithm used for classification and regression tasks. SVMs are effective in dealing with both linearly separable and non-linearly separable datasets. They aim to find an optimal hyperplane that separates the data points into different classes with the maximum margin [34] [35].

The basic idea behind SVMs is to map the input data points into a high-dimensional feature space, where a linear separation is more likely to exist. In this feature space, SVMs search for the hyperplane that maximally separates the classes while minimizing the classification error [34] [35].

Here's a step-by-step explanation of how SVM works:

Data Preparation: Given a labeled dataset with input features and corresponding class labels, SVM requires the data to be properly preprocessed and scaled. It's important to ensure that the features are on a similar scale to prevent bias towards any particular feature during the training process [34] [35].

Feature Mapping: SVMs use a technique called the kernel trick to implicitly map the input data points into a higher-dimensional feature space. The kernel function calculates the similarity between two data points without explicitly computing the transformation [34] [35].

Margin Maximization: SVMs aim to find the hyperplane that maximizes the margin between the support vectors (data points closest to the decision boundary) of different classes. The decision boundary is defined as the hyperplane that separates the classes, and the margin is the distance between the decision boundary and the support vectors. Maximizing the margin helps to improve the generalization ability of the model [34] [35].

Soft Margin Classification: In cases where the data is not linearly separable, SVMs employ a soft margin approach. Soft margin classification allows for some misclassifications by introducing a penalty parameter (C) that controls the trade-off between maximizing the margin and minimizing the classification errors. Higher values of C lead to a smaller margin but fewer misclassifications, while lower values of C prioritize a larger margin with more misclassifications [34] [35].

Optimization: The process of finding the optimal hyperplane is formulated as an optimization problem. The goal is to minimize the classification error while maximizing the margin. This is typically achieved using quadratic programming techniques or optimization algorithms that solve the constrained optimization problem [34] [35].

4.4.KNN:

K-Nearest Neighbors (KNN) is a simple and intuitive classification algorithm that can be used for both regression and classification tasks. It operates based on the principle of similarity, where an unlabeled instance is assigned a label based on the labels of its k nearest neighbors in the training data [36].

Here is an explanation of the KNN algorithm:

Training Phase:

- The training phase of KNN involves storing the feature vectors and corresponding labels of the training instances.
- The feature vectors represent the attributes or characteristics of each instance, while the labels indicate their class or category.
- The training data serves as the reference points for classification in the prediction phase.

Prediction Phase:

- When a new unlabeled instance is presented for classification, KNN identifies its k nearest neighbors in the training data based on some similarity metric (usually Euclidean distance or cosine similarity).
- The value of k is a parameter set by the user and determines the number of neighbors to consider.
- Once the nearest neighbors are identified, the majority class among them is assigned to the unlabeled instance as its predicted class.

5. Dataset used:

5.1.FER dataset:

The FER (Facial Expression Recognition) dataset is a commonly used dataset in the field of facial expression recognition. It consists of a collection of facial images labeled with corresponding emotion labels. The dataset is used for training and evaluating machine learning algorithms and models for facial expression recognition tasks [15].

The source of the FER dataset can vary depending on the specific version or variant of the dataset being referred to. However, one popular version of the FER dataset, known as the "FER2013" dataset, was introduced in the paper "Emotion Recognition in the Wild" by Goodfellow et al. (2013). This dataset was created by collecting facial images from the internet and crowdsourcing the annotation of emotion labels for each image.

The FER2013 dataset contains approximately 35,887 grayscale images of size 48x48 pixels. Each image is labeled with one of seven emotion categories: angry, disgust, fear, happy, sad, surprise, or neutral. The dataset is commonly split into training and testing subsets, with a standard split ratio used in research publications.

The FER dataset, including the FER2013 version, has been widely used in research and serves as a benchmark for evaluating the performance of facial expression recognition algorithms. It

provides a diverse range of facial expressions and is a valuable resource for developing and testing models in this field.



Figure 7: FER dataset [16]

Conclusion:

In this chapter, we have covered different elements of facial expression recognition. We delved into the training approach and methods for preprocessing. We examined techniques for extracting features, and we introduced the architecture of the machine learning system we propose. To train and test our system, we utilized a specific dataset. Moving forward to the next chapter, we will showcase the experimental results, which will demonstrate how well our proposed methods perform in accurately recognizing facial expressions.

Chapter 4: The experimental results

1. Introduction

In the preceding chapter, we outlined the overall architecture of the system and provided a comprehensive breakdown of each stage and step involved in the process. In this chapter, we focus on discussing the tools utilized in the development of our work, alongside showcasing the empirical results that have been achieved.

2. Development Tools

2.1.Hardware Tools

This program was created in a type of DELL laptop with the following specifications:

• Processor: INTEL I3 6300U/2.50 GHz

• Installed memory (RAM): 4.0GB.

• System type: 64-bit operating system.

• Operating system: Windows 10.

2.2.Software Tools

2.2.1. Anaconda Distribution

The Anaconda Distribution, an open-source platform, provides the most straightforward approach for conducting Python-based data science and machine learning tasks on Linux, Windows, and Mac OS X. Trusted by more than 11 million users globally, it has become the industry norm for development, testing, and training processes on a single machine [37].

2.2.2. Spyder IDE

Spyder is a Python-based scientific environment that is freely available and open source. It has been specifically developed by and for scientists, engineers, and data analysts. This software offers a distinctive blend of powerful features, including advanced editing, analysis, debugging, and profiling tools typically found in comprehensive development environments. Additionally, it provides data exploration, interactive execution, deep inspection, and visually appealing visualization capabilities commonly associated with scientific packages [38].

2.2.3. Google colab

Colaboratory, or "Colab" for convenience, is a product developed by Google Research. It offers users the ability to write and execute Python code directly from their web browser. It serves as an excellent platform for individuals interested in machine learning, data analysis, and educational purposes. Colab functions as a hosted Jupyter notebook service, eliminating the need for any setup or configuration. Furthermore, it provides users with free access to computational resources, including GPUs, enhancing the overall computing experience [39].

2.2.4. Python language

The utilized version is 3.9. Python is a programming language known for its simplicity and power, making it easy to learn and use. It offers efficient high-level data structures and employs a straightforward yet effective approach to object-oriented programming. Python's elegant syntax, dynamic typing, and interpreted nature make it a versatile language suitable for scripting and rapid application development across various domains and platforms [40].

3. Data split:

In this research, we split the data into two portions. The first part consisted of the original FER dataset, which we used to train our system. The second part also comprised the original FER dataset, but we applied the Gabor filter to it. We adjusted the filter settings multiple times, resulting in a new dataset each time. This particular dataset was utilized for our experimental

study, aiming to enhance the performance and accuracy achieved during the initial training of our system with the original dataset.

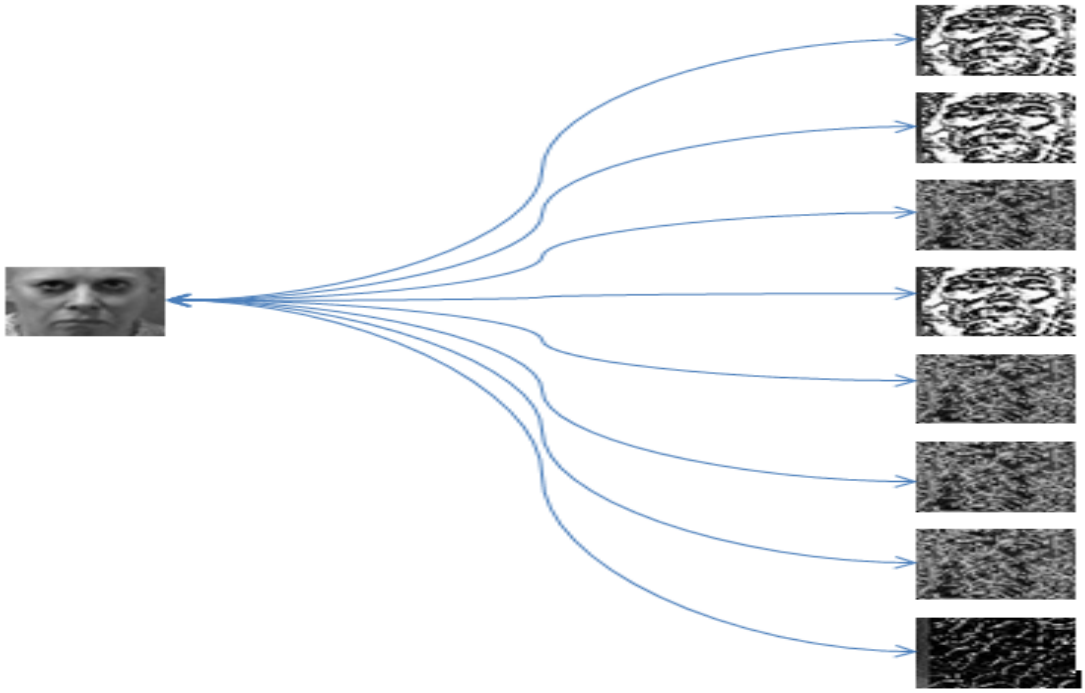


Figure 8: : Gabor filter

4. Experimentations and Results

In this section, we will systematically examine and discuss the various findings derived from our methodology, as presented in the previous chapter. Each finding will be presented in a step-by-step manner, accompanied by thorough explanations and justifications.

In our initial experiment, we trained and classified our facial expression recognition (FER) dataset without utilizing any specific features extracted from the images.

Our system was trained using the SVM classifier, and the corresponding outcomes are presented in the table below:

	SVM	
	accuracy	**Error Rate**
FER 2013 dataset	35.27%	64.73%

Table 3: SVM result

The KNN classifier was employed to explore the same concept, and the outcomes are depicted in the table provided:

	KNN	
	accuracy	**Error Rate**
FER 2013 dataset	40.28%	59.72%

Table 4: KNN result

The initial trial yielded unsatisfactory and significantly low outcomes. The accuracy rate did not surpass 35.27% when employing SVM, and similarly, it was 40.28% when utilizing KNN.

Subsequently, the HOG feature extraction technique was employed in conjunction with both the SVM and KNN classifiers. The obtained outcomes were as outlined below:

	HOG + SVM		HOG + KNN	
	Accuracy	**Error Rate**	**accuracy**	**Error Rate**
FER 2013 dataset	31.86%	68.14%	51.11%	48.89%

Table 5: HOG + (SVM/KNN) result

Regarding the second experiment involving the integration of the feature extractor HOG with the KNN and SVM classifiers, the outcomes were unsatisfactory for the SVM classifier. The accuracy of the SVM classifier dropped significantly to 31.86%. On the other hand, the KNN classifier exhibited improvement, with the accuracy increasing to 51.11%.

In light of the enhanced accuracy achieved by employing the HOG feature extractor, we opted to explore an alternative feature extractor, namely LBP, to further improve our results. The obtained outcomes were as outlined below:

	LBP + SVM		LBP + KNN	
	Accuracy	**Error Rate**	**accuracy**	**Error Rate**
FER 2013 dataset	-	-	25.34%	74.66%

Table 6: KNN + (SVM/KNN) result

The LBP feature extractor's performance was underwhelming as it failed to produce any output for the SVM classifier, and the accuracy decreased when using KNN.

In the initial phase of our experiment, we observed a range of outcomes spanning from unfavorable to moderate. Moving on to the second phase, we employed a system built upon HOG (Histogram of Oriented Gradients) and KNN (K-Nearest Neighbors). This particular system was chosen based on its superior performance in previous experiments. However, in this current experiment, we incorporated datasets that had undergone Gabor filtering as a preprocessing step.Also, in this experiment, we conducted our study on datasets that were subjected to Gabor filtering as a pre-processing step.

	HOG + KNN	
	accuracy	**Error Rate**
FER with GABOR (GDP2)	80.56%	19.40%
FER with GABOR (LDIP)	76.88%	23.11%
FER with GABOR (LDTP15)	79.63%	20.36%
FER with GABOR (LGIP)	79.60%	20.39%
FER with GABOR (LGtrP)	80.25%	19.74%
FER with GABOR (LMP)	80.19%	19.80%

Table 7: Gabor filter with (HOG+KNN) result

The provided table presents the accuracy and error percentage for different datasets that were trained using our system. These datasets were subjected to experiments, and the table showcases the results obtained from those experiments. The results are positive, indicating progress in the system's performance.

The accuracy values mentioned in the table range from 76% to 80%. This means that the system was able to classify the data correctly in the given datasets with a high degree of accuracy. The closer the accuracy values are to 100%, the better the system's performance.

Among the datasets, the highest accuracy achieved was 80.56%. This occurred when the system utilized the GDP2 Gabor filter, a specific technique for processing images or signals. The use of this filter resulted in the highest accuracy compared to other datasets.

Additionally, the error percentage is mentioned for each dataset in the table. The error percentage represents the proportion of incorrect classifications made by the system. In this case, the dataset with the GDP2 Gabor filter had a relatively low error rate of 19.40%. This means that only 19.40% of the classifications made by the system using this dataset were incorrect, which is comparatively low when compared to the other datasets.

Overall, the results from the experiments presented in the table indicate positive progress and improved performance of the system, with the dataset utilizing the GDP2 Gabor filter demonstrating the highest accuracy and a relatively low error rate.

We enhanced our findings by incorporating a confusion matrix, which serves as a concise depiction of the classification model's effectiveness. The confusion matrix is a tabular summary that outlines the model's predictions compared to the real labels or categories within the dataset.

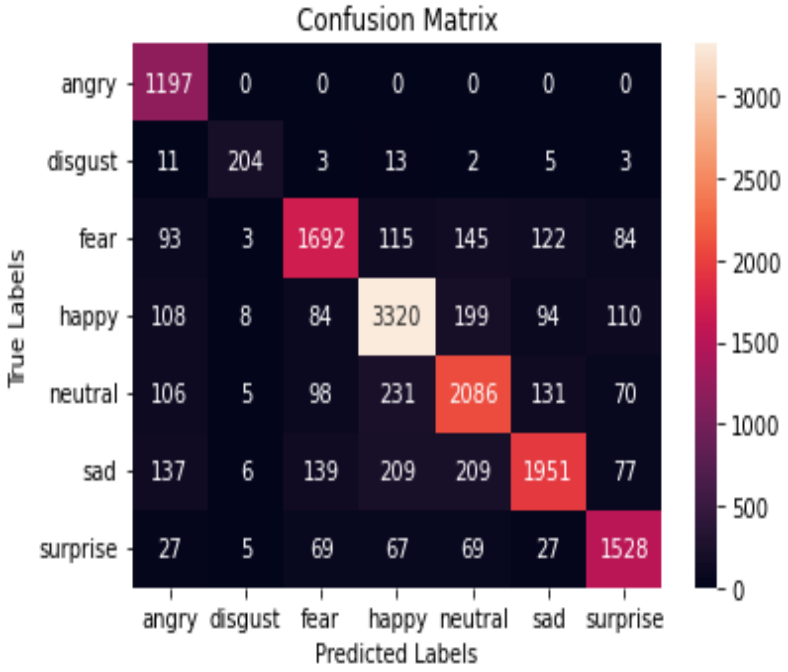


Figure 9:confusion matrix for GDP2 filtre

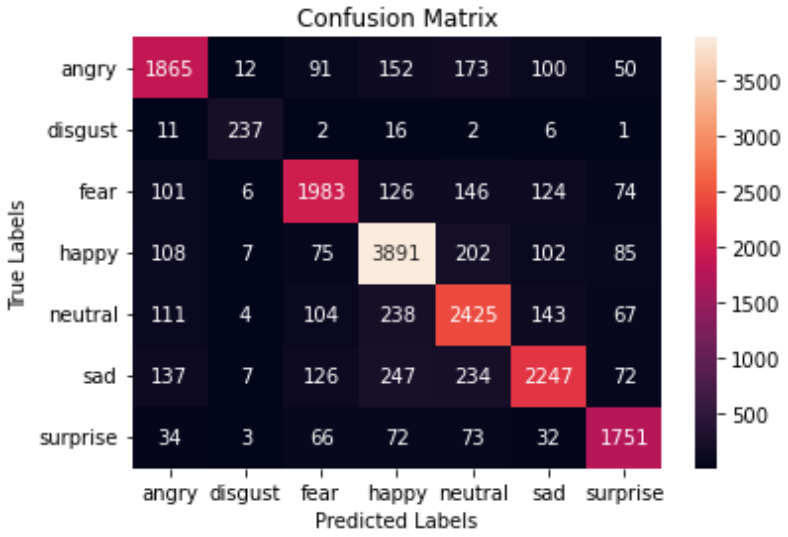


Figure 10: confusion matrix for LGtrP filtre

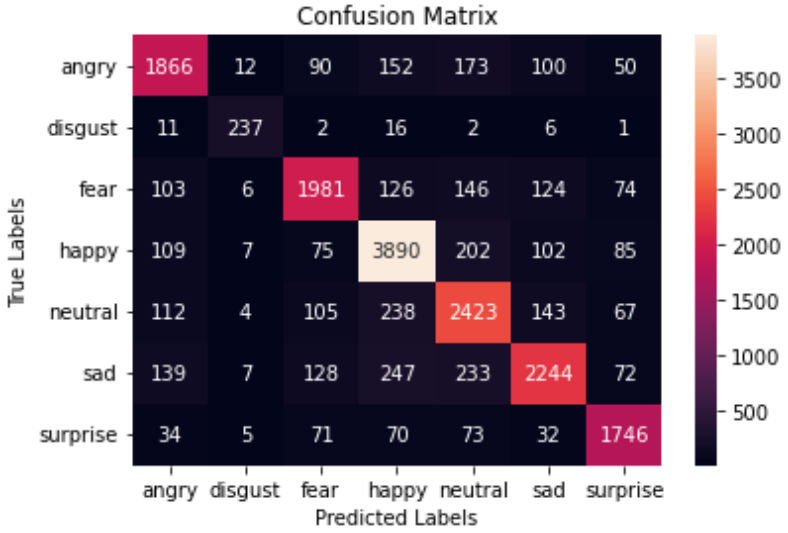


Figure 11: confusion matrix for LMP filtre

5. Additional tests:

In this section, we will present the experimental outcomes of all the conducted experiments in our research. In the previous section, we showcased the key findings we obtained, whereas here we will provide a comprehensive display of all the results obtained throughout this study. These results are derived from the data sets obtained through the application of the Gabor filter, as well as the systems that were not utilized in the latter part of our experiments. Thus, this section is dedicated to presenting all the experimental results obtained in this study.

		Accuracy	Error rate
FER 2013	SVM	35.27%	64.73%
FER 2013	KNN	40.28%	59.72%
FER 2013	SVM + HOG	31.86%	68.14%
FER 2013	KNN + HOG	**51.11%**	**48.89%**
FER 2013	SVM + LBP	-	-
FER 2013	KNN + LBP	25.34%	74.66%
FER with GABOR (GDP)	KNN + HOG	63.56%	36.43%
FER with GABOR (GDP2)	KNN + HOG	**80.59%**	**19.40%**
FER with GABOR (GDPN)	KNN + HOG	74.94%	25.05%
FER with GABOR (LDIP)	KNN + HOG	76.88%	23.11%
FER with GABOR (LDiPv)	KNN + HOG	64.11%	35.88%
FER with GABOR (LDTP9)	KNN + HOG	79.57%	20.42%
FER with GABOR (LDTP11)	KNN + HOG	79.60%	20.39%
FER with GABOR (LDTP13)	KNN + HOG	77.47%	22.52%

FER with GABOR (LDTP15)	KNN + HOG	79.63%	20.36%
FER with GABOR (LDTP17)	KNN + HOG	79.58%	20.41%
FER with GABOR (LGIP)	KNN + HOG	79.60%	20.39
FER with GABOR (LGP)	KNN + HOG	79.56%	20.43%
FER with GABOR (LGtrP)	KNN + HOG	**80.25%**	**19.74%**
FER with GABOR (LMP)	KNN + HOG	**80.19%**	**19.80%**

Table 8: Additional tests

The table provided includes all the experiments conducted, including both successful and unsuccessful attempts. Initially, we conducted experiments using only the two classifiers. However, the results were not satisfactory. To enhance our approach, we incorporated feature extractors from the FER dataset. Unfortunately, these experiments also yielded disappointing outcomes.

Subsequently, we employed the Gabor filter on the dataset along with the KNN classifier and the HOG feature extractor. These experiments showed significant improvement, with some results achieving an accuracy rate of over 80% and an error rate below 20%.

The application of the Gabor filter to the dataset proved to be effective in enhancing the accuracy of our system. It resulted in a notable improvement of 30%, suggesting that the use of the Gabor filter set contributed significantly to the accuracy enhancement.

Conclusion:

In conclusion, this chapter presents the outcomes of the facial expression recognition system discussed earlier. It offers a thorough evaluation of the achieved results and the techniques employed to enhance prediction accuracy. The successful outcomes observed in certain experiments demonstrate the efficacy of the applied filters, fulfilling the primary objective of this thesis. Additionally, comparing various trials and approaches has provided a deeper understanding of their individual advantages and drawbacks. Altogether, this chapter has provided valuable insights into the performance and limitations of the facial expression recognition system.

General Conclusion

Facial expression recognition is a wide-ranging and significant field that has seen notable contributions. This thesis focuses on developing a machine learning-based method for offline facial expression recognition using Texture features. The research delves into the domains of face recognition and machine learning, discussing common feature extraction methods, classifiers, and datasets utilized in the field. Various pre-processing techniques are also explored.

To provide an encompassing overview of the field as of 2023, an extensive review of recent studies was conducted. These studies were analyzed for their datasets, methodologies, and outcomes, and a summary of relevant papers was compiled in a table.

The research encompasses multiple aspects of facial expression recognition, including training methods, pre-processing techniques, and the proposal of a custom machine learning architecture tailored to this task. Relevant datasets were employed in the study, which were divided into two parts. The first part consisted of the original FER dataset used for training the system. The second part involved applying a Gabor filter to the original dataset, modifying the filter settings multiple times to create new datasets. This experimental study aimed to enhance the performance and accuracy achieved during the initial training.

The facial expression recognition (FER) dataset was trained and classified without using any specific features extracted from the images. SVM and KNN were employed as classifiers, and the results were compared. The extracted features were subsequently incorporated with the classifiers, and the outcomes were documented in tables. The findings demonstrated that using KNN and HOG (Histogram of Oriented Gradients) was effective and yielded satisfactory results.

In the second stage, a system based on HOG and KNN was utilized, given its superior performance in previous trials. The datasets used in this experiment underwent Gabor filtering as a pre-processing step. The trials showed significant improvement, with some achieving an accuracy rate of over 80% and an error rate of less than 20%.

The application of a Gabor filter to the dataset proved effective in enhancing the system's accuracy. It resulted in a notable 30% improvement, indicating that the use of the Gabor filter significantly enhanced accuracy.

In conclusion, a table was created to summarize all the experiments conducted in the dissertation and their respective results. Additionally, a comparison of different experiments and approaches was presented to provide a comprehensive understanding of their individual advantages and disadvantages.

Given the broad scope of the field, this work serves as a proposal based on the findings presented in the thesis. To fully benefit from this research, it is recommended to employ a system with a powerful feature extractor and classifier.

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